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An improved impulsive control approach to nonlinear systems with time-varying delays*

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A scheme for the impulsive control of nonlinear systems with time-varying delays is investigated in this paper. Based on the Lyapunov-like stability theorem for impulsive functional differential equations (FDEs), some sufficient conditions are presented to guarantee the uniform asymptotic stability of impulsively controlled nonlinear systems with time-varying delays. These conditions are more effective and less conservative than those obtained. Finally, two numerical examples are provided to demonstrate the effectiveness of the proposed method.

Keywords: impulsive control, time-varying delays, impulsive functional differential equations, Lyapunov-like stability theorem

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1. Introduction

In recent years, impulsive systems and impulsive control have attracted considerable attention due to their great potential applications in engineering, economics, medicine and biology.[1] The main idea of impulsive control is to change the states of continuous dynamic systems via discontinuous control inputs at certain time moments. In chaotic secure communication systems, impulsive control is attractive since it allows the stabilization and the synchronization of chaotic systems by using only small control impulses. In Ref.[2], the concept and the principles of impulsive control were first presented, and the asymptotic stability of the impulsively controlled nonlinear system was studied. Based on the control idea in Ref.[2], some less conservative sufficient conditions for asymptotic stability of impulsively controlled nonlinear systems were derived in Refs.[3,4]. The scheme for the stabilization and the synchronization of the unified chaotic systems via an impulsive control method was discussed in Ref.[5]. However, the aforementioned and most studies[2−13] have considered only the nonlinear systems described by impulsive ordinary differential equations (ODEs). On the other hand, time delays, as a source of oscillation and instability, are ubiquitous in both nature and man-made systems. This motivates the increasing investigation of the stabilization of nonlinear delayed systems via impulsive control,[14−19] which corresponds to the stability problem of impulsive functional differential equations (FDEs).[20−25]

Motivated by the aforementioned discussion, in the present paper we address the impulsive control scheme for nonlinear systems with time-varying delays as described in Ref.[16]. The criteria derived in this paper are more effective and less conservative than those given in Ref.[16] and easy to verify. In addition, some unnecessary restrictions are removed from the criteria. Furthermore, our results can be applied to a wider class of nonlinear delayed systems and are helpful for improving the existing technologies used in chaotic secure communication systems and chaotic spread spectrum communications.

The rest of the present paper is organized as follows. In Section 2, some necessary preliminaries on impulsive FDEs are provided. Main results for im-

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pulsive control of nonlinear systems with time-varying delays are derived in Section 3. In Section 4, two illustrative examples are presented to demonstrate the effectiveness and the feasibility of the proposed method. Finally, concluding remarks are given in Section 5.

Throughout the paper, the notations $\mathbb{R}$ and $\mathbb{R}^n$ denote the real number and n-dimensional Euclidean space, respectively. $\mathbb{R}^{n\times m}$ is the set of all real matrices. $\mathbb{R}^+ := [0,\infty)$. $\mathbb{N} := \{1,2,\ldots\}$. $\mathcal{C}(X,Y)$ denotes the space of continuous mappings from the topological space X to the topological space Y. $\boldsymbol{I}$ represents the identity matrix with appropriate dimensions. $\|\cdot\|$ refers to Euclidean vector norm or the induced matrix 2-norm. $\lambda_{\max}(\boldsymbol{A})$ ($\lambda_{\min}(\boldsymbol{A})$) denotes the maximal (minimal) eigenvalue of matrix $\boldsymbol{A}$.

2. Preliminaries

To derive our main results, some notations, definitions and lemma about impulsive FDEs should be introduced.

