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Topological aspect of disclinations in two-dimensional crystals*

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By using topological current theory, this paper studies the inner topological structure of disclinations during the melting of two-dimensional systems. From two-dimensional elasticity theory, it finds that there are topological currents for topological defects in homogeneous equation. The evolution of disclinations is studied, and the branch conditions for generating, annihilating, crossing, splitting and merging of disclinations are given.

Keywords: topological current theory, disclinations, two-dimensional crystals
PACC: 1110, 6170E, 6470D

1. Introduction

Topological defects, which are a necessary consequence of broken continuous symmetry, play an important role in two-dimensional (2D) phase transition. In 1973, Kosterlitz and Thouless (KT) constructed a detailed and complete theory of superfluidity on two-dimensions.[1] This theory indicates that the unbinding of vortices pair will lead to a second-order phase transition in superfluid films. Later, a microscopic scenario of 2D melting has been posited in the form of the Kosterlitz–Thouless–Halperin–Nelson–Young (KTHNY) theory.[2−4] The KTHNY theory predicts a new phase, the so-called hexatic phase, which exists between the solid and liquid phases in 2D melting.[4,5]

In 2D colloidal systems, topological defects have been studied in experiments and computer simulations. A serial experiments were performed to calculate dislocations and disclinations dynamic of 2D colloidal systems, and dissociation of dislocations and disclinations were observed.[6−11] During the years, a large number of computer simulations indicated that there exists a two-stage melting as prescribed by KTHNY theory, however, results are still controversial.[12−15] Although the KTHNY theory is currently preferred, a different theoretical approach, evokes grain-boundary-induced melting, which was a first-order transition suggested by Chui.[16] One may note that the condensation of geometrical defects is also a first-order transition.[17,18] Our previous work found that there exists a hexatic–isotropic liquid phase coexistence during the melting of soft Yukawa systems.[19,20] By Voronoi polygons analysis, the behaviour of point defects in the coexistence is very complicated. The evolution of topological defects during the melting of 2D system is still an open question.

Recently, a topological field theory for topological defects is developed by Duan *et al.*[21−24] By using Duan's topological current theory, the inner topological structure and bifurcation of topological defects, such as disclination and dislocation in liquid crystal and solid, were studied. In KT phase transition, there also exists an elementary vortex topological current constructed by the superfluid order parameter.[25] By using the topological current theory, we can give the branch conditions for generating, annihilating, crossing, splitting and merging topological defects.

In this paper, we will discuss the topological quantization and bifurcation of topological defects in 2D crystals. This work is based on the so-called Duan's topological current theory. The organization of this paper is as follows. We describe the elasticity theory in Section 2. Using Duan's topological current theory, we discussed the topological structure of disclination in 2D crystals in Section 3. The evolution of disclinations discussed in Section 4. In the last section, we summarize our results.

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2. Two-dimensional elasticity theory

In continuum elasticity theory, the elastic Hamiltonian in 2D triangular solid is given by[26]

$$F = \frac{1}{2}\int \mathrm{d}^2 r(2\mu u_{ij}^2 + \lambda u_{kk}^2), \tag{1}$$

where λ and μ are the 2D Lamé coefficients. The strain tensor is

$$u_{ij}(r) = \frac{1}{2}\left[\frac{\partial u_i(r)}{\partial r_j} + \frac{\partial u_j(r)}{\partial r_i}\right]. \tag{2}$$

A deformation is represented by a displacement vector field $\boldsymbol{u}(r) = (u_1, u_2)$, which maps the point $\boldsymbol{r} = (x, y)$ to $\boldsymbol{r} + \boldsymbol{u}$. If there are no defects, the deformation is a single-valued mapping of the plane onto itself. But $\boldsymbol{u}$ becomes a multi-valued function when there is a dislocation. A single dislocation corresponds to an extra half lattice plane, which is characterized by a Burger's vector $\boldsymbol{b}$. Another type of defect in 2D crystal is disclination, which is defined in terms of the bond angle field θ. $\theta(x)$ is the angle between local lattice bonds and a reference axis.

