Vol 18 No 3, March 2009
1674-1056/2009/18(03)/0952-06

Chinese Physics B

© 2009 Chin. Phys. Soc.
and IOP Publishing Ltd

Stochastic resonance in a single-mode laser driven by frequency modulated signal and coloured noises*

Jin Guo-Xiang(金国祥)[a)†], Zhang Liang-Ying(张良英)[b)], and Cao Li(曹 力)[c)]

[a)] School of Computer Science and Engineering, Wuhan Institute of Technology, Wuhan 430073, China
[b)] Department of Physics & Electronics Information Technology, Xiangfan University, Xiangfan 441053, China
[c)] State Key Laboratory of Laser Technology, Huazhong University of Science and Technology, Wuhan 430074, China

(Received 3 July 2008; revised manuscript received 19 August 2008)

By adding frequency modulated signals to the intensity equation of gain–noise model of the single-mode laser driven by two coloured noises which are correlated, this paper uses the linear approximation method to calculate the power spectrum and signal-to-noise ratio (SNR) of the laser intensity. The results show that the SNR appears typical stochastic resonance with the variation of intensity of the pump noise and quantum noise. As the amplitude of a modulated signal has effects on the SNR, it shows suppression, monotone increasing, stochastic resonance, and multiple stochastic resonance with the variation of the frequency of a carrier signal and modulated signal.

Keywords: Gaussian coloured noise, single-mode laser, stochastic resonance, frequency modulated signal

PACC: 0540, 4260K

1. Introduction

In the last twenty years, stochastic resonance (SR) has been an object of great scientific interest. As one of the most pioneering studies in nonlinear problems, the study of SR has achieved great advances both theoretically and experimentally.[1−12] Most of the former studies have been focused on single-frequency signals, but signals in practice are mostly modulated signals, hence studying SR of modulated signals has a variety of practical applications. For example, low frequency signal was produced by the difference frequency component of amplitude modulated signal, and weak periodic signals were detected from strong noises, which were in a wide frequency range in Ref.[13]. The base-band signal needs to be modulated into a form which is more suitable to transform in channels in most communication systems, so modulated signals are often used in communications, as it has a powerful resistance to interference, it is widely used in occasions which require high quality. Therefore, the study of SR of modulated signals is a matter of significant realistic meaning. Laser communication, in which we use laser as a tool for transforming information by loading signals onto the source of the laser, has a promising future. The study of the SR of single-mode laser, in which single-frequency signal was replaced by modulated signals, is shown in Ref.[14, 15]. Reference [16] focuses on SR when the input signal is frequency modulated signal and two white noises are related in the form of δ function. In this paper, we report SR of a single-mode laser for gain–noise model when two coloured noises are related in the form of exponential function. The result shows that not only typical SR occurs when signal-to-noise ratio (SNR) changes as noise intensity changes, generalized SR also occurs when SNR changes as carrier signal frequency and modulated signals frequency change. Generalized SR is greatly influenced by amplitude of the modulated signals. The curve of SNR versus signal frequency has results of suppression, monotone increasing, stochastic resonance, multiple stochastic resonance, as amplitude of the modulated signals increases.

*Project supported by the Key Project Scientific Research Foundation from the Education Department of Hubei Province of China (Grant No D200725001).
†E-mail: gxjinwitedu@yahoo.com.cn

2. Intensity correlation function in gain–noise model of single-mode laser with frequency modulated wave

The light intensity equation for gain–noise model of single-mode laser[17] with frequency modulated wave is given by

$$\frac{\mathrm{d}I}{\mathrm{d}t} = -2K\,I + \frac{2\Gamma}{1+\beta\,I}I + D + \frac{2I}{1+\beta\,I}\xi(t) + 2\sqrt{I}\eta(t) + A\cos(\omega\,t + \beta_{\mathrm{FM}}\sin(\Omega\,t)). \quad (1)$$

The statistical properties of the two Gaussian noises $\xi(t)$ and $\eta(t)$ are as follows:

$$\langle\xi(t)\rangle = \langle\eta(t)\rangle = 0,$$
$$\langle\xi(t)\xi(s)\rangle = \frac{Q}{2\tau_1}\exp\left(-\frac{|t-s|}{\tau_1}\right),$$
$$\langle\eta(t)\eta(s)\rangle = \frac{D}{2\tau_2}\exp\left(-\frac{|t-s|}{\tau_2}\right),$$
$$\langle\xi(s)\eta(t)\rangle = \langle\xi(t)\eta(s))\rangle = \frac{\lambda\sqrt{DQ}}{2\tau_3}\exp\left(-\frac{|t-s|}{\tau_3}\right), \quad (-1 \le \lambda \le 1).$$