$\mathcal{PC} := \{\varphi : [-\tau,0] \to \mathbb{R}^n,\ \varphi(t)$ is continuous everywhere except at the finite number of points $\bar{t}$ at which $\varphi(\bar{t}^+)$ and $\varphi(\bar{t}^-)$ exist.$\}$,

$S(\rho) := \{x \in \mathbb{R}^n : \|x\| < \rho\}$,

$\mathcal{K} := \{\alpha \in \mathcal{C}(\mathbb{R}^+,\mathbb{R}^+) : \alpha(s)$ is strictly increasing and $\alpha(0) = 0\}$,

$\mathcal{K}^* := \{\psi \in \mathcal{K} : \psi(s) < s,\ \forall s > 0\}$,

$\Omega := \{H \in \mathcal{C}(\mathbb{R}^+,\mathbb{R}^+) : H(0) = 0,\ H(s) > 0,\ \forall s > 0\}$.

Consider the following impulsive FDEs:

$$\begin{aligned} &\dot{x}(t) = \hat{f}(t,x_t),\ t \geq t_0,\\ &x(t_k^+) = J_k(x(t_k^-)),\ k \in \mathbb{N}, \end{aligned} \tag{1}$$

where $\hat{f} : [t_0,\infty) \times PC \to \mathbb{R}^n$ and $J_k(x) : S(\rho) \to \mathbb{R}^n$ for each $k \in \mathbb{N}$. For any $t \geq t_0$, $x_t \in PC$ is defined as $x_t(s) = x(t+s)$, $-\tau \leq s \leq 0$. Suppose that discrete instant set $\{t_k\}$ satisfies $0 < t_1 < t_2 < \ldots < t_k < t_{k+1} < \ldots$, $\lim\limits_{k\to\infty} t_k = \infty$, $0 \leq t_0 < t_1$, and $\lim\limits_{t\to t_k^+} x(t) = x(t_k)$, i.e. $x(t)$ is right-continuous at $t = t_k$, $\lim\limits_{t\to t_k^-} x(t) = x(t_k^-)$ exists, $k \in \mathbb{N}$.

Definition 1[1,2] Function $V(t,x) : [t_0,\infty) \times S(\rho) \to \mathbb{R}^+$ belongs to class V_0 if

(i) $V(t,x)$ is continuous in each of sets $[t_{k-1},t_k) \times S(\rho)$. $\lim\limits_{(t,y)\to(t_k^-,x)} V(t,y) = V(t_k^-,x)$ exists for all $x \in S(\rho)$ and $k \in \mathbb{N}$;

(ii) $V(t,x)$ is locally Lipschitzian in $x \in S(\rho)$, and $\forall t \geq t_0$, $V(t,0) = 0$.

Here, function $V(t,x)$ is similar to the Lyapunov-like function although it might be piecewise continuous in the time domain, which implies that $V(t,x)$ is generally not differentiable with respect to time t.

Definition 2[1,2] Let $V \in V_0$. For any $(t,x) \in [t_{k-1},t_k) \times S(\rho)$, the upper right (Dini) derivative of $V(t,x(t))$ along the solution $x(t)$ of Eq.(1) is defined as

$$\begin{aligned} D^+V(t,x(t)) := \lim_{h\to 0^+} \sup \frac{1}{h}(&V(t+h,x(t+h))\\ &- V(t,x(t))). \end{aligned}$$

Lemma 1[20] The zero solution of Eq.(1) is uniformly asymptotically stable if there exist $V \in V_0$, $\alpha_1,\ \alpha_2 \in \mathcal{K}$, $\psi \in \mathcal{K}^*$ and $H \in \Omega$ such that

(i) $\alpha_1(\|x\|) \leq V(t,x) \leq \alpha_2(\|x\|)$ for $(t,x) \in [t_0,\infty) \times S(\rho)$;

(ii) for all $x \in S(\rho_1)$, $0 < \rho_1 \leq \rho$ and $k \in \mathbb{N}$, $V(t_k,J_k(x)) \leq \psi(V(t_k^-,x))$;

(iii) for any solution $x(t)$ of Eq.(1), $V(t+s,x(t+s)) \leq \psi^{-1}(V(t,x))$, $-\tau \leq s \leq 0$, which implies that $D^+V(t,x(t)) \leq g(t)H(V(t,x(t)))$, where $g : [t_0,\infty) \to \mathbb{R}^+$ is locally integrable, with ψ^{-1} being the inverse function of ψ;

(iv) H is nondecreasing and there exist constants $\bar{\lambda} \geq \lambda > 0$ and $\eta > 0$ such that, for all $k \in \mathbb{N}$ and $\mu > 0$, $\lambda \leq t_k - t_{k-1} \leq \bar{\lambda}$ and $\int_{\psi(\mu)}^{\mu} \frac{\mathrm{d}u}{H(u)} - \int_{t_{k-1}}^{t_k} g(s)\mathrm{d}s \geq \eta$.

