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Performance characteristics of an energy selective electron refrigerator with double resonances*

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This paper establishes the energy selective electron (ESE) engine with double resonances as a refrigerator in one dimensional (1D) system. It consists of two infinitely large electron reservoirs with different temperatures and chemical potentials, and they are perfectly thermally insulated from each other and interaction only via a double 'idealized energy filter' whose widths are all finite. Taking advantage of the density of state and Fermi distribution in the 1D system, the heat flux into each reservoir may then be calculated. Moreover, the coefficient of performance may be derived from the expressions for the heat flux into the hot and cold reservoirs. The performance characteristic curves are plotted by numerical analysis. The influences of the resonances widths, the energy position of resonance and the space of two resonances on performance of the ESE refrigerator are discussed. The results obtained here have theoretical significance for the understanding of thermodynamic performance of the micro–nano devices.

Keywords: electron transport, double resonances, ESE refrigerator, performance parameter
PACC: 0570L, 7210, 7980, 0720

1. Introduction

So-called Brownian electron refrigerator utilizes a temperature difference between two reservoirs of electrons to transport high-energy electrons against an electrochemical potential gradient.[1] This issue is currently attracting considerable interest,[2−15] including reversible quantum Brownian refrigerators and heat engines for electrons,[1] reversible thermoelectric nanomaterials,[16] thermionic refrigerators and power generators,[17−20] quantum tunnelling ratchets, quantum dot ratchets,[21−26]and so on. A number of very interesting results have been obtained. For example, two electron reservoirs with different temperatures and electrochemical potentials can exchange electrons reversibly at the energy where the Fermi–Dirac distributions in the two reservoirs have the same value. Using suitable energy filters, Brownian electron refrigerator can attain Carnot coefficient of performance (COP) value in principle.[1]

The energy selective electron (ESE) refrigerator and heat engine in 1D–0D–1D system was proposed by Humphrey *et al*[1] electron refrigerators and heat engines work by employing energy selectivity in electron transport direction. Specially, the ESE refrigerators and heat engines can be viewed as operating as one half-period of the rocked resonant tunnelling ratchet. The refrigeration load and COP of the ESE refrigerator can be calculated.

Now, double resonances idealized filter are implemented theoretically for the ESE refrigerator in the 1D–0D–1D system such that the quantum dot connected to the reservoirs via one-dimensional (1D) leads. In this paper, the refrigeration load and COP of the ESE refrigerator with double resonances are obtained by numerical calculation. The influences of resonances widths, the energy position of resonance and the energy space of two resonances on the performance of the ESE refrigerator are investigated. Importantly, the results obtained here are expected to have significant implications for design of practical multi-resonances electron refrigerator such as thermionic refrigerators and power generators.

2. The model of the ESE refrigerator with double resonances

The ESE refrigerator with double resonances consists of two infinitely large electron reservoirs, which

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have different temperatures and chemical potentials. Two different reservoirs are thermally insulated from each other, and interact only via two 'idealized energy filter', which freely transmits electrons in a specified energy range and blocks the transport of all others. Schematic diagram of the ESE refrigerator with double resonances is shown in Fig.1, where $T_{\rm C}$ and $T_{\rm H}$ are, respectively, the temperatures of cold and hot reservoirs, $\mu_{\rm C}$ and $\mu_{\rm H}$ are, respectively, the chemical potential of cold and hot reservoirs, E_1 and E_2 are, respectively, the energy positions of the first and second resonances, ΔE is energy space of the double resonances, eV_0 is a bias voltage that is applied to the right (hot) reservoir. Now, considering double resonances, we find that the energy level position of one resonance is at least lower than E_0 where two electron reservoirs with arbitrary temperatures and electrochemical potentials can exchange electrons reversibly. The ESE refrigerator can attain Carnot COP value in principle.[1] This special energy level position E_0 is equal to $\dfrac{T_{\rm H}\mu_{\rm C} - T_{\rm C}\mu_{\rm H}}{T_{\rm H} - T_{\rm C}}$.

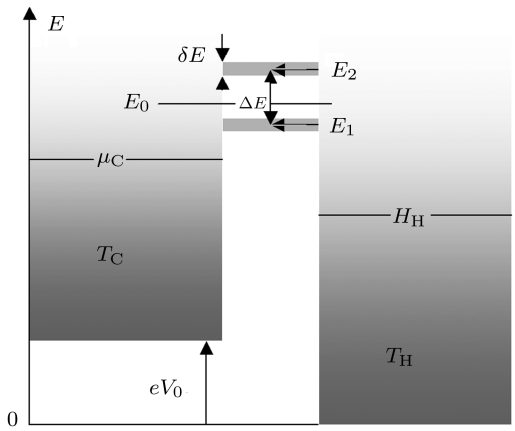


Fig.1. Schematic diagram of the ESE refrigerator with double resonances.