In Eq.(1) I is the laser intensity, λ represents the noise correlation coefficient, Q and D are the intensities of the pump noise and quantum noise respectively, τ_1 is the auto-correlation time for pump noise, τ_2 for quantum noise, τ_3 is the cross-correlation time for $\xi(t)$ and $\eta(t)$, in $\beta = \dfrac{\tilde{A}}{\Gamma}$, $\tilde{A}$ and Γ represent the self-saturation and gain coefficient, K for loss coefficient, A and β_{FM} are the amplitudes of the periodical signal and the modulated signal respectively, ω and Ω are frequencies of the periodical signal and the modulated signal respectively.

Linearize Eq.(1) around the deterministic steady-state intensity $I_0 = \dfrac{\Gamma - K}{\beta K}$, and let $I = I_0 + \varepsilon(t)$. Substitute the above equation into Eq.(1), we can obtain the linearized equation:

$$\frac{\mathrm{d}\varepsilon(t)}{\mathrm{d}t} = -\gamma\varepsilon(t) + D + \frac{2I_0}{1+\beta\,I_0}\xi(t) + 2\sqrt{I_0}\eta(t) + A\cos(\omega\,t + \beta_{\mathrm{FM}}\sin(\Omega\,t)), \quad (2)$$

where $\varepsilon(t)$ is the perturbation term, $\gamma = 2K(\Gamma - K)/\Gamma$.

The frequency modulated signal can be expanded into a series:

$$A\cos(\omega\,t + \beta_{\mathrm{FM}}\sin(\Omega\,t)) = A\sum_{n=-\infty}^{\infty} J_n(\beta_{\mathrm{FM}})\cos(\omega + n\Omega)\,t,$$

where $J_n(\beta_{\mathrm{FM}})$ is the nth order of the first kind Bessel function

$$J_n(\beta_{\mathrm{FM}}) = \sum_{m=0}^{\infty}\frac{(-1)^m\left(\dfrac{1}{2}\beta_{\mathrm{FM}}\right)^{n+2m}}{m!(n+m)!}.$$

According to the definition of average laser intensity correlation function:

$$C(\tau) = \lim_{t\to\infty}\frac{\Omega}{2\pi}\int_t^{t+\frac{2\pi}{\Omega}}\langle I(\zeta+\tau)I(\zeta)\rangle\,\mathrm{d}\zeta,$$

we can obtain the average laser intensity correlation function. When it is transformed by the Fourier transform, we obtain the power spectrum as:

$$S(\omega') = S_1(\omega') + S_2(\omega'), \quad (3)$$

where $S_1(\omega')$ and $S_2(\omega')$ are the output power spectrum of the signal and the noise, respectively. The specific expressions are

$$S_1(\omega') = \frac{\pi A^2}{2}\sum_{n=-\infty}^{\infty}\frac{J_n^2(\beta_{\mathrm{FM}})}{\gamma^2 + (\omega+n\Omega)^2}[\delta(\omega' - (\omega+n\Omega)) + \delta(\omega' + (\omega+n\Omega))],$$
$$S_2(\omega') = \left[\frac{4I_0^2Q}{(1+\beta I_0)^2(\omega'^2\tau_1^2+1)} + \frac{8I_0^{3/2}\lambda\sqrt{DQ}}{(1+\beta I_0)(\omega'^2\tau_3^2+1)} + \frac{4I_0D}{\omega'^2\tau_2^2+1}\right]\frac{1}{(\gamma^2+\omega'^2)}.$$

$S_1(\omega')$ includes two delta functions, we only take the positive spectrum to indicate the power spectrum of output signals:

$$S_1(\omega') = \frac{\pi A^2}{2}\sum_{n=-\infty}^{\infty}\frac{J_n^2(\beta_{\mathrm{FM}})}{\gamma^2 + (\omega+n\Omega)^2}\delta(\omega' - (\omega+n\Omega)).$$

The unit noise power for each frequency in output signal power spectrum is

$$S_2(\omega+n\Omega) = \frac{1}{\gamma^2+(\omega+n\Omega)^2}\left[\frac{4I_0^2Q}{(1+\beta\, I_0)^2((\omega+n\Omega)^2\tau_1^2+1)}+\frac{8I_0^{3/2}\lambda\sqrt{DQ}}{(1+\beta I_0)((\omega+n\Omega)^2\tau_3^2+1)}+\frac{4I_0D}{(\omega+n\Omega)^2\tau_2^2+1}\right].$$