3. Main results

Consider a plant $\mathcal{P}$ described by the following nonlinear systems with time-varying delays:

$$\begin{aligned} &\dot{x}(t) = \boldsymbol{A}x(t) + f(t,x(t),x(t-r_1(t)),\ldots x(t-r_m(t))),\\ &y(t) = \boldsymbol{C}x(t), \quad t \geq t_0 = 0, \end{aligned} \tag{2}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ is the state vector, $\boldsymbol{y}(t) \in \mathbb{R}^m$ is the output vector, $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ and $\boldsymbol{C} \in \mathbb{R}^{m\times n}$are the constant matrices, the delay r_i may be unknown (constant or time-varying) but is bounded by a known constant, i.e. $0 \leq r_i(t) \leq \tau_i$, $i = 1,2,\ldots m$, $f : \mathbb{R}^+ \times \underbrace{\mathbb{R}^n \times \ldots \mathbb{R}^n}_{m+1} \to \mathbb{R}^n$ is a continuous function satisfying

$$\begin{aligned} &\|f(t,x(t),x(t-r_1(t)),\ldots,x(t-r_m(t)))\|\\ \leq &Kx(t) + \sum_{i=1}^{m} L_i \|x(t-r_i(t))\|,\\ &K,\ L_i \geq 0, \ \text{ for any } x(t),\\ &x(t-r_i(t)) \in \mathbb{R}^n,\ t \geq 0. \end{aligned} \tag{3}$$

To study the impulsive control of system (2), we design an impulsive control law:

$$\Delta x(t_k) := x(t_k) - x(t_k^-) = \boldsymbol{B}_k y(t_k^-) = \boldsymbol{B}_k \boldsymbol{C} x(t_k^-),$$

where $k \in \mathbb{N}$. Then, we can obtain an impulsively controlled system with time-varying delays as follows:

$$\begin{aligned} \dot{x}(t) =& \boldsymbol{A}x(t) + f(t, x(t), x(t - r_1(t)), \ldots, \\ & x(t - r_m(t))), \ t \neq t_k, \\ x(t_k) =& (\boldsymbol{I} + \boldsymbol{C}_k)x(t_k^-), \ k \in \mathbb{N}, \\ x(t) =& \varphi(t), \ -\tau \leq t \leq 0, \end{aligned} \tag{4}$$

where $\boldsymbol{C}_k = \boldsymbol{B}_k \boldsymbol{C}$, $\tau = \max_{1 \leq i \leq m}\{\tau_i\}$, and the initial condition φ is continuous for $t \in [-\tau, 0]$.

Our goal is to design the control gain $\boldsymbol{B}_k$ and the inter impulse interval $\Delta_k = t_k - t_{k-1}$ ($k \in \mathbb{N}$) such that the origin of system (4) is asymptotically stable. Next, we give the following results on the design of impulsive control of nonlinear systems with time-varying delays.

Theorem 1 The origin of system (4) is uniformly asymptotically stable if there exists a constant $\gamma > 0$ such that

$$2 \ln \omega + \lambda_0 (t_k - t_{k-1}) < 0, \tag{5}$$

where $\lambda_0 = ||\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}} + \gamma \boldsymbol{I} + \gamma^{-1} \omega_0^2 \boldsymbol{I}|| > 0$, $\omega_0 = K + \omega^{-1} \sum_{i=1}^{m} L_i$, and $\omega = \max_{k \in \mathbb{N}}\{||\boldsymbol{I} + \boldsymbol{C}_k||\}$.

