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Tolerance of edge cascades with coupled map lattices methods*

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This paper studies the cascading failure on random networks and scale-free networks by introducing the tolerance parameter of edge based on the coupled map lattices methods. The whole work focuses on investigating some indices including the number of failed edges, dynamic edge tolerance capacity and the perturbation of edge. In general, it assumes that the perturbation is attributed to the normal distribution in adopted simulations. By investigating the effectiveness of edge tolerance in scale-free and random networks, it finds that the larger tolerance parameter λ can more efficiently delay the cascading failure process for scale-free networks than random networks. These results indicate that the cascading failure process can be effectively controlled by increasing the tolerance parameter λ. Moreover, the simulations also show that, larger variance of perturbation can easily trigger the cascading failures than the smaller one. This study may be useful for evaluating efficiency of whole traffic systems, and for alleviating cascading failure in such systems.

Keywords: cascading failures, coupled map lattice methods, tolerance
PACC: 0590, 0200

1. Introduction

The study of cascading failure process in the framework of complex networks has attracted considerable attention as an interdisciplinary subject. Cascading failures[1−15] can occur in many physical systems, such as the traffic and transportation networks,[16−25] the internet[26,27] and power grids,[28] and so on. Generally, many researchers studied cascading failures in complex networks, triggered by attacks (e.g. removing some hub nodes or important edges). Removal of nodes or edges, in general, can cause redistribution of loads over the whole complex networks. Recently, there have been a number of studies on the dynamics of cascading failure process.[1−15] Motter and Lai[1] showed that for such networks where loads can be redistributed among the nodes, intentional attacks can cause the entire or a substantial part of the network to collapse. Watts[2] presented the explanation that cascading failure phenomenon is due to interacting with the actions of their neighbours according to a simple threshold rule in complex networks. Moreover, it showed that the heterogeneity structure of the network plays an important role in determining the system's stability. Heterogeneous thresholds and heterogeneous degree distribution have a close relationship with the system global cascades. Crucitti *et al*[3] presented a dynamical redistribution of flow model for cascading failures based on edge's efficiency dynamics. Buzna *et al*[4] studied the effectiveness of recovery strategies for the dynamic model of failure spreading in complex networks. These strategies control the distribution of resources based on information about current network state and network topology. Zheng and Gao *et al*[5,6] considered the congested effects on cascading failures. Furthermore, the efficient dynamics of cascades is also investigated in the complex networks. Wang and Xu[7,8] studied the cascading failures based on coupled map lattices (CMLs) methods in different topologies. They discovered that cascading failures are much easier to occur in small-world and scale-free coupled map lattices than in globally coupled map lattices. Cui and Gao *et al*[9,10] studied the cascading failure process on

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the community networks according to the degree information. Wang and Chen[14] proposed an edge-based cascading failure model with a local weighted flow redistribution rule on weighted complex networks. In this paper, we intend to build the connection between edge-based cascading failures and perturbation with CML methods on complex networks. In detail, we introduce the parameter of edge's capacity, which is assumed to be controlled by perturbation and a tolerance parameter.

This paper is organized as follows. In Section 2, the prototype of edge-based cascading failure is introduced in terms of the CML methods. In Section 3, the simulation results and the corresponding discussions are presented. A summary and conclusion is given in Section 4.

2. Edge-based cascade with CML methods

The CML methods[29] are very suitable for computational studies of complex networks. In CML methods, dynamical elements are situated at discrete points in space, time is discrete and state is considered as a continuous variable. Each spatial unit is coupled to its neighbours. The selection of neighbours is determined by topological structure of complex networks. In most studies, regular diffusive coupling rules (e.g. global interaction and nearest neighbour interaction) are considered, where the topological structure is usually assumed to be adopted. For simplicity, we apply the CML methods to describe dynamics of cascading failures by considering tolerance effect of edge breakdown in a macroscopically manner as follows:

$$\begin{aligned} x_{ij}(t+1) = \Big| & (1-\varepsilon_1-\varepsilon_2) f(x_{ij}(t)) \\ & + \varepsilon_1 \Big(\sum_{p_v \in \Gamma_i} a_{p_v i} \frac{f(x_{p_v i}(t))}{k_{p_v i}} \Big) \\ & + \varepsilon_2 \Big(\sum_{q_w \in \Gamma_j} b_{j q_w} \frac{f(x_{j q_w}(t))}{k_{j q_w}} \Big) \Big|, \end{aligned} \tag{1}$$

($i,\ j$=1, 2, …, n; p_v and q_w are nodes in the nearest neighbours sets Γ_i and Γ_j, respectively, see Fig.1).