3. Performance parameters of the ESE refrigerator with double resonances

In order to calculate the refrigeratory load and COP of the refrigerator, we must obtain the expression for the heat flux into each reservoir, and the heat flux $\dot{Q}$ which occurs upon particles flux $\dot{N}$ with energy E into the reservoir. According to the first law of thermodynamics for an open system at constant volume and pressure,

$$\dot{U}_{\rm C/H} = \dot{Q}_{\rm C/H} + \mu_{\rm C/H}\dot{N}, \tag{1}$$

the change of magnitudes in the internal energy of the cold reservoir is $\dot{U}_{\rm C} = E\dot{N}$ and that of the hot reservoir $\dot{U}_{\rm H} = E\dot{N}$. An expression for $\dot{Q}_{\rm C/H}$ (where C/H indicates the cold (C) or hot (H) reservoirs) can be solved from Eq.(1) as following,

$$\dot{Q}_{\rm C/H} = \left(E - \mu_{\rm C/H}\right)\dot{N}. \tag{2}$$

Based on statistical mechanics, for the 1D electron reservoir the number of states over a small energy range E to $E + {\rm d}E$ per unit length is

$$D\left(E\right){\rm d}E = \frac{2}{h}\left(\frac{m}{2E}\right)^{1/2}{\rm d}E, \tag{3}$$

where m is mass of electron, h is Plank constant. The particles flux over a small energy range ${\rm d}E$ from the cold to the hot reservoir, $\dot{N}\left(E\right)_{\rm C\to H}$ can be obtained by the product of number of states in the electron reservoir, the velocity in the direction of transport, and the Fermi distribution $f_{\rm C}\left(E,\mu_{\rm C},T_{\rm C}\right)$ in the cold reservoir,

$$\begin{aligned}&\dot{N}\left(E\right)_{\rm C\to H}\\ &= \frac{2}{h}\left(\frac{m}{2E}\right)^{1/2}\left(\frac{2E}{m}\right)^{1/2} f_{\rm C}\left(E,\mu_{\rm C},T_{\rm C}\right){\rm d}E\\ &= \frac{2}{h} f_{\rm C}\left(E,\mu_{\rm C},T_{\rm C}\right){\rm d}E.\end{aligned} \tag{4}$$

3.1. In the case of $E_0 > E_2 = E_1 + \Delta E$

Because $E_1 < E_0$, the direction of net particles flux $\dot{N}$ at the energy level E_1 of the first resonance must be from cold reservoir to hot reservoir. Using Eq.(4), we can get net particles flux $\dot{N}$

$$\begin{aligned}\dot{N}\left(E_1\right) = \frac{2}{h}&(f_{\rm C}\left(E_1,\mu_{\rm C},T_{\rm C}\right)\\ &-f_{\rm H}\left(E_1,\mu_{\rm H},T_{\rm H}\right)){\rm d}E.\end{aligned} \tag{5}$$

Substituting Eq.(5) into Eq.(2) we obtain the expressions for the heat change of magnitudes per unit time in each of the reservoirs as a function of the energy of the filter E_1. The refrigeratory load of refrigerator per unit time for E_1 resonance is

$$\begin{aligned}\dot{Q}_{\rm C}^{-}\left(E_1\right) = \frac{2}{h}&\int_{E_1-\delta E/2}^{E_1+\delta E/2}\left(E-\mu_{\rm C}\right)\left(f_{\rm C}\left(E,\mu_{\rm C},T_{\rm C}\right)\right.\\ &\left.-f_{\rm H}\left(E,\mu_{\rm H},T_{\rm H}\right)\right){\rm d}E,\end{aligned} \tag{6}$$

and the increased heat of hot reservoir per unit time is

$$\begin{aligned}\dot{Q}_{\rm H}^{+}\left(E_1\right) = \frac{2}{h}&\int_{E_1-\delta E/2}^{E_1+\delta E/2}\left(E-\mu_{\rm H}\right)(f_{\rm C}\left(E,\mu_{\rm C},T_{\rm C}\right)\\ &-f_{\rm H}\left(E,\mu_{\rm H},T_{\rm H}\right)){\rm d}E.\end{aligned} \tag{7}$$

Similarly, when $E_2 < E_0$, the direction of net particles flux $\dot{N}$ at the energy level E_2 of the second resonance must also be from cold reservoir to hot reservoir, using Eq.(4) we obtain the net particle flux $\dot{N}$

$$\dot{N}(E_2) = \frac{2}{h}(f_{\rm C}(E_2,\mu_{\rm C},T_{\rm C}) - f_{\rm H}(E_2,\mu_{\rm H},T_{\rm H}))\mathrm{d}E. \quad (8)$$

Thus, the increased amount of heat of hot reservoir per unit time is

$$\dot{Q}^{+}_{\rm H}(E_2) = \frac{2}{h}\int_{E_2-\delta E/2}^{E_2+\delta E/2}(E-\mu_{\rm H})(f_{\rm C}(E,\mu_{\rm C},T_{\rm C}) - f_{\rm H}(E,\mu_{\rm H},T_{\rm H}))\mathrm{d}E, \quad (9)$$

and the refrigeratory load of refrigerator per unit time for E_2 resonance is

$$\dot{Q}^{-}_{\rm C}(E_2) = \frac{2}{h}\int_{E_2-\delta E/2}^{E_2+\delta E/2}(E-\mu_{\rm C})(f_{\rm C}(E,\mu_{\rm C},T_{\rm C}) - f_{\rm H}(E,\mu_{\rm H},T_{\rm H}))\mathrm{d}E. \quad (10)$$