If we minimize F_s with respect to variations in $\boldsymbol{u}$, we obtain the equation

$$\partial_i \sigma_{ij} = 0, \tag{3}$$

where the stress tensor σ_{ij} is defined by

$$\sigma_{ij} = 2\mu u_{ij} + \lambda u_{kk}\delta_{ij}. \tag{4}$$

Because σ_{ij} is symmetric, it can be written as

$$\sigma_{ij} = \epsilon_{ik}\epsilon_{jl}\nabla_k\nabla_l\chi, \tag{5}$$

The function χ is called the Airy stress function. Although any choice for χ yields a stress tensor which satisfies Eq.(3), the choice cannot be arbitrary. The strain u_{ij} is related to the stress as

$$\begin{aligned} u_{ij} &= \frac{1}{2\mu}\sigma_{ij} - \frac{\lambda}{4(\lambda+\mu)}\delta_{ij}\sigma_{kk} \\ &= \frac{1+\sigma_0}{Y}\epsilon_{ik}\epsilon_{kl}\nabla_k\nabla_l\chi - \frac{\sigma_0}{Y}\nabla^2\chi\delta_{ij}, \end{aligned} \tag{6}$$

where $Y = 4\mu(\mu+\lambda)/(2\mu+\lambda)$ is the 2D Young's modulus and $\sigma_0 = \lambda/(2\mu+\lambda)$ the 2D Poisson ratio. Applying $\epsilon_{ik}\epsilon_{jl}\nabla_k\nabla_l$ to both of this equation, we find

$$\begin{aligned} \frac{1}{Y}\nabla^4\chi &= \frac{1}{2}\epsilon_{ik}\epsilon_{jl}\nabla_k\nabla_l(\partial_i u_j + \partial_j u_i) \\ &= \frac{1}{2}\epsilon_{ik}\epsilon_{jl}\nabla_k\nabla_l(\partial_i u_j - \partial_j u_i) \\ &\quad + \frac{1}{2}\epsilon_{ik}\epsilon_{jl}\nabla_k\nabla_l\partial_j u_i \\ &= \epsilon_{kl}\partial_k\partial_l\theta + \epsilon_{ik}\partial_k(\epsilon_{jl}\partial_l\partial_j u_i). \end{aligned} \tag{7}$$

The defects associated with the continuum elastic theory of a solid are dislocations and disclinations. Dislocations and disclinations can be introduced into the theory in a way similar to the discussion of superfluid vortices.[25] In the following we only consider the triangular lattice since it is the most densely packed one in two-dimension and favoured by nature.

3. Topological current in 2D crystal

The KTHNY theory predicts that the unbinding of topological defects breaks the symmetry in the two-stage transitions. The physical driving force behind the two-stage transitions is the dissociation of bound defect pairs, specifically pairs of dislocation (solid→hexatic) and pairs of disclinations (hexatic→liquid). Two-dimensional crystals are characterized by two different order parameters, namely, the orientational and translational order, corresponding to the two types of topological defects. Dissociation of the dislocation pairs causes the translational symmetry to be broken, while at the same time dissociation of free dislocations causing the orientational symmetry to be broken.

Disclinations, which are characterized by a topological charge, have a much higher energy than dislocations (see Fig.1).

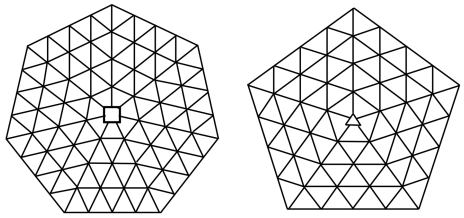


Fig.1. Disclination in triangular lattices. A $-\pi/3$ disclination with its sevenfold coordinated site in the centre (left). A $\pi/3$ disclination with its fivefold coordinated site (right).