The overall output signal power is

$$P_s=\int_0^\infty S_1(\omega')\mathrm{d}\omega', \tag{4}$$

SNR is defined as the ratio of the output overall power of the signal and the sum of unit noise powers for each frequency in output signal:

$$R=\frac{P_{\rm S}}{\sum\limits_{n=-\infty}^{\infty}S_2(\omega+n\Omega)}. \tag{5}$$

Where

$$P_{\rm S}=\frac{\pi A^2}{2}\sum_{n=-\infty}^{\infty}\frac{J_n^2(\beta_{\rm FM})}{\gamma^2+(\omega+n\Omega)^2},$$

$$\sum_{n=-\infty}^{\infty}S_2(\omega+n\Omega) = \sum_{n=-\infty}^{\infty}\frac{1}{\gamma^2+(\omega+n\Omega)^2}\times\left[\frac{4I_0^2Q}{(1+\beta\, I_0)^2((\omega+n\Omega)^2\tau_1^2+1)}+\frac{8I_0^{3/2}\lambda\sqrt{DQ}}{(1+\beta\, I_0)((\omega+n\Omega)^2\tau_3^2+1)}+\frac{4I_0D}{(\omega+n\Omega)^2\tau_2^2+1}\right].$$

3. SR of the output laser intensity

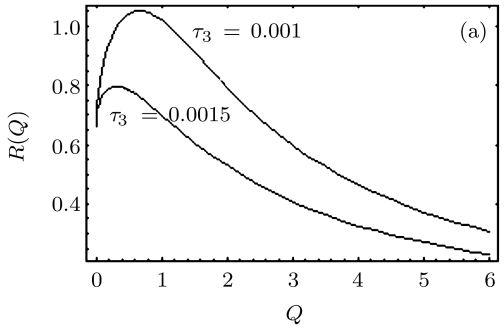
There are many frequency component in the output signal. $J_n^2(\beta_{\rm FM})$ drops rapidly as n increases, so we can take out finite spectrum from the approximately calculated modulated signal. The spectrum of frequency modulated signals is the same as the spectrum of amplitude modulated signals when the value of $\beta_{\rm FM}$ is small. We will discuss the SR phenomenon of frequency modulated signals when $\beta_{\rm FM}\geq 1$ in succession.

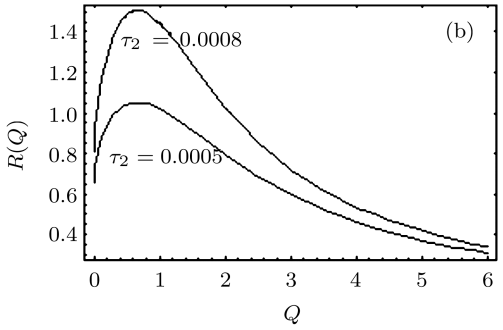
3.1. SNR changes as the intensities of pump noise Q and quantum noise D

According to Eq.(5) and taking τ_3 as the parameter, when τ_3 is larger than τ_2 and τ_1, we can obtain $R-Q$ curve as shown in Fig.1(a). From it, we can see that SR appears in SNR as the varying of the intensity of pump noise Q. With the increase of τ_3, the position of the peak turns left and goes down.

According to Eq.(5) and taking τ_2 as the parameter, when τ_3 is larger than τ_2 and τ_1, we can get $R-Q$ curve as shown in Fig.1(b). From it, we can see that SR appears in SNR as the varying of the pump noise intensity Q. With the increase of τ_2, the position of the peak goes higher.

According to Eq.(5) and taking τ_1 as the parameter, when τ_3 is larger than τ_2 and τ_1, we can get $R-Q$ curve as shown in Fig.1(c). From it, we can see that, with the increase of τ_1, the position of the peak turns right and goes higher.





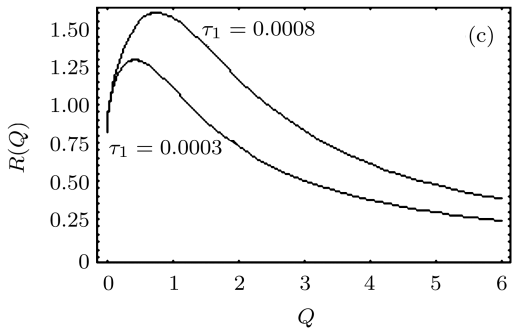


Fig.1. SNR varies as the intensity of the pump noise. ($\beta = 1, I_0 = 1, \gamma = 400, \Omega = 30, \omega = 800, \beta_{\rm FM} = 1, n = 3, \lambda = -0.8, D = 0.4, A = 2$ (a) $\tau_1 = 0.0007, \tau_2 = 0.0005$; (b) $\tau_3 = 0.001, \tau_1 = 0.0007$; (c)$\tau_3 = 0.001, \tau_2 = 0.0008$).