Using Eqs.(6), (7), (9) and (10) we obtain the COP and refrigeratory load of refrigerator of the ESE refrigerator with double resonances

$$\varepsilon(E_1,E_2) = \frac{\left(\dot{Q}^{-}_{\rm C}(E_1)+\dot{Q}^{-}_{\rm C}(E_2)\right)}{\left(\dot{Q}^{+}_{\rm H}(E_1)+\dot{Q}^{+}_{\rm H}(E_2)\right)-\left(\dot{Q}^{-}_{\rm C}(E_1)+\dot{Q}^{-}_{\rm C}(E_2)\right)}, \quad (11)$$

and

$$\dot{Q}^{-}_{\rm C}(E_1,E_2) = \dot{Q}^{-}_{\rm C}(E_1)+\dot{Q}^{-}_{\rm C}(E_2), \quad (12)$$

respectively.

3.2. In the case of $E_0 < E_2 = E_1 + \Delta E$

When $E_2 > E_0$, the direction of net particles flux $\dot{N}$ at the energy level E_2 of the second resonance must be from hot reservoir to cold reservoir. The Eq.(8) should be changed as

$$\dot{N}(E_2) = \frac{2}{h}(f_{\rm H}(E_2,\mu_{\rm H},T_{\rm H}) - f_{\rm C}(E_2,\mu_{\rm C},T_{\rm C}))\delta E. \quad (13)$$

Thus, the expressions for the heat change of magnitudes per unit time in each of the reservoirs as a function of the energy of the filter E_2 may be acquired. Firstly, the lost amount of heat of hot reservoir per unit time at E_2 is

$$\dot{Q}^{-}_{\rm H}(E_2) = \frac{2}{h}\int_{E_2-\delta E/2}^{E_2+\delta E/2}(E-\mu_{\rm H})(f_{\rm H}(E,\mu_{\rm H},T_{\rm H}) - f_{\rm C}(E,\mu_{\rm C},T_{\rm C}))\mathrm{d}E, \quad (14)$$

and the increased amount of heat of cold reservoir per unit time is

$$\dot{Q}^{+}_{\rm C}(E_2) = \frac{2}{h}\int_{E_2-\delta E/2}^{E_2+\delta E/2}(E-\mu_{\rm C})(f_{\rm H}(E,\mu_{\rm H},T_{\rm H}) - f_{\rm C}(E,\mu_{\rm C},T_{\rm C}))\mathrm{d}E. \quad (15)$$

Using Eqs.(6), (7), (14) and (15) we obtain the COP and refrigeratory load of refrigerator of the ESE refrigerator with double resonances

$$\varepsilon(E_1,E_2) = \frac{\left(\dot{Q}^{-}_{\rm C}(E_1)-\dot{Q}^{+}_{\rm C}(E_2)\right)}{\left(\dot{Q}^{+}_{\rm H}(E_1)-\dot{Q}^{-}_{\rm H}(E_2)\right)-\left(\dot{Q}^{-}_{\rm C}(E_1)-\dot{Q}^{+}_{\rm C}(E_2)\right)}, \quad (16)$$

and

$$\dot{Q}^{-}_{\rm C}(E_1,E_2) = \dot{Q}^{-}_{\rm C}(E_1)-\dot{Q}^{+}_{\rm C}(E_2), \quad (17)$$

respectively.

4. Performance characteristics of the ESE refrigerator

For the ESE refrigerator, two cases about the space of two resonances ΔE must be considered due to $E_0/k - \mu_{\rm C}/k = 2\,{\rm K}$ (k is Boltzmann's constant, K is absolute units of temperature here).

4.1. In the case of $\Delta E/k < E_0/k - \mu_{\rm C}/k = 2\,{\rm K}$

According to Eqs.(12) and (17), the refrigeratory load of the ESE refrigerator $\dot{Q}^{-}_{\rm C}(E_1,E_2)$ can be numerically calculated for giving a series of parameters ($T_{\rm C} = 1\,{\rm K}$, $T_{\rm H} = 2\,{\rm K}$, $\mu_{\rm C}/k = 12\,{\rm K}$ and $\mu_{\rm H}/k = 10\,{\rm K}$, so $E_0/k = 14\,{\rm K}$) in this case. The curves of the refrigeratory load of the ESE refrigerator versus energy level E_1 of the first resonance are plotted, as shown in Figs.2 and 3, where different values of ΔE or δE are given. It can be seen that the refrigeratory load exhibits the maximum value at a particular value $E_{1\rm Q}$ of E_1 , on the other hand, energy level E_1 of the first resonance must lower than the maximum value $E_{1\rm Q\,max}$. When $E_1 > E_{1\rm Q\,max}$, the refrigeratory load of the ESE refrigerator is smaller than zero. The refrigerator loses

its role. Thus the feasible region of energy level E_1 should be

$$E_1 < E_{1\mathrm{Q}\,\max}, \tag{18}$$

where $E_{1\mathrm{Q}\,\max}$ may be obtained by numerical computation. $E_{1\mathrm{Q}\,\max}$ is one important parameter of the ESE refrigerator and determines the maximum value of energy level E_1. At the given the energy space of double resonances, both of $\dot{Q}^{-}_{\mathrm{C}\,\max}$ and $E_{1\mathrm{Q}}$ will decrease as δE decreases. And at the given resonance width, both of $\dot{Q}^{-}_{\mathrm{C}\,\max}$ and $E_{1\mathrm{Q}}$ will decrease as ΔE increases.