They are defined in terms of the bond angle field $\theta(r)$, which measures the bonds orientational order. It is convenient to define an order parameter for bond orientational order, which is $\psi(r) = \psi_0 \mathrm{e}^{\mathrm{i}6\theta(r)}$ for the triangular lattices. However, the bond angle field is undefined at the disclination cores, i.e. the zero points of the order parameter field. We rewrite the orientation order parameter into $\boldsymbol{\psi}(r) = \phi_6^1 + \mathrm{i}\phi_6^2$ instead of

$\boldsymbol{\psi}(r) = \psi_0 \mathrm{e}^{\mathrm{i}6\theta(r)}$. Let us define the unit vector field $\boldsymbol{n}$ as

$$n^a = \frac{\phi_6^a}{||\phi_6||}, \quad ||\phi_6|| = \sqrt{\phi_6^a \phi_6^a}, \ a = 1, 2. \tag{8}$$

Obviously, $n^a n^a = 1$. The topological defect is related to the zero points of the two-component vector parameter $\boldsymbol{\Psi}$, i.e.,

$$\phi_6^1(x, y) = 0, \ \phi_6^2(x, y) = 0. \tag{9}$$

Suppose there is a defect located at z_i, the topological charge of the defect is defined by the Gauss map n: $\partial \sum_i \to S^1$,

$$W(\phi_6, z_i) = \frac{1}{2\pi} \oint_{\partial \sum_i} \epsilon_{ab} n^a \mathrm{d}n^b. \tag{10}$$

Using the stokes' theorem in the exterior differential form, one can deduce

$$W(\phi_6, z_i) = \frac{1}{2\pi} \oint_{\sum_i} \epsilon_{ab} \epsilon^{ij} \partial_i n^a \partial_j n^b \mathrm{d}^2 x. \tag{11}$$

We can construct a topological current of disclinations in 2D crystal,

$$\begin{aligned} j^k_{\mathrm{disc}} &= \frac{1}{6} \epsilon^{ijk} \epsilon_{ab} \partial_i n_6^a \partial_j n_6^b \\ &= \frac{2\pi}{6} \delta^2(\boldsymbol{\phi}_6) J^k \left(\frac{\phi_6}{x} \right). \end{aligned} \tag{12}$$

It is the ϕ-mapping current for disclinations. Where $J^k(\phi_6/x)$ is the vector Jacobians of $\boldsymbol{\phi}_6$,

$$J^k \left(\frac{\phi_6}{x} \right) = \frac{1}{2} \epsilon^{ijk} \epsilon_{ab} \partial_i \phi_6^a \partial_j \phi_6^b, \tag{13}$$

It is easy to see that this topological current is identically conserved, i.e.,

$$\partial_k j^k = 0. \tag{14}$$

According to the implicit function theorem, if Jacobian determinant

$$J^0 \left(\frac{\phi_6}{x} \right) = J \left(\frac{\phi_6}{x} \right) \neq 0, \tag{15}$$

the solutions of Eq.(9) can be generally expressed as

$$x = x_l(t), \quad y = y_l(t), \quad l = 1, 2, \ldots, N, \tag{16}$$

which represent N zero points $z_l(t)(l = 1, 2, \ldots, N)$ or world line of N disclinations D_l in space–time.

With the δ-function theory, $\delta^2(\phi)$ can be expanded as

$$\delta^2(\phi_6) = \sum_{l=1}^{N} \frac{\beta_l}{|J(\phi_6/x)_{z_l}|} \delta^2(\boldsymbol{r} - \boldsymbol{z}_l(t)), \tag{17}$$

where the positive integer β_l is called the Hopf index of map $x \to \boldsymbol{\phi}$. The meaning of β_l is that when the point $\boldsymbol{r}$ covers the neighbourhood of the zero $\boldsymbol{z}_l$ once, the vector field $\boldsymbol{\phi}_6$ covers the corresponding region for β_l times. Using the implicit function theorem and the definition of vector Jacobians (13), we can obtain the velocity of the lth defect,

$$\begin{aligned} \boldsymbol{v}_l &= \frac{\mathrm{d}\boldsymbol{z}_l}{\mathrm{d}t} = \left[\frac{\boldsymbol{J}(\phi_6/x)}{J(\phi_6/x)} \right]_{\boldsymbol{z}_l}, \\ \boldsymbol{J} \left(\frac{\phi_6}{x} \right) &= \left[J^1(\phi_6/x), J^2(\phi_6/x) \right]. \end{aligned} \tag{18}$$