In Eq.(1), $x_{ij}(t+1)$ is a state variable of edge ij at time step $t+1$. The adjacency matrices $A = \{a_{p_v i}\}$ and $B = \{b_{j q_w}\}$ represent right and left connection information of two endpoints for edge ij, respectively. If there exist the nearest right or left neighbouring edges, the $a_{p_v i}$ (or $b_{j q_w}$) is presented by number 1; otherwise, by number 0. Especially, there are no multiple edges between two nodes and no looped edges with identical ends. The $k_{p_v i}$ and $k_{j q_w}$ are defined as the degrees of neighbouring edges which connect left and right endpoints of edge ij, respectively. The $\varepsilon_1,\ \varepsilon_2 \in (0,1)$ represent different coupled strengths of the two endpoints for one edge. Here mapping function $f(x)$ defines local dynamics which is usually chosen to be a chaotic logistic map $f(x) = dx(1-x)$, $d \in (0,4]$.

To guarantee the nonnegative state of each node, we use the absolute value notation as follows:

$$\begin{aligned} x_{ij}(l+1) = \Big| & (1-\varepsilon_1-\varepsilon_2) f(x_{ij}(l)) \\ & + \varepsilon_1 \Big(\sum_{p_v \in \Gamma_i} a_{p_v i} \frac{f(x_{p_v i}(l))}{k_{p_v i}} \Big) \\ & + \varepsilon_2 \Big(\sum_{q_w \in \Gamma_j} b_{j q_w} \frac{f(x_{j q_w}(l))}{k_{j q_w}} \Big) \Big| + R_{ij}, \end{aligned} \tag{2}$$

where R_{ij} is the external perturbation which is added to edge ij. Different from previous study, the perturbation of each edge is assigned randomly with a normal distribution $R_{ij} \sim N(\mu, \sigma^2)$, where μ is the mean value of perturbation and σ^2 is variance of perturbation.

To introduce dynamic cascading failure process, we assign a capacity C_{ij} on the edge (i,j), which shows the maximum possible crossing flows on the edge (i,j). The edge ij is in a normal state at the lth time step, when the condition $0 < x_{ij}(t) < C_{ij}$, $t \le l$ is satisfied. Otherwise, if $0 < x_{ij}(t) < C_{ij}$, $t < l$ and $x_{ij}(l) \ge C_{ij}$, then edge ij will be failed at the lth time step. For such failed edge, we have $x_{ij}(t) \equiv 0$, for all $t > l$. The total number of failed edges can be denoted by the parameter $I(t)$ before the $(t+1)$th time step.

In many previous studies, the edge capacity is characterized by a capacity as the maximum load that the edge can handle.[3] And the capacity C_{ij} of edge ij is assumed to be proportional to its initial load. Generally, the load is evolving in a dynamical way. And the load dynamics can be considered as a Markov chain, which is independent on the initial condition. In this paper, we assume that the capacity C_{ij} of edge ij is proportional to its external perturbation. And it is described as follows:

$$C_{ij} = (1+\lambda) R_{ij}, \tag{3}$$

where λ is a tolerance parameter. The larger value of edge's capacity C_{ij} can transport the more flow of

edge ij than the smaller one. The tolerance parameter $\lambda(\lambda > 0)$ can dynamically control and efficiently adjust the edge's capacity.

3. The simulation

Due to the existence of a broad degree distribution, transportation on the network is different when the edge is between two hub nodes with a large number of connections or between low-degree nodes. We therefore characterize the transport co-efficiencies by the explicit dependence on two endpoints' degrees $k_{p_v i}$ and k_{jq_w} (see Eq.(1) and Fig.1). Such dependence is not significant in the homogeneous systems, but it may become very important in the networks where the node degree spans a wide range of values, such as in scale-free networks.

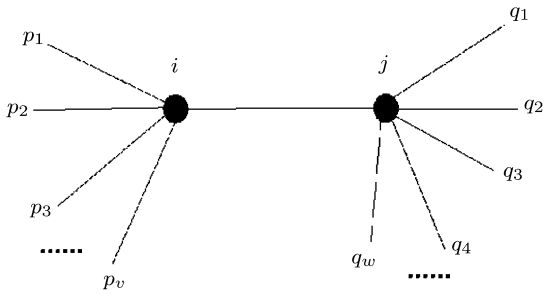


Fig.1. The schematic illustration of edge cascading failures process in terms of the CML methods. Edge ij is broken and the perturbation is redistributed to neighbouring edges which are connected by two endpoints of edge ij. The p_1, p_2, $p_3, ...$, p_v are left endpoints of edge ij. At the same time, the q_1, q_2, $q_3, ...$, q_w are right endpoints of edge ij. The edge degree ij of two endpoints are $k_{p_v i}$ and k_{jq_w}, respectively.