The curve of SNR versus D is similar to the curve of SNR versus Q and is omitted here.

3.2. SNR varies as the frequencies of carrier signal ω and modulated signal Ω

When $\beta_{\rm FM}$ is 1 and n is 3 and taking Ω as the parameter, we can obtain the curve of $R-\omega$ as shown in Fig.2(a). From it, we can see that there is only a minimum when the value of Ω is smaller. When Ω is larger, the minimum of the curve turns right and widens to a deep valley, then the giant suppression appears, and there is a small peak at the low valley.

When $\beta_{\rm FM}$ is 1.6 and n is 4 and taking Ω as the parameter, we obtain the curve of $R-\omega$ as in Fig.2(b). There is only a minimum when the value of Ω is smaller, a peak and a deep valley appear in the curve as the value of Ω is big, and two small peaks exist at the peak and the deep valley, respectively.

Taking Ω as the parameter, Fig.2(c) shows the curve of $R-\omega$ when $\beta_{\rm FM}$ is 2.2 and n is 5. There are a maximum and a minimum in it when the value of Ω is small. When the value of Ω is larger the curve has a maximum and a deep valley, and two small peaks exist at the maximum and the deep valley, respectively.

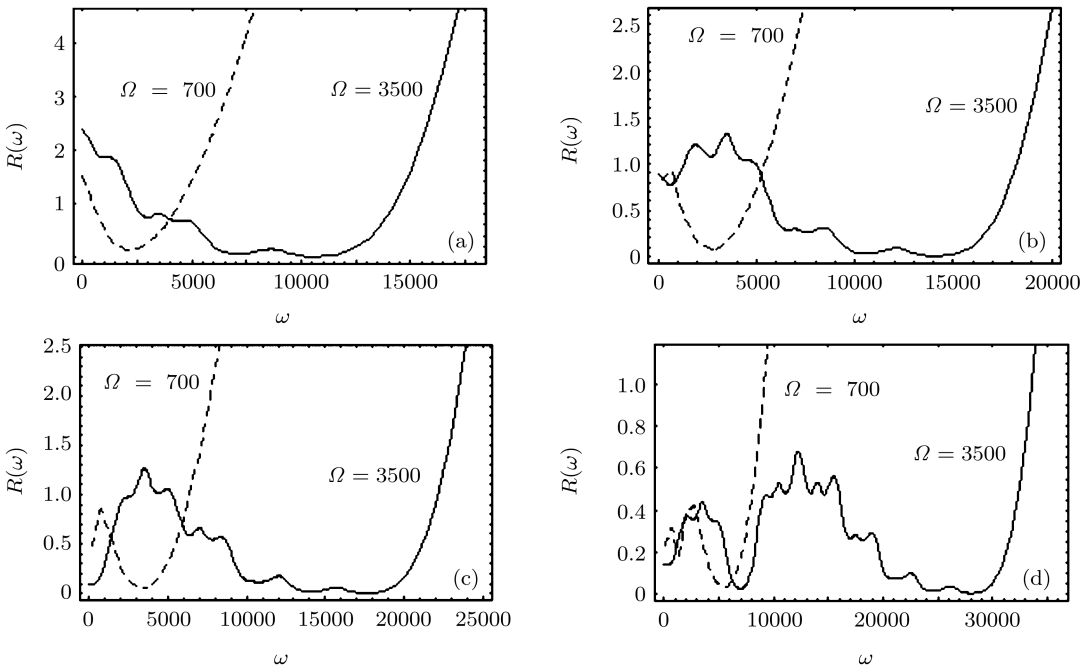


Fig.2. SNR varies as the frequency of carrier signal ($\beta = 1, I_0 = 1, A = 2, \lambda = -0.8, Q = 3, D = 0.5, \gamma = 400, \tau_1 = 0.0007, \tau_2 = 0.0005, \tau_3 = 0.001$). (a) $n = 3, \beta_{\rm FM} = 1.0$; (b) $n = 4, \beta_{\rm FM} = 1.6$; (c) $n = 5, \beta_{\rm FM} = 2.2$; (d) $n = 8, \beta_{\rm FM} = 5.0$.