Then the spatial and temporal components of the defect current j^u can be written as the form of the current and the density of the system of N classical point particles with topological charge $W_l = \beta_l \eta_l$ moving in the (2+1)-dimensional space–time,

$$\boldsymbol{j} = \sum_{l=1}^{N} \beta_l \eta_l \boldsymbol{v}_l \delta^2(\boldsymbol{r} - \boldsymbol{z}_l(t)), \tag{19}$$

$$\rho = \sum_{l=1}^{N} \beta_l \eta_l \delta^2(\boldsymbol{r} - \boldsymbol{z}_l(t)), \tag{20}$$

where η_l is Brouwer degree,

$$\eta_l = \frac{J(\phi_6/x)}{|J(\phi_6/x)|} \bigg|_{\boldsymbol{z}_l} = \pm 1. \tag{21}$$

For disclinations, using Duan's topological current theory we can write the homogeneous equation as

$$\frac{1}{Y} \nabla^4 \chi = \frac{2\pi}{6} \delta^2(\boldsymbol{\phi}_6) J \left(\frac{\phi_6}{x} \right). \tag{22}$$

Similar results are got by Nelson in the KTHNY theory.[27] In our theory, the topological charge of a disclination D_l is

$$Q_l = \frac{1}{6} \oint_{\Sigma_i} \epsilon_{ab} \epsilon^{ij} \partial_i n_6^a \partial_j n_6^b \mathrm{d}^2 x = \frac{2\pi}{6} W_l = \beta_l \eta_l, \tag{23}$$

where W_l is the winding number of $\boldsymbol{\Psi}$ around D_l, the above expression reveals distinctly that the topological charge of disclination is not only the winding number, but also expressed by the Hopf indices and Brouwer degrees. The topological inner structure showed in Eq.(23) is more essential than that in Eq.(7), this is the advantage of our topological description of the disclination.

It is clearly seen that Eq.(19) shows the movement of 2D crystal topological defects in space–time.

According to Eq.(14), the topological charge of defects in 2D crystal are conserved,

$$\frac{\partial \rho}{\partial t} + \nabla \boldsymbol{j} = 0. \tag{24}$$

In addition, there is a constraint of 'charge neutrality',

$$\int \rho \mathrm{d}^2 x = \frac{1}{2\pi} \sum_{l=1}^{N} \beta_l \eta_l = 0. \tag{25}$$

It indicates that the defect in 2D crystal appears in pair.

4. The generation and annihilation of disclinations

Analogy to vortices in superfluid films, the zero point of the order parameter field plays an important role in describing the topological defects in 2D crystal. Now we study the properties of these zero points. If the Jacobian determinant $J^0(\phi_6/x) \neq 0$, we will have the isolated solution of the zeros of the order parameter field. But when $J^0(\phi_6/x) = 0$, the above results will change in some way, and will lead to the branch process of defects. We denote one of the vectors Jacobian at zero points as $(t^*, \boldsymbol{z}_l)$. According to the values of the vector Jacobian at zero points of the order parameter, there are limit points and bifurcation points. Each kind corresponds to different cases of branch processes.