We first construct the complex networks in practice by considering two artificial network topologies: (I) Erdös–Rényi (ER) random graphs;[29] (II) scale-free networks, i.e., graphs with the degree distribution $P(k) \sim k^{-\gamma}$ and the degree exponent $\gamma = 3$ generated according to the Barabási–Albert (BA) model.[30] We illustrate the networks of the edge size $E = 5000$ with the average degree $< k >= 4$ both for the random networks and scale-free networks.

In Fig.2, we report the number of total failed edge I i.e., the total failed edge after the whole complex network has reached to a stationary state, as a function of the tolerance parameter λ for the different mean value of μ. To facilitate comparison, we study the tolerance effect of edge capacity on the networks in the case of fixed variance $\sigma^2 = 0.01$. And we show

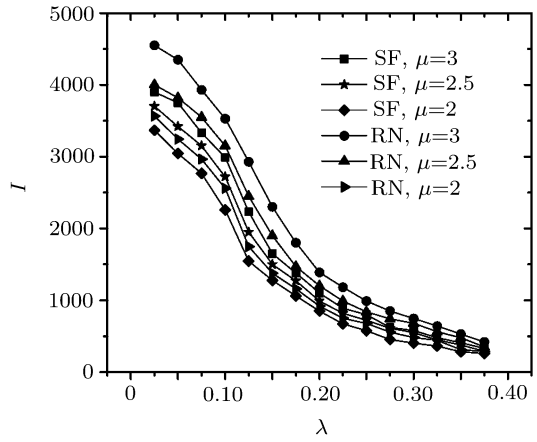


Fig.2. The plot of tolerance capacity parameter λ against the total failed edges number I for different perturbation expectation μ in random networks and scale-free networks. Cascading failure process with total edge number $E = 5000$, the coupled strength $\varepsilon_1 = \varepsilon_2 = 0.6$, triggered by adding the external perturbation with the normal distribution $R_{ij} \sim N(\mu, \sigma^2)$ and $\sigma^2 = 0.01$. Each curve corresponds to the average over 20 realizations of networks.

the results for three mean values of perturbation, namely, $\mu = 2$, 2.5, 3. According to the Fig.2, we can see that the number of failed edge I decreases with the increasing tolerance parameter λ. It is easy to be understood because the larger tolerance parameter λ can enlarge the edge capacity to burden the more flow. Moreover, the results also indicate that the larger mean value μ can fasten the cascading failure process. Figure 3 shows the number of final failed

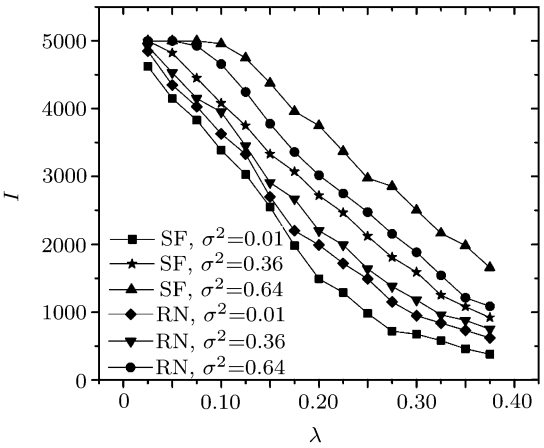


Fig.3. The plot of tolerance capacity parameter λ against the total failed edges number I for the different variance σ^2 in random networks and scale-free networks. Cascading failure process with total edge number $E = 5000$, the coupled strength $\varepsilon_1 = \varepsilon_2 = 0.6$, triggered by adding the external perturbation with the normal distribution $R_{ij} \sim N(\mu, \sigma^2)$ and $\mu = 3$. Each curve corresponds to the average over 20 realizations of networks.

edges I for different values of variance σ^2 and tolerance capacity parameter λ in scale-free and random networks. From Fig.3, it can be observed that the scale-free networks are more robust than the random networks for the smaller variance σ^2. At the same time, for the larger variance σ^2, the scale-free networks are more fragile than the random networks. It is owing to topological structure of scale-free structure, which exist the more hub-nodes with larger degrees. Figure 4 presents the time series of $I(t)$ with the same tolerance parameter $\lambda = 0.1$ and the fixed variance $\sigma^2 = 0.01$ in random networks and scale-free networks for the different mean value μ. It is clearly seen that the smaller mean value μ can availably delay the cascading failures process, which is consistent with Fig.2. For comparison, we plot the time series of failed edges I for different variance σ^2 by considering small mean value μ and the large mean value μ in Fig.5 and Fig.6, respectively. Interestingly, according to Fig.5, when the mean value μ is small, random networks appear to be more resistant to cascading failures than the scale-free networks for larger variance σ^2. Figure 6 has shown that, when the mean value μ is large, for the scale-free networks cascading failure process occurs more easily than the random networks for larger variance σ^2. As is illustrated, the simulation results of Fig.6 are well consistent with the results of Fig.3 by the CML methods.