Taking Ω as the parameter, Fig.2(d) shows the curve of $R-\omega$ when $\beta_{\rm FM}$ is 5 and n is 8. From the picture, it can be seen that two large peaks and two deep valleys are in the curves. When the value of Ω is larger at both the second large peak and deep valley, there are several small peaks. That is, the multiple SR appears.

From Fig.2 we see that, in brief, the $R-\omega$ curve is greatly affected by $\beta_{\rm FM}$, and there is only a minimum when $\beta_{\rm FM}$ is small, as the value of $\beta_{\rm FM}$ increases, the curve shows a maximum and a minimum, and when $\beta_{\rm FM}$ increases further, the multiple SR appears.

When $\beta_{\rm FM}$ is 1 and n is 3 and taking ω as the parameter, we obtain the curve of $R-\Omega$ as shown in Fig.3(a). From it, we can see that when the value of ω is small, the curve of $R-\Omega$ monotonically rises and then tends to a definite value. When the value of ω is larger the $R-\Omega$ curve presents a deep valley, and a small peak at the valley.

When $\beta_{\rm FM}$ is 1.6 and n is 4 and taking ω as the parameter, we obtain the curve of $R-\Omega$ as shown in Fig.3(b). When the value of ω is small, the curve of $R-\Omega$ monotonically rises and then tends to a definite value, as ω is larger, the curve first presents a deep valley and then a peak, at last a big SR peak appears.

Taking ω as the parameter, Fig.3(c) shows the curve of $R-\Omega$ when $\beta_{\rm FM}$ is 2.2 and n is 5. There is a SR peak in the curve when ω is small, while ω is larger, the curve first presents a deep valley and then presents a peak, at last a big SR peak appears.

Taking ω as the parameter, Fig.3(d) shows the curve of $R-\Omega$ when $\beta_{\rm FM}$ is 5 and n is 8. There is also a SR peak in the curve when ω is small, as ω is larger, the curve presents multiple SR.

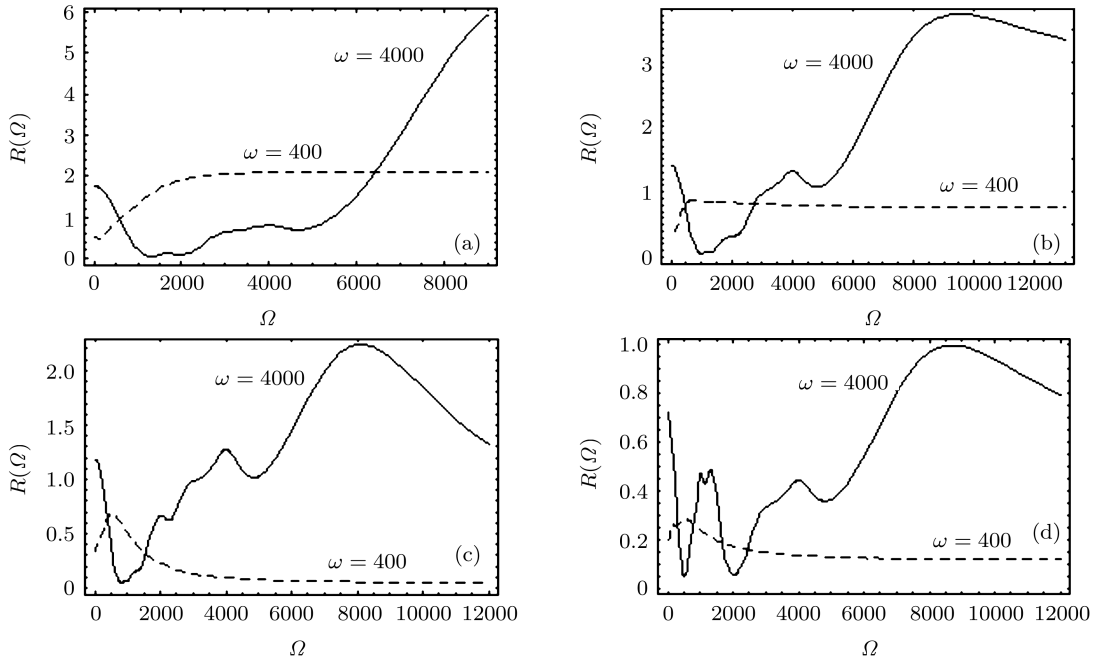


Fig.3. SNR varies as the frequency of modulated signal ($\beta=1$, $I_0=1$, $A=2$, $\lambda=-0.8$, $Q=3$, $D=0.5$, $\gamma=400$, $\tau_1=0.0007$, $\tau_2=0.0005$, $\tau_3=0.001$). (a) $n=3$, $\beta_{\rm FM}=1.0$; (b) $n=4$, $\beta_{\rm FM}=1.6$; (c) $n=5$, $\beta_{\rm FM}=2.2$; (d) $n=8$, $\beta_{\rm FM}=5.0$.