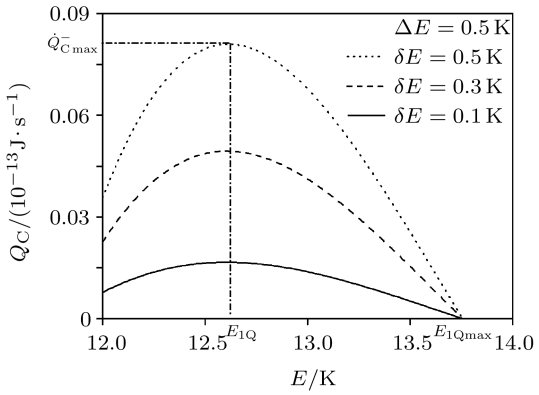


Fig.2. When $\Delta E/k < (E_0 - \mu_{\mathrm{C}})/k = 2.0\,\mathrm{K}$, the refrigeratory load as a function of E_1 for different δE at the given parameters $T_{\mathrm{C}} = 1\,\mathrm{K}$, $T_{\mathrm{H}} = 2\,\mathrm{K}$, $\mu_{\mathrm{C}}/k = 12\,\mathrm{K}$, $\mu_{\mathrm{H}}/k = 10\,\mathrm{K}$ and $\Delta E/k = 0.5\,\mathrm{K}$.

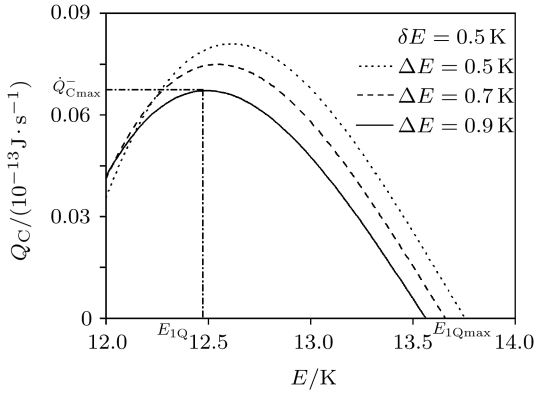


Fig.3. When $\Delta E/k < (E_0 - \mu_{\mathrm{C}})/k = 2.0\,\mathrm{K}$, the refrigeratory load as a function of E_1 for different ΔE at the given parameters $T_{\mathrm{C}} = 1\,\mathrm{K}$, $T_{\mathrm{H}} = 2\,\mathrm{K}$, $\mu_{\mathrm{C}}/k = 12\,\mathrm{K}$, $\mu_{\mathrm{H}}/k = 10\,\mathrm{K}$ and $\delta E/k = 0.5\,\mathrm{K}$.

Similarly, according to Eqs.(11) and (16), the COP of the ESE refrigerator can be numerically calculated for giving the same series of parameters as above. The curves of the COP versus energy level E_1 of the first resonance are plotted, as shown in Figs.4 and 5, where different values of ΔE or δE are given. It is shown that the COP of the refrigerator exhibits the maximum value $\varepsilon_{\max}$ at a particular value $E_{1\varepsilon}$ of E_1. At the given ΔE or δE, the characteristic curves show the same characteristics of exhibiting $\varepsilon_{\max}$, and $E_{1\varepsilon}$ will increase as the ΔE or δE decreases. The energy level E_1 of the first resonance must lower than the maximum value $E_{1\varepsilon\,\max}$. When $E_1 > E_{1\varepsilon\,\max}$, the COP of the ESE refrigerator is smaller than zero. In addition, when the resonance is very narrow and the energy space of double resonances is very small, reversible behaviour is approached, then the COP is close to Carnot value.

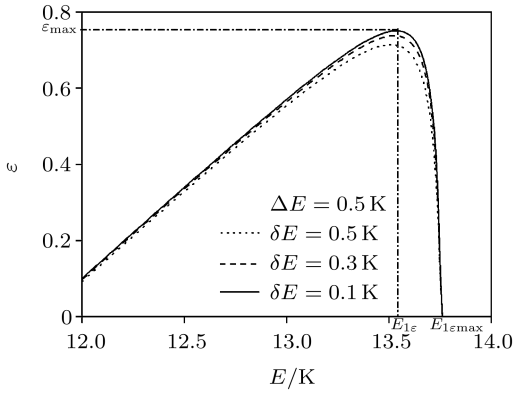


Fig.4. When $\Delta E/k < (E_0 - \mu_{\mathrm{C}})/k = 2.0\,\mathrm{K}$, the coefficient of the performance as a function of E_1 for different δE, the values of the other parameters are the same as those in Fig.2.