Proof Construct a Lyapunov function $V(t, x) = x^{\mathrm{T}} x$, and for $t \in [t_{k-1}, t_k)$, $k \in \mathbb{N}$, the Dini derivative of $V(t, x)$ along the trajectory of system (4) is obtained as follows:

$$\begin{aligned} D^+ V(t, x(t)) =& 2x^{\mathrm{T}}(t)\dot{x}(t) \\ =& 2x^{\mathrm{T}}(t)(\boldsymbol{A} + f(t, x(t), x(t - r_1(t)), \ldots, x(t - r_m(t)))) \\ =& x^{\mathrm{T}}(t)(\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}})x(t) + 2x^{\mathrm{T}}(t) f(t, x(t), x(t - r_1(t)), \ldots, x(t - r_m(t))) \\ \leq& x^{\mathrm{T}}(t)(\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}})x(t) + \gamma x^{\mathrm{T}}(t)x(t) + \gamma^{-1} ||f(t, x(t), x(t - r_1(t)), \ldots, x(t - r_m(t)))||^2. \end{aligned} \tag{6}$$

For all $x \in S(\rho_1)$, $0 < \rho_1 \leq \rho$ and $k \in \mathbb{N}$,

$$\begin{aligned} V(t_k, x(t_k)) =& x^{\mathrm{T}}(t_k^-)(\boldsymbol{I} + \boldsymbol{C}_k)^{\mathrm{T}}(\boldsymbol{I} + \boldsymbol{C}_k)x(t_k^-) \\ \leq& ||\boldsymbol{I} + \boldsymbol{C}_k||^2 V(t_k^-, x(t_k^-)) \\ \leq& \omega^2 V(t_k^-, x(t_k^-)). \end{aligned}$$

Let $\psi(s) = \omega^2 s$. It follows from inequality (5) that $0 < \omega < 1$ holds. Then we have $\psi \in \mathcal{K}^*$. For any solution of system (4), if

$$V(t+s, x(t+s)) \leq \psi^{-1}(V(t, x(t))), \ -\tau \leq s \leq 0, \tag{7}$$

then we have

$$V(t - r_i(t), x(t - r_i(t))) \leq \psi^{-1}(V(t, x(t))), \tag{8}$$

i.e.,

$$x^{\mathrm{T}}(t - r_i(t))x(t - r_i(t)) \leq \omega^{-2} x^{\mathrm{T}}(t)x(t), \tag{9}$$

or

$$||x(t - r_i(t))|| \leq \omega^{-1} ||x(t)||. \tag{10}$$

From inequality (10), we obtain

$$\begin{aligned} & ||f(t, x(t), x(t - r_1(t)), \ldots, x(t - r_m(t)))|| \\ \leq& K||x(t)|| + \sum_{i=1}^{m} L_i ||x(t - r_i(t))|| \\ \leq& \left(K + \omega^{-1} \sum_{i=1}^{m} L_i\right) ||x(t)|| \\ =& \omega_0 ||x(t)||. \end{aligned} \tag{11}$$

Substituting inequality (11) into inequality (6) yields

$$\begin{aligned} & D^+ V(t, x(t)) \\ \leq& x^{\mathrm{T}}(t)(\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}})x(t) + \gamma x^{\mathrm{T}}(t)x(t) + \gamma^{-1}\omega_0^2 ||x(t)||^2 \\ \leq& \lambda_0 V(t, x(t)). \end{aligned} \tag{12}$$

Let $g(s) = 1$ and $H(s) = \lambda_0 s$. Condition (iv) of Lemma 1 implies that

$$\begin{aligned}&\int_{\psi(\mu)}^{\mu}\frac{\mathrm{d}u}{H(u)}-\int_{t_{k-1}}^{t_k}g(s)\mathrm{d}s\\&=\int_{\omega^2\mu}^{\mu}\frac{\mathrm{d}u}{\lambda_0 u}-(t_k-t_{k-1})\\&=-\frac{2}{\lambda_0}\ln\omega-(t_k-t_{k-1})\\&=-\frac{1}{\lambda_0}\left[2\ln\omega+\lambda_0(t_k-t_{k-1})\right]>0.\end{aligned}\tag{13}$$

It is easy to see that the conditions of Lemma 1 are all satisfied. Therefore, we may conclude that the origin of system (4) is uniformly asymptotically stable. This completes the proof.