We explore what will happen to the disclination at the limit point $(t^*, \boldsymbol{z}_l)$. The limit points are determined by

$$J^0\left(\frac{\phi_6}{x}\right)\Big|_{t^*,\boldsymbol{z}_l} = 0, \quad J^1\left(\frac{\phi_6}{x}\right)\Big|_{t^*,\boldsymbol{z}_l} \neq 0, \tag{26}$$

$$J^0\left(\frac{\phi_6}{x}\right)\Big|_{t^*,\boldsymbol{z}_l} = 0, \ J^2\left(\frac{\phi_6}{x}\right)\Big|_{t^*,\boldsymbol{z}_l} \neq 0. \tag{27}$$

Considering the condition (26) and making use of the implicit function theorem, we find the solution of Eq.(9) in the neighbourhood of the point $(t^*, \boldsymbol{z}_l)$

$$t = t(x), \quad y = y(x), \tag{28}$$

where $t^* = t(z_l^1)$. In this case, one can see that

$$\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{(t^*,\boldsymbol{z}_l)} = \left.\frac{J^1(\phi_6/x)}{J(\phi_6/x)}\right|_{(t^*,\boldsymbol{z}_l)} = \infty, \tag{29}$$

or

$$\left.\frac{\mathrm{d}t}{\mathrm{d}x}\right|_{(t^*,\boldsymbol{z}_l)} = 0. \tag{30}$$

The Taylor expansion of $t = t(x)$ at the limit points $(t^*, \boldsymbol{z}_l)$ is

$$t - t^* = \frac{1}{2}\left.\frac{\mathrm{d}^2 t}{\mathrm{d}x^2}\right|_{t^*,\boldsymbol{z}_l} (x - z_l^1)^2, \tag{31}$$

which is a parabola in the $x - t$ plane. From this equation, we can obtain two solutions $x_1(t)$ and $x_2(t)$, which give two branch solutions (world lines of disclinations). If $\left.\frac{\mathrm{d}^2 t}{\mathrm{d}x^2}\right|_{(t^*,\boldsymbol{z}_l)} > 0$, then we have the branch solutions for $t > t^*$. It is related to the origin of a disclination pair. Otherwise, we have the branch solutions for $t < t^*$, which are related to the annihilation of a disclination pair (see Fig.2).

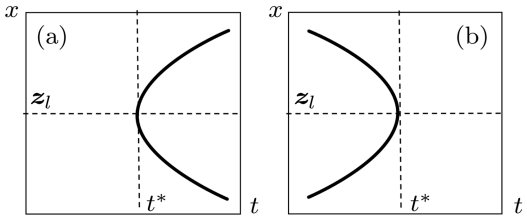


Fig.2. Generating and annihilating of disclination pairs. (a) The origin of two disclinations. (b) Two disclinations annihilate in collision at the limit point.

Since the topological current is identically conserved, the topological charges of these two generated or annihilated disclinations must be opposite at the limit points, say

$$\beta_1 \eta_1 + \beta_2 \eta_2 = 0. \tag{32}$$

For a limit point it is required that $J^1(\phi/x)|_{t^*,\boldsymbol{z}_l} \neq 0$. As to bifurcation point, it must satisfy a more complex condition. This case will be discussed in the following.

5. Encountering, splitting and merging of disclinations

Now, let us turn to consider the restrictions on zero point $(t^*, \boldsymbol{z}_l)$,

$$J^k\left(\frac{\phi_6}{x}\right)\Big|_{(t^*,\boldsymbol{z}_l)} = 0, \quad k = 0, 1, 2, \tag{33}$$

which imply an important fact that the function relationship between t and x or y is not unique in the neighbourhood of the bifurcation point $(t^*, \boldsymbol{z}_l)$. This fact is easily seen from

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{J^1(\phi_6/x)}{J(\phi_6/x)}\Big|_{t^*,\boldsymbol{z}_l}, \quad \frac{\mathrm{d}y}{\mathrm{d}t} = \frac{J^2(\phi_6/x)}{J(\phi_6/x)}\Big|_{t^*,\boldsymbol{z}_l}, \tag{34}$$

which under Eq.(33) directly shows the indefiniteness of the direction of integral curve of Eq.(34) at $(t^*, \boldsymbol{z}_l)$. This is why the very point $(t^*, \boldsymbol{z}_l)$ is called a bifurcation point of the orientation order parameter.