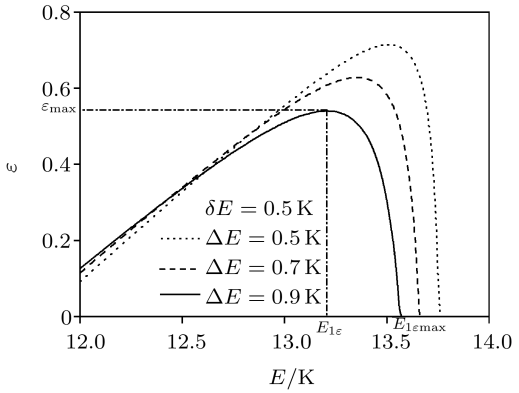


Fig.5. When $\Delta E/k < (E_0 - \mu_{\mathrm{C}})/k = 2.0\,\mathrm{K}$, the coefficient of the performance as a function of E_1 for different ΔE, the values of the other parameters are the same as those in Fig.3.

Especially, the curves of the refrigeratory load versus the COP are plotted, as shown in Figs.6 and 7. it is found that the characteristic curves between the refrigeratory load and the COP are the open loop-shaped ones. The maximum refrigeratory load $\dot{Q}^{-}_{\mathrm{C}\,\max}$

and the corresponding COP $\varepsilon_{\rm Q}$, as well as the maximum COP $\varepsilon_{\rm max}$ and the corresponding refrigeratory load $\dot{Q}^{-}_{{\rm C}\varepsilon}$ may be numerically calculated at given other parameters. The optimal operating regions of the refrigerator should be located in those of the $\dot{Q}^{-}_{\rm C} \sim \varepsilon$ curve with a negative slope. It is clearly shown that when the refrigeratory load is in this region, there are two different COPs for a given refrigeratory load, where one is smaller than $\varepsilon_{\rm Q}$ and the other is larger than $\varepsilon_{\rm Q}$. Obviously, one always wants to obtain the COP as large as possible for a given refrigeratory load. Thus, the optimal COP of the refrigerator should be

$$\varepsilon_{\rm Q} \leq \varepsilon \leq \varepsilon_{\rm max}. \tag{19}$$

$\varepsilon_{\rm Q}$ and $\varepsilon_{\rm max}$ are two important performance parameters which determine the lower and upper bounds of the optimal COP. Moreover, the optimal energy level E_1 of the first resonance should be

$$E_{1{\rm Q}} \leq E_1 \leq E_{1\varepsilon}. \tag{20}$$

Obviously, $E_{1{\rm Q}}$ and $E_{1\varepsilon}$ are also two important performance parameters which determine the lower and upper bounds of the energy level E_1 of the first resonance. The values of the several important performance parameters may be obtained by the numerical calculation, as listed in Table 1.

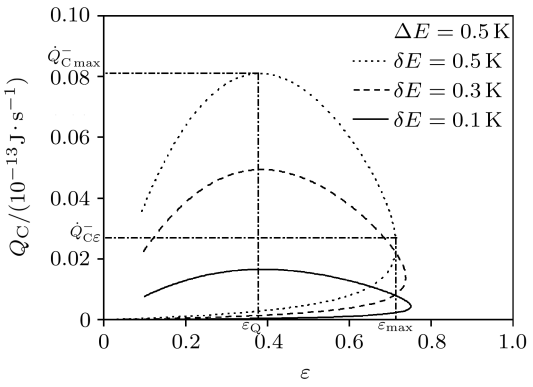


Fig.6. When $\Delta E/k < (E_0 - \mu_{\rm C})/k = 2.0\,{\rm K}$, the refrigeratory load versus the COP for different δE, the values of the other parameters are the same as those in Fig.2.

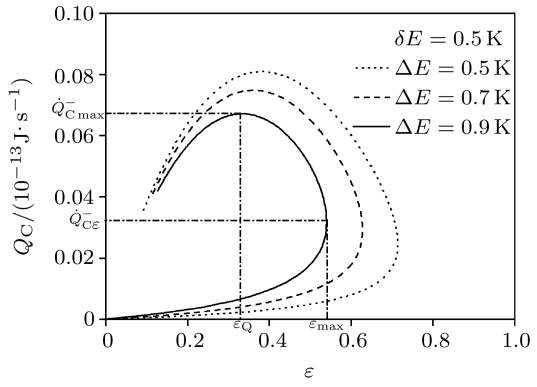


Fig.7. When $\Delta E/k < (E_0 - \mu_{\rm C})/k = 2.0\,{\rm K}$, the refrigeratory load versus the COP for different ΔE, the values of the other parameters are the same as those in Fig.3.

Table 1. the values of the several important performance parameters at different ΔE and δE for $\Delta E/k < (E_0 - \mu_{\rm C})/k = 2.0\,{\rm K}$.