In practice, for convenience, the gain $\boldsymbol{B}_k$ is always selected as a constant matrix and the inter impulse interval $\Delta_k = t_k - t_{k-1}(k \in \mathbb{N})$ is set to be a positive constant, then we have the following corollary.

Corollary 1 Assume that $\Delta_k = t_k - t_{k-1} = \Delta$ and $\boldsymbol{B}_k = \boldsymbol{B}(k \in \mathbb{N})$. The origin of system (4) is uniformly asymptotically stable if there exists a constant $\gamma > 0$ such that

$$2\ln\omega + \lambda_0\Delta < 0,\tag{14}$$

where $\lambda_0 = ||\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}} + \gamma\boldsymbol{I} + \gamma^{-1}\omega_0^2\boldsymbol{I}|| > 0$, $\omega_0 = K + \omega^{-1}\sum_{i=1}^{m} L_i$, and $\omega = ||\boldsymbol{I} + \boldsymbol{BC}||$.

Remark 1 For the same impulsive control scheme in the present paper, in Ref.[16] the impulsive-type comparison principles[2,3] were used and a series of different sufficient conditions was presented. Unfortunately, in Ref.[16], the referential control method (i.e. the impulsive-type comparison principles for the case without time delay) is unsupported for the case with time-varying delays and the sufficient conditions have some restrictions, which can be summarized as follow:

(i) The comparison principles, first introduced in Refs.[2, 3], are applicable just to the impulsive control scheme of nonlinear systems without time delay. For the case with time delay (constant or time-varying delay) studied in Ref.[16], the above results cannot be used directly and new analytical method needs developing.

(ii) From expression (13) of Ref.[16], the time delay term is estimated to be $||(t - r_i(t))|| \leq M_i||(t)||$. However, positive constant M_i is hard to identify in practice, and the corresponding selection rule cannot be found in the whole paper.

(iii) In Ref.[16], the time delay need satisfy $\tau \leq t_{k+1} - t_k$, $k \in \mathbb{N}$. However, Fig.2(b) in Ref.[16] shows the trajectory of $x(t)$ via impulsive control, from which it follows that the time delay is much larger than the inter impulse interval ($\tau = 17 >> t_{k+1} - t_k \approx 0.005$), which contradicts the result of Theorem 1 in Ref.[16].

(iv) In Theorem 1 of Ref.[16], the matrix $\boldsymbol{BC}$ need be symmetric.

Compared with the results in Ref.[16], in the present paper an improved impulsive control method is presented and the restrictions mentioned above are removed. Correspondingly, the improved points in the present paper can be summarized as follow:

(i) Lemma 1 (the so-called Lyapunov-like stability theorem) plays an important role for asymptotic stability of impulsive FDEs, which provides a powerful mathematical tool to deal with the impulsive control scheme studied in the present paper.

(ii) From inequalities (7)–(10), the time delay term can be estimated, which is strict in theory and easy to execute.

(iii) In the present paper, the restriction on time delay, i.e. $\tau \leq t_{k+1} - t_k$, is removed.

(iv) In the present paper, the restriction on symmetry of matrix $\boldsymbol{BC}$ is removed.

4. Illustrative example

In this section, we will give two examples to demonstrate the effectiveness and the feasibility of the proposed method.