With the aim of finding the different directions of all branch curves at the bifurcation point, we suppose

$$\left.\frac{\partial \phi_6^1}{\partial y}\right|_{t^*, \boldsymbol{z}_l} \neq 0. \tag{35}$$

According to the ϕ-mapping theory, the Taylor expansion of the solution of the zeros of the order parameter field in the neighbourhood of $(t^*, \boldsymbol{z}_l)$ can be expressed as

$$\begin{aligned} &A(x - z_l^1)^2 + 2B(x - z_l^1)(t - t^*) \\ &+C(t - t^*)^2 + \ldots = 0, \end{aligned} \tag{36}$$

which leads to

$$A\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + 2B\frac{\mathrm{d}x}{\mathrm{d}t} + C = 0, \tag{37}$$

and

$$C\left(\frac{\mathrm{d}t}{\mathrm{d}x}\right)^2 + 2B\frac{\mathrm{d}t}{\mathrm{d}x} + A = 0, \tag{38}$$

where A, B, and C are constants determined by the order parameter. The solutions of Eq.(37) or Eq.(38) give different directions of the branch curves (world line of vortices) at the bifurcation point. There are four possible cases, which will show the physical meanings of the bifurcation points.

Case 1 ($A \neq 0$). For $\Delta = 4(B^2 - AC) > 0$, from Eq.(37) we obtain two different directions of motion of the core of disclination

$$\frac{\mathrm{d}x}{\mathrm{d}t}\Big|_{1,2} = \frac{-B \pm \sqrt{B^2 - AC}}{A}, \tag{39}$$

which is shown in Fig.3, where two world lines of two disclination intersect with different directions at the bifurcation point. This shows that two disclinations encounter and depart at the bifurcation point.

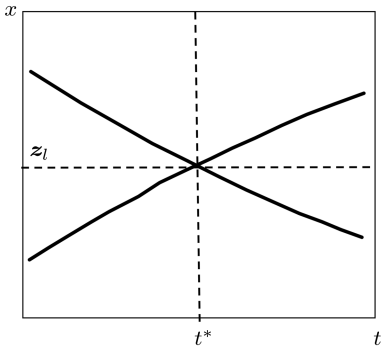


Fig.3. Two disclinations collide with different directions of motion at the bifurcation point in (2+1)-dimensional space–time. Two world lines intersect with different directions at the bifurcation point, i.e., two disclinations encounter at the bifurcation point.

Case 2 ($A \neq 0$). For $\Delta = 4(B^2 - AC) = 0$, form Eq.(37), we obtain only one direction of motion of the core of disclination

$$\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{1,2} = -\frac{B}{A}, \tag{40}$$

which includes three important cases (see Fig.4). (i) Two world lines tangentially contact, i.e., two disclinations tangentially encounter at the bifurcation point. (ii) Two world lines merge into one world line, i.e., two disclinations merge into one disclination at the bifurcation point. (iii) One world line resolves into two world lines, i.e., one disclination splits into two disclinations at the bifurcation point.

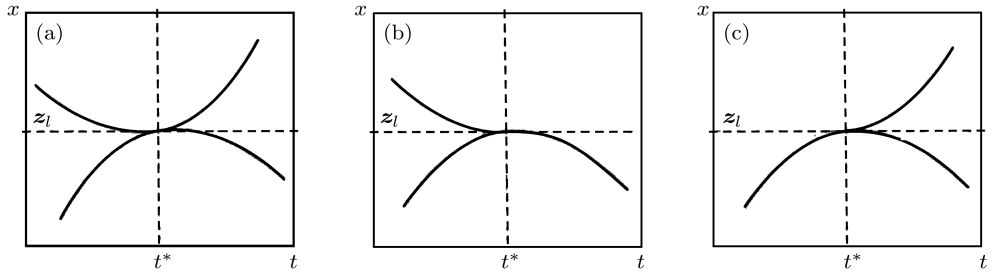


Fig.4. Disclinations have the same direction of motion. (a) Two world lines tangentially contact, i.e., two disclinations tangentially encounter at the bifurcation point. (b) Two world lines merge into one world line, i.e., two disclinations merge into one disclination at the bifurcation point. (c) One world line resolves into two world lines, i.e., one disclination splits into two disclinations at the bifurcation point.