ΔE	δE	$E_{1{\rm Q\,max}}$	$E_{1\varepsilon\,{\rm max}}$	$E_{1{\rm Q}}$	$E_{1\varepsilon}$
0.5	0.1	13.75	13.75	12.61	13.54
	0.3	13.75	13.75	12.61	13.53
	0.5	13.75	13.75	12.62	13.51
0.7	0.1	13.65	13.65	12.54	13.37
	0.3	13.65	13.65	12.54	13.37
	0.5	13.65	13.65	12.55	13.35
0.9	0.1	13.56	13.56	12.49	13.22
	0.3	13.56	13.56	12.49	13.22
	0.5	13.56	13.56	12.50	13.21
ΔE	δE	$\dot{Q}^{-}_{\rm C\,max} \times 10^{-13}({\rm J\cdot s^{-1}})$	$\varepsilon_{\rm max}$	$\dot{Q}^{-}_{\rm C\varepsilon} \times 10^{-13}({\rm J\cdot s^{-1}})$	$\varepsilon_{\rm Q}$
0.5	0.1	0.01661	0.75060	0.00435	0.39109
	0.3	0.04940	0.73780	0.01362	0.38716
	0.5	0.08096	0.71403	0.02454	0.38398
0.7	0.1	0.01537	0.65487	0.00559	0.37355
	0.3	0.04572	0.64560	0.01671	0.36532
	0.5	0.07489	0.62791	0.02947	0.35779
0.9	0.1	0.01380	0.56141	0.00623	0.34797
	0.3	0.04104	0.55421	0.01861	0.34024
	0.5	0.06717	0.54028	0.03152	0.33299

4.2. In the case of $\Delta E/k \geq E_0/k - \mu_C/k = 2$K

According to Eqs.(12) and (17), the refrigeratory load of the ESE refrigerator $\dot{Q}_C^-(E_1, E_2)$ can be numerically calculated for giving a series of parameters ($T_C = 1$ K, $T_H = 2$ K, $\mu_C/k = 12$ K and $\mu_H/k = 10$ K, so $E_0/k = 14$ K) in this case. The curves of the refrigeratory load of the ESE refrigerator versus energy level E_1 of the first resonance are plotted, as shown in Figs.8 and 9, where different values of ΔE or δE are given. It can be seen that the refrigeratory load exhibits the maximum value at a particular value E_{1Q} of E_1, on the other hand, energy level E_1 of the first resonance must lower than the maximum value $E_{1Q\,max}$ and larger than the minimum value $E_{1Q\,min}$. When $E_1 > E_{1Q\,max}$ and $E_1 < E_{1Q\,min}$, the refrigeratory load of the ESE refrigerator is smaller than zero. The refrigerator loses its role. Thus the feasible region of energy level E_1 should be

$$E_{1Q\,min} < E_1 < E_{1Q\,max}. \tag{21}$$

Where $E_{1Q\,min}$ and $E_{1Q\,max}$ may be obtained by numerical computation. $E_{1Q\,min}$ and $E_{1Q\,max}$ are two important parameters of the ESE refrigerator and determine the minimum and maximum values of energy level E_1. At the given energy space of double resonances, both of $\dot{Q}_{C\,max}^-$ and E_{1Q} will decrease as δE decreases. And at the given width of resonance, $\dot{Q}_{C\,max}^-$ will decrease while E_{1Q} will increase as ΔE increases.

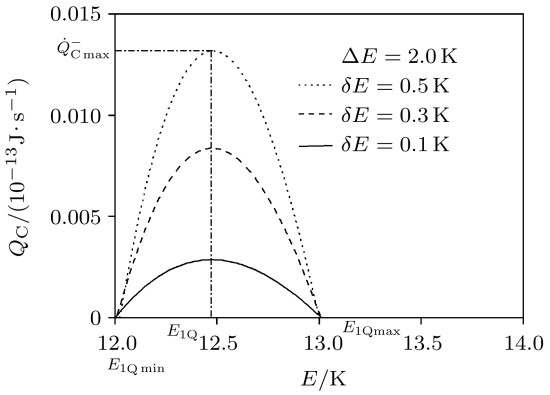


Fig.8. When $\Delta E/k \geq (E_0 - \mu_C)/k = 2.0$ K, the refrigeratory load as a function of E_1 for different δE, the values of the other parameters are the same as those in Fig.2.

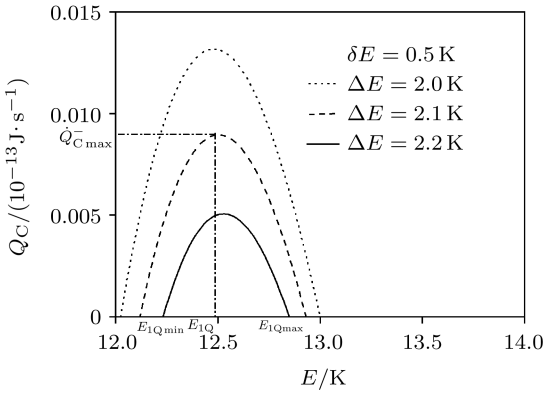


Fig.9. When $\Delta E/k \geq (E_0 - \mu_C)/k = 2.0$ K, the refrigeratory load as a function of E_1 for different ΔE, the values of the other parameters are the same as those in Fig.3.