Example 1 Consider the following Mackey–Glass model:[15,16]

$$\begin{aligned}&\dot{x}(t) = -ax(t) + \frac{bx(t-\tau)}{1 + x^c(t-\tau)},\\&y(t) = x(t),\ t \geq 0.\end{aligned}\tag{15}$$

When $a = 0.1$, $b = 0.2$, $c = 10$, and $\tau = 17$, the model turns into a commonly known chaotic system and the origin is unstable. Figure 1 shows its chaotic behaviour. From the given parameters in Eq.(15), we can obtain $m = 1$, $\boldsymbol{A} = -0.1$, $\boldsymbol{C} = 1$, $K = 0$, and $L_1 = 0.2$. We choose $\boldsymbol{B} = -1.5$, then $\omega = 0.5$. If we further choose $\gamma = 0.4$, then we have $\lambda_0 = 0.6$. Hence, from inequality (14), the inter impulse interval is estimated to be

$$\Delta < \frac{-2\ln\omega}{\lambda_0} = 2.3105.$$

Using these parameters, conditions in Corollary 1 are satisfied for $\Delta < 2.3105$. Let the initial value of x_0 be 1.8, i.e. $x_0 = 1.8$. Impulsive control of Mackey–Glass model with inter impulse interval $\Delta = 2$ is shown in Fig.2. As can be seen, the origin of Mackey–Glass model is uniformly asymptotically stable.

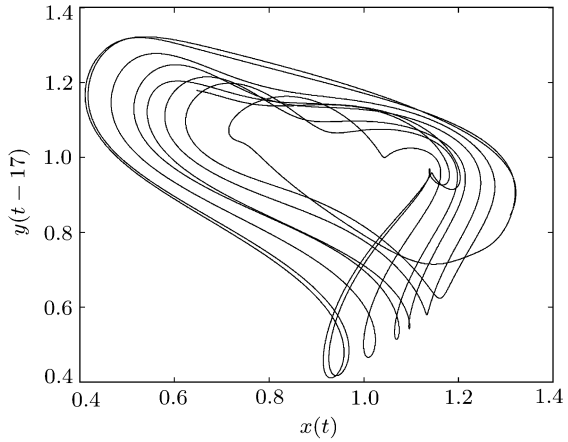


Fig.1. Chaotic attractor of Mackey–Glass model (15).

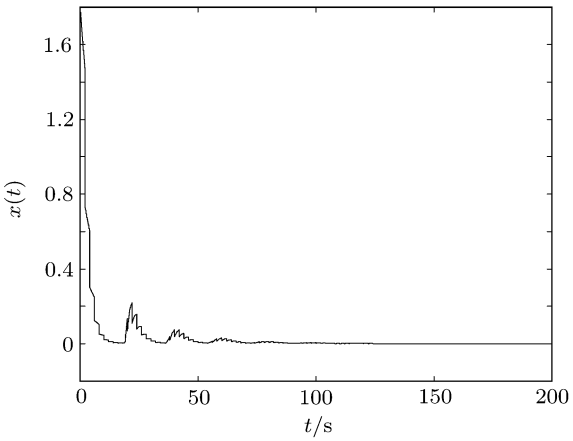


Fig.2. Impulsive time response of the Mackey–Glass model with $\boldsymbol{B} = -1.5$ and $\Delta = 2$.

Example 2 Consider the following chaotic delayed neural networks (DNNs):[26]

$$\begin{aligned}\dot{x}_i(t) =& - w_i x_i(t) - \sum_{j=1}^{n} \hat{w}_{ij} f_j(x_j(t)) \\ & - \sum_{j=1}^{n} \tilde{w}_{ij} f_j(x_j(t-\tau)), \\ & i = 1, 2, \ldots, n, \end{aligned} \tag{16}$$

or the compact form

$$\dot{x}(t) = -\boldsymbol{W}x(t) - \hat{\boldsymbol{W}} f(x(t)) - \tilde{\boldsymbol{W}} f(x(t-\tau)), \tag{17}$$

where $\tau = 1$, $f_i(x_i) = (|x_i + 1| - |x_i - 1|)/2$, $i, j = 1, 2$,

$$\boldsymbol{W} = \mathrm{diag}\{w_1, w_2\} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\hat{\boldsymbol{W}} = (\hat{w}_{ij})_{2\times 2} = \begin{bmatrix} 1+\pi/4 & 20 \\ 0.1 & 1+\pi/4 \end{bmatrix},$$

$$\tilde{\boldsymbol{W}} = (\tilde{w}_{ij})_{2\times 2} = \begin{bmatrix} -1.3\sqrt{2}\pi/4 & 0.1 \\ 0.1 & -1.3\sqrt{2}\pi/4 \end{bmatrix}.$$