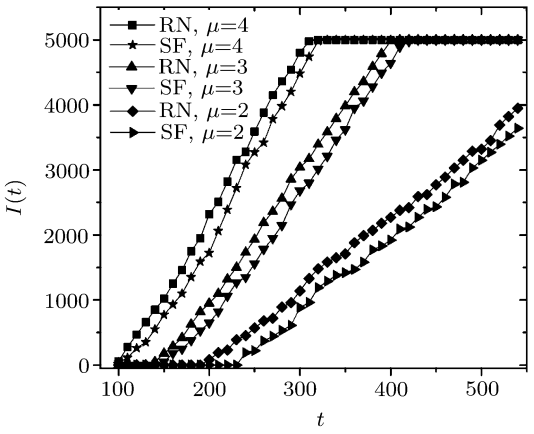


Fig.4. The plot of total failed edges number $I(t)$ versus the time step t with different mean value μ for random networks and scale-free networks. Each curve corresponds to the average over 20 realizations of networks with total edge number $E = 5000$, the tolerance parameter $\lambda = 0.1$, the variance $\sigma^2 = 0.01$ and the coupled strength $\varepsilon_1 = \varepsilon_2 = 0.6$.

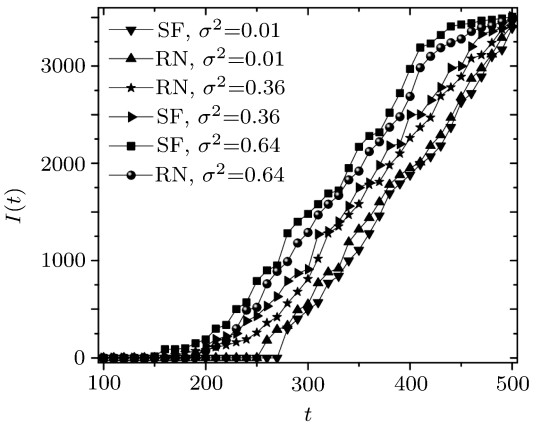


Fig.5. The plot of total failed edges number $I(t)$ versus the time step t with different variance σ^2 for random networks and scale-free networks. Each curve corresponds to the average over 20 realizations of networks with total edge number $E = 5000$, the tolerance parameter $\lambda = 0.1$, the small mean value $\mu = 2$ and the coupled strength $\varepsilon_1 = \varepsilon_2 = 0.6$.

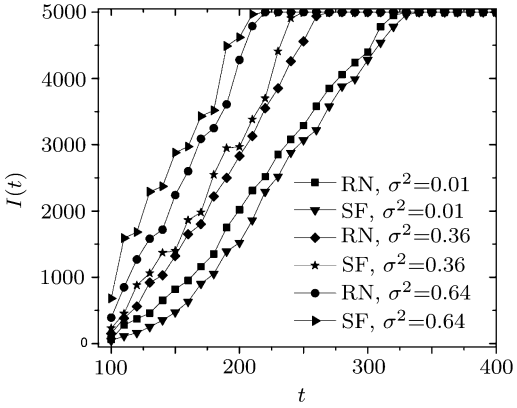


Fig.6. The plot of total failed edges number $I(t)$ versus the time step t with different variance σ^2 for random networks and scale-free networks. Each curve corresponds to the average over 20 realizations of networks with total edge number $E = 5000$, the tolerance parameter $\lambda = 0.1$, the large mean value $\mu = 4$ and the coupled strength $\varepsilon_1 = \varepsilon_2 = 0.6$.

In our simulations, we can see that the larger variance σ^2 can have a more obvious influence to the scale-free networks than the random networks for the cascading failures process. As a matter of fact, the small value of variance ($\sigma^2 = 0.01$) is similar to an uniform perturbation system, and the large value of variance ($\sigma^2 = 0.64$) corresponds to a system with different perturbation for each edge. Moreover, scale-free networks are greatly different to random networks in the aspect of topological structure. A scale-free network has heterogeneous topological structure in nature: most nodes have small degree but a small number of hub nodes have many connections. It is this heterogeneous feature that makes the connectivity of a scale-free network which can bear much more steady external perturbation but vulnerable to perturbation with large variance σ^2. More precisely, the connectivity of such networks is highly robust against steady uniform perturbation system (small variance σ^2).

4. Conclusion

Summing up, we investigate the tolerance effects of edge cascading failure in scale-free networks and random networks based on the CML methods. The results reveal that the tolerance parameter λ can greatly influence the cascading failures process. Furthermore, the small variance σ^2 can delay and reduce the cascading failure effects. We believe that the simple CML methods have illuminated the general work of cascades in dynamics, and have allowed us to understand the broad features of some universal cascading failure phenomena in these complex networks. Our results will motivate further work both in dynamics and interpretation of cascading failure process.

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