Case 3 ($A = 0, C \neq 0$). For $\Delta = 4(B^2 - AC) > 0$ from Eq.(37) we have

$$\left.\frac{\mathrm{d}t}{\mathrm{d}x}\right|_{1,2} = \frac{-B \pm \sqrt{B^2 - AC}}{C} = 0, \quad -\frac{2B}{C}. \tag{41}$$

There are two important cases (see Fig.5). (i) One world line resolves into three world lines, i.e., one disclination splits into three disclinations at the bifurcation point. (ii) Three world line merge into one world line, i.e., three disclinations merge into one disclination at the bifurcation point.

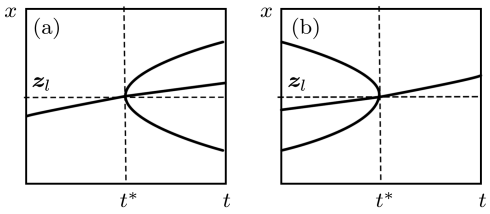


Fig.5. Two important cases of Eq.(41). (a) One world line resolves into three world lines, i.e., one disclination resolves into three disclinations at the bifurcation point. (b) Three world lines merge into one world line, i.e., three disclinations merge into one disclination at the bifurcation point.

Case 4 ($A = C = 0$). Equations (37) and (38) give respectively

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 0, \quad \frac{\mathrm{d}t}{\mathrm{d}x} = 0. \tag{42}$$

This case shows that two worldlines intersect normally at the bifurcation point, which is similar to case 3. It is not surprising that both parts of Eq.(42) are correct because they give the slope coefficients of two different curves at the same point $(t^*, \boldsymbol{z}_l)$.

Since the topological current of disclinations is identically conserved, the sum of the topological charge of final disclinations must be equal to the initial disclinations at the bifurcation point, i.e.

$$\sum_f \beta_{lf}\eta_{lf} = \sum_i \beta_{li}\eta_{li} \tag{43}$$

for fixed l. It indicates that, disclinations with a higher value of Burgers vector can evolve to the lower value of Burgers vector, or disclinations with a lower value of Burgers vector can evolve to the higher value of Burgers vector through the bifurcation process. Furthermore, we see that the generation, annihilation, and bifurcation of disclinations are not gradual changes, but start at a critical value of argument, i.e., a sudden change. We must point out that further bifurcations are possible during the evolution of disclinations besides cases studied here. We need to assume that the terms in the Taylor series considered above vanish and the expansion of the field is dominated by higher order terms in the expansion.

6. Conclusion

In this paper, we have studied the evolution of disclinations by making use of Duan's topological current theory and obtain the inner topological structure of disclinations in 2D systems. First, a more essential topological formulary of charge density of disclinations in 2D crystals has been studied. We revealed that the inner topological relationship of the charge of disclinations is characterized by the Hopf index and the Brouwer degree. The generation and annihilation of disclinations have been found at every zero point of the order parameter field under the condition of the Jacobian determinant $J(\phi/x) \neq 0$. Second, we concluded that there exist crucial cases of branch processes in the evolution of disclinations when $J(\phi/x) = 0$, i.e, η_l is indefinite. It means that disclinations are generated or annihilated at the limit points and encounter, split, or merge at the bifurcation points, which shows that the disclination is unstable at these branch points. We would like to point that all the results in this paper obtain from the viewpoint of topology without any hypothesis, and they are not depended on the property of systems, such as interaction between particles.

We mainly concern about the topologic aspect of disclinations in this paper. On the other hand, the topologic aspect of dislocations and how the topologic aspect of defects affected the nature of 2D melting also have been discussed in Ref.[28].

Acknowledgments

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