Similarly, according to Eqs.(11) and (16), the COP of the ESE refrigerator can be numerically calculated for giving the same series of parameters as above. The curves of the COP versus energy level E_1 of the first resonance are plotted, as shown in Figs.10 and 11, where different values of ΔE or δE are given. It is shown that the COP of the refrigerator exhibits the maximum value ε_{max} at a particular value $E_{1\varepsilon}$ of E_1. At the given ΔE, both of ε_{max} and $E_{1\varepsilon}$ will increase as δE decreases. And at the given δE, both of ε_{max} and $E_{1\varepsilon}$ will decrease as ΔE increases. The energy level E_1 of the first resonance must lower than the maximum value $E_{1\varepsilon\,max}$ and larger than the minimum value $E_{1\varepsilon\,min}$. When $E_1 > E_{1\varepsilon\,max}$ and $E_1 < E_{1\varepsilon\,min}$, the COP of the ESE refrigerator is smaller than zero. In addition, the reversible behaviour can not be approached, and the COP is not close to Carnot value due to $\Delta E/k \geq 2$K.

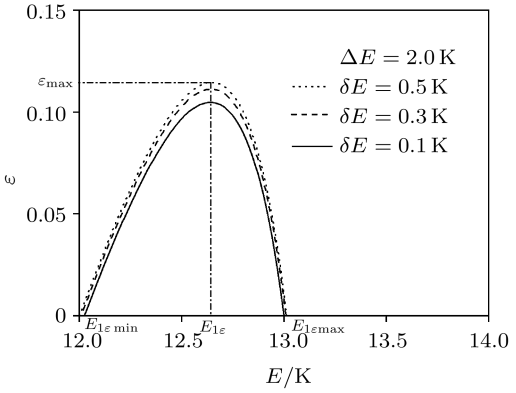


Fig.10. When $\Delta E/k \geq (E_0 - \mu_C)/k = 2.0$ K, the coefficient of the performance as a function of E_1 for different δE, the values of the other parameters are the same as those in Fig.2.

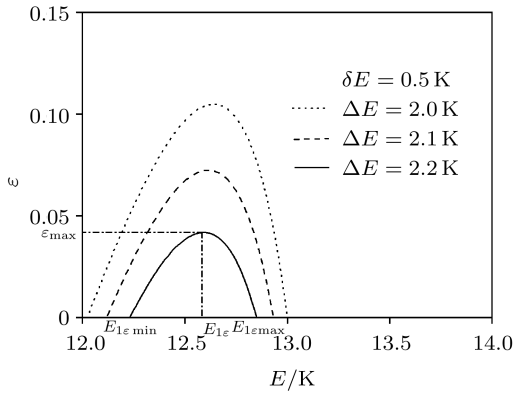


Fig.11. When $\Delta E/k \geq (E_0 - \mu_{\mathrm{C}})/k = 2.0\,\mathrm{K}$, the coefficient of the performance as a function of E_1 for different ΔE, the values of the other parameters are the same as those in Fig.3.

Especially, the curves of the refrigeratory load versus the COP are plotted, as shown in Figs.12 and 13. It is found that the characteristic curves between the refrigeratory load and the COP are the closed loop-shaped ones. The maximum refrigeratory load $\dot{Q}^{-}_{\mathrm{C}\,\max}$ and the corresponding COP ε_{Q}, as well as the maximum COP $\varepsilon_{\max}$ and the corresponding refrigeratory load $\dot{Q}^{-}_{\mathrm{C}\varepsilon}$ may be numerically calculated at other given parameters. The optimal operating regions of the refrigerator in this case should be located in those of the $\dot{Q}^{-}_{\mathrm{C}} \sim \varepsilon$ curve with a negative slope. Similarly, it is clearly shown that when the refrigeratory load is in this region, there are two different COPs for a given refrigeratory load, where one is smaller than ε_{Q} and the other is larger than ε_{Q}. Obviously, one always wants to obtain the COP as large as possible for a given refrigeratory load. Thus, the optimal COP of the refrigerator should be

$$\varepsilon_{\mathrm{Q}} \leq \varepsilon \leq \varepsilon_{\max}. \tag{22}$$

ε_{Q} and $\varepsilon_{\max}$ are two important performance parameters which determine the lower and upper bounds of the optimal COP. Moreover, the optimal energy level E_1 of the first resonance should be

$$E_{1\mathrm{Q}} \leq E_1 \leq E_{1\varepsilon}. \tag{23}$$

Obviously, $E_{1\mathrm{Q}}$ and $E_{1\varepsilon}$ are also two important performance parameters which determine the lower and upper bounds of the energy level E_1 of the first resonance. The values of the several important performance parameters may be obtained by the numerical calculation, as listed in Table 2.

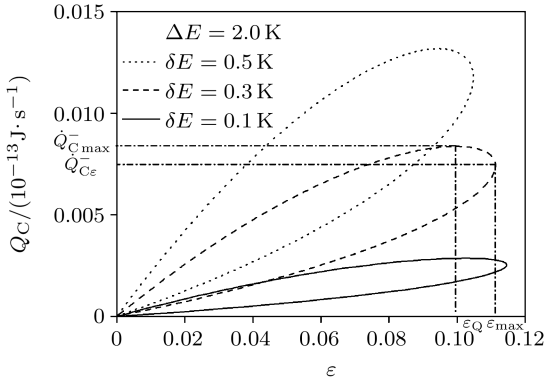


Fig.12. When $\Delta E/k \geq (E_0 - \mu_{\mathrm{C}})/k = 2.0\,\mathrm{K}$, the refrigeratory load versus the COP for different δE, the values of the other parameters are the same as those in Fig.2.