In Fig.3 the $R-\Omega$ curve is greatly affected by $\beta_{\rm FM}$ and ω. When the values of ω and $\beta_{\rm FM}$ are small, the curve monotonically rises and then tends to a definite value, while the value of $\beta_{\rm FM}$ is increasing, the curve has a maximum, at the same time if the ω is increased, the curve has a deep valley and a peak, when $\beta_{\rm FM}$ and ω are both bigger, multiple SR appears in the curve.

As generalized SR is greatly affected by the amplitude of modulated signal $\beta_{\rm FM}$, it is specially discussed: from Figs.2 and 3 we can see that when $\beta_{\rm FM}$ is small, neither the curves of $R-\omega$ nor $R-\Omega$ present generalized SR, as the value of $\beta_{\rm FM}$ is increasing, both of the two curves present generalized SR, when $\beta_{\rm FM}$ increases further, multiple SR occurs in both the two curves. It shows that when the amplitude of modulated signal $\beta_{\rm FM}$ is larger, the nonlinear phenomenon is more sensible. The value of the peak of generalized

SR decreases as $\beta_{\rm FM}$ increases.

4. Conclusion

We have three conclusions: first, when the input signal is frequency modulated and the value of τ_3 is larger than that of τ_2 and τ_1, typical SR occurs in the curves of SNR versus Q and SNR versus D; secondly, the curve of SNR changes with the frequencies of carrier signal and modulated signal is greatly influenced by the amplitude of modulated signal $\beta_{\rm FM}$, when $\beta_{\rm FM}$ is small, there is no generalized SR, and generalized SR exists when $\beta_{\rm FM}$ is big enough; thirdly, when both $\beta_{\rm FM}$ and ω are big enough, multiple SR occurs in the $R-\Omega$ curve, and when both $\beta_{\rm FM}$ and Ω are big enough, multiple SR occurs in the $R-\omega$ curve.

References

[1] Zhang L Y, Cao L, Wu D J and Wang J 2003 *Chin. Phys. Lett.* **20** 25

[2] Zhu H J, Li R and Wen X D 2003 *Acta Phys. Sin.* **52** 2404 (in Chinese)

[3] Chen D Y and Wang Z L 2008 *Acta Phys. Sin.* **57** 3333 (in Chinese)

[4] Kang Y M, Xu J X and Xie Y 2003 *Acta Phys. Sin.* **52** 2712 (in Chinese)

[5] Li J H, Huang Z Q and Wang C Y 1998 *Acta Phys. Sin.* **47** 382 (in Chinese)

[6] Luo X Q and Zhu S Q 2004 *Chin. Phys.* **13** 1201

[7] Lin M, Fang L M and Zhu R G 2008 *Acta Phys. Sin.* **57** 2638 (in Chinese)

[8] Barykin A V and Seki K 1998 *Phys. Rev.* E **57** 6555

[9] Berdichevsky V and Gitterman M 1999 *Phys. Rev.* E **60** 1494

[10] Li J L and Xu B H 2006 *Chin. Phys.* **15** 2867

[11] Zhou B C and Xu W 2007 *Acta Phys. Sin.* **56** 5623 (in Chinese)

[12] Ning L J, Xu W and Yao M L 2008 *Chin. Phys.* B **17** 486

[13] Lin M and Huang Y M 2006 *Proc. CSEE.* **26** 128 (in Chinese)

[14] Zhang L Y, Cao L and Jin G X 2006 *Acta Phys. Sin.* **55** 6238 (in Chinese)

[15] Jin G X, Cao L and Zhang L Y 2007 *Acta Phys. Sin.* **56** 3739 (in Chinese)

[16] Zhang L Y, Jin G X and Cao L 2008 *Acta Phys. Sin.* **57** (in Chinese)

[17] Hernandez-Garcia E, Toral R and San Miguel M 1990 *Phys. Rev.* A **42** 6823