As shown in Fig.3, system (16) possesses a chaotic behaviour. Let the output $y(t) = x(t)$ and from the above parameters, we have $m = 1$, $\boldsymbol{A} = -\boldsymbol{I}$, $\boldsymbol{C} = \boldsymbol{I}$, $K = 20.1589$, and $L_1 = 1.5439$. We choose the following impulsive control gain matrix:

$$\boldsymbol{B} = \begin{bmatrix} -1.1 & 0.1 \\ 0.2 & -1.1 \end{bmatrix},$$

then $\omega = 0.2618$. Obviously, matrix $\boldsymbol{BC}$ is not symmetric. If we further choose $\gamma = 26$, then we have $\lambda_0 = 50.1123$. Hence, from inequality (14), the inter impulse interval is estimated to be

$$\Delta < \frac{-2\ln\omega}{\lambda_0} = 0.0535.$$

Using these parameters, conditions in Corollary 1 are satisfied for $\Delta < 0.0535$. Let the initial value of x_0 be $[2,\ 1]^{\mathrm{T}}$, i.e. $x_0 = [2,\ 1]^{\mathrm{T}}$.

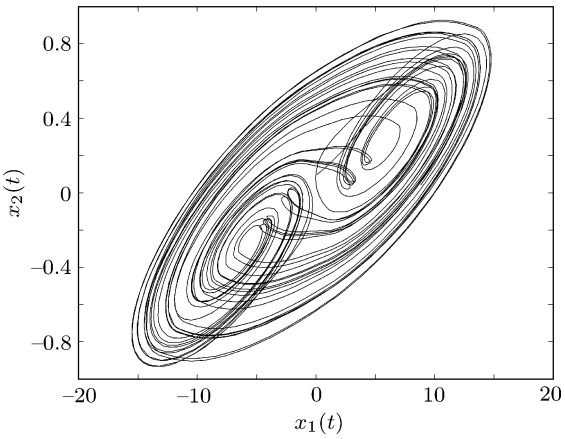


Fig.3. Chaotic attractor of chaotic DNNs (16).

Impulsive control of chaotic DNNs with inter impulse interval $\Delta = 0.05$ is shown in Fig.4. As seen in the figure, the origin of chaotic DNNs is uniformly asymptotically stable.

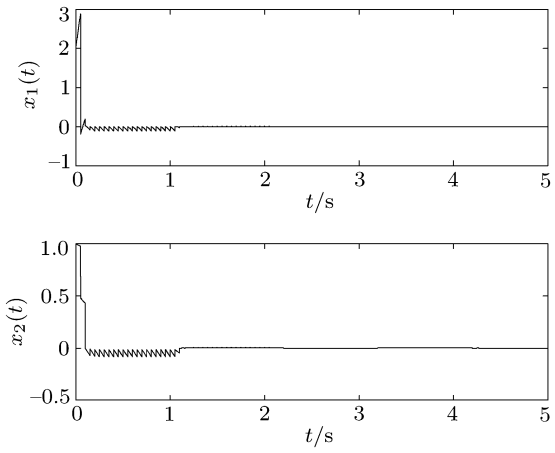


Fig.4. Impulsive time response of chaotic DNNs (16) with $\Delta = 0.05$.

5. Conclusions

In this paper, we investigate the scheme for impulsive control of nonlinear systems with time-varying delays. Based on the Lyapunov-like stability theorem for impulsive FDEs, some new sufficient conditions are established. Furthermore, a comparison between theoretical results obtained in the present study and the existing results shows that our results are more effective and less conservative. It should be pointed out that the methods of impulsively controlled nonlinear delayed systems with stochastic and impulsive effects are far from being straightforward due to the difficulty in investigating the stability of stochastic impulsive FDEs. Moreover, the conditions we give are only sufficient ones, and less conservative conditions still need developing. These issues are interesting and important, and continued research is desirable.

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