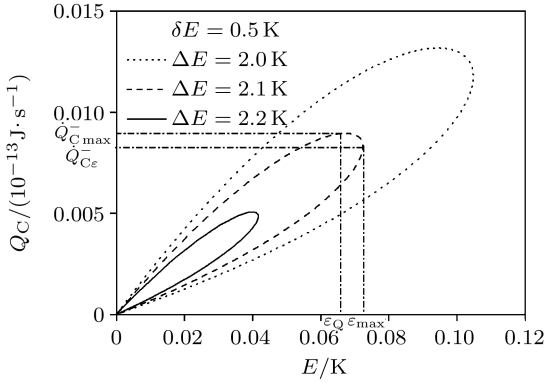


Fig.13. When $\Delta E/k \geq (E_0 - \mu_{\mathrm{C}})/k = 2.0\,\mathrm{K}$, the refrigeratory load versus the COP for different ΔE, the values of the other parameters are the same as those in Fig.3.

Table 2. The values of the several important performance parameters at different ΔE and δE for $\Delta E/k \geq (E_0 - \mu_{\mathrm{C}})/k = 2.0\,\mathrm{K}$.

ΔE	δE	$E_{1\mathrm{Q}\,\min}$	$E_{1\mathrm{Q}\,\max}$	$E_{1\varepsilon\,\min}$	$E_{1\varepsilon\,\max}$	$E_{1\mathrm{Q}}$
2.0	0.1	12.01	13.01	12.01	13.01	12.48
	0.3	12.01	13.00	12.01	13.00	12.48
	0.5	12.03	12.99	12.03	12.99	12.48
2.1	0.1	12.10	12.94	12.10	12.94	12.50
	0.3	12.11	12.94	12.11	12.94	12.51
	0.5	12.12	12.93	12.12	12.93	12.51
2.2	0.1	12.20	12.87	12.20	12.87	12.52
	0.3	12.21	12.86	12.21	12.86	12.53

Table 2. *Continued*

	0.5	12.24	12.84	12.24	12.84	12.53
ΔE	δE	$E_{1\varepsilon}$	$\dot{Q}^{-}_{\mathrm{C}\max} \times 10^{-13}(\mathrm{J}\cdot\mathrm{s}^{-1})$	$\varepsilon_{\max}$	$\dot{Q}^{-}_{\mathrm{C}\varepsilon} \times 10^{-13}(\mathrm{J}\cdot\mathrm{s}^{-1})$	ε_{Q}
2.0	0.1	12.65	0.00287	0.11457	0.00252	0.10322
	0.3	12.64	0.00837	0.11129	0.00747	0.10025
	0.5	12.64	0.01318	0.10480	0.01178	0.09437
2.1	0.1	12.62	0.00202	0.08186	0.00185	0.07588
	0.3	12.62	0.00582	0.07868	0.00534	0.07381
	0.5	12.61	0.00896	0.07239	0.00834	0.06790
2.2	0.1	12.60	0.00123	0.05092	0.00117	0.04948
	0.3	12.60	0.00348	0.04781	0.00329	0.04598
	0.5	12.59	0.00507	0.04169	0.00487	0.04007

5. Conclusion

The performance of the ESE refrigerator with double resonances is studied here. Due to the double resonances, we discuss the influences of the resonances widths, the energy level position of resonance and the space of two resonances on performance of the ESE refrigerator. In the paper, two cases about ΔE are analysed for the ESE refrigerator. In the $\Delta E/k < 2.0\,\mathrm{K}$ case, as δE and ΔE decrease, reversible behaviour is approached, and then the COP is close to Carnot value and $E_{1\mathrm{Q}\max}$, $E_{1\varepsilon\max}$ are found, so the energy level E_1 of the first resonance must lower than them. When $E_1 > E_{1\mathrm{Q}\max}$ or $E_1 > E_{1\varepsilon\max}$, the refrigerator loses its role. In the $\Delta E/k \geq 2.0\,\mathrm{K}$ case, it can be seen from Figs.8 to 13 and Table 2 that the COP is not close to Carnot value. $E_{1\mathrm{Q}\min}$, $E_{1\mathrm{Q}\max}$, $E_{1\varepsilon\min}$ and $E_{1\varepsilon\max}$ are found, so the energy level E_1 of the first resonance must be located at the regions $E_{1\mathrm{Q}\min} < E_1 < E_{1\mathrm{Q}\max}$ and $E_{1\varepsilon\min} < E_1 < E_{1\varepsilon\max}$. When E_1 is not in these regions, the refrigerator loses its role. Importantly, $E_{1\mathrm{Q}}$ and $E_{1\varepsilon}$ are found in two cases, respectively, so the optimal region $E_{1\mathrm{Q}} \leq E_1 \leq E_{1\varepsilon}$ for the operation of the ESE refrigerator can be ascertained in the two different cases.

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