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Controlling the motion of solitons in BEC by a weakly periodic potential*

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By developing multiple-scale method combined with Wentzel–Kramer–Brillouin expansion, this paper analytically studies the modulating effect of weakly periodic potential on the dynamical properties of the Bose–Einstein condensates (BEC) trapped in harmonic magnetic traps. A black–grey soliton transition is observed in the BEC trapped in harmonic magnetic potential, due to the weakly periodic potential modulating effect. Meanwhile, it finds that with the slight increase of the weakly periodic potential strength, the velocity of the soliton decreases, while its width firstly decreases then increases, a minimum exists there. These results show that the amplitude, velocity, and width of matter solitons can be effectively managed by means of a weakly periodic potential.

Keywords: Bose–Einstein condensates, solitons, modulating effect
PACC: 0530J, 0290, 1110L

1. Introduction

Several spectacular experiments on the creations of dark solitons[1−4] in Bose–Einstein condensates (BEC) of trapped alkali-atom gases open an unprecedented possibility to study the nonlinear dynamics of such macroscopically excited Bose-condensed states. In the experiments, dark solitons were observed to have a density dip and a phase slip in one direction[1] when the interatomic interactions of the BEC trapped in a harmonic magnetic potential are repulsive. Subsequently, many dynamical characteristics of the BEC are extensively investigated.[5−18] In optical fibres, usually, the dark solitons are divided into two typologies.[19] One is black soliton, which is defined as a dark soliton with zero minimum light intensity at its centre on a stable continuous background. The other is grey soliton with a nonzero minimum value of light intensity. In the field of the atomic matter wave, the s-wave scattering length (which represents the interatomic interactions) can be controlled by utilizing the Feshbach resonance. When the scattering length of ^{87}Rb atoms varies from $110a_0$ to $140a_0$ (where a_0 is the Bohr radius), the minimum value of the dark soliton varies from zero to nonzero (the full details have been given in Ref.[20]). It means that black and monotonically solitons can be exchanged by varying the scattering length via the Feshbach resonance.[20]

With the development of the experimental technique, neutral atoms confined in an array of magnetic or optical traps offer a scalable system for the application in quantum computation and quantum information processing.[21] So far, BEC have been successfully loaded in optical lattices.[22−24] In these experiments, the optical lattice is created by the interference of laser beams. The lattice potential can be modulated from very low to very high strength. The study of BEC in optical lattices is a very active field of research, both from the theoretical and experimental sides.[25−30] For a repulsive interatomic interaction, it is shown that there occurs bright gap solitons or/and bright gap soliton trains when the condensates are confined in optical lattices.[21,31]

In this paper, we study the solitary excitation of the BEC trapped in a combined potential consisting of a periodic potential and a harmonic magnetic

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trap. It is shown that the motion of dark solitons can be controlled by means of weakly periodic potentials. The mechanism is realized in terms of cigar-shaped BEC confined in a harmonic magnetic potential, in the presence of an optical lattice. Under consideration of weakly periodic potential modulating effect, there exhibit dark solitons for the repulsive interactions, different from that of the BEC trapped in optical lattices. It is worthwhile to point out that a black soliton of the BEC in harmonic magnetic traps can be transformed to a grey soliton by means of a weakly periodic potential modulating effect.

2. Hydrodynamical model

In mean-field approximation, the full BEC dynamical properties are well described by the time-dependent Gross–Pitaevskii (GP) equation[32]

$$\mathrm{i}\hbar\frac{\partial \Psi}{\partial T}=\left[-\frac{\hbar^2}{2m}\nabla^2+V\left(X,Y,Z\right)+g\left|\Psi\right|^2\right]\Psi, \quad (1)$$

where $\Psi\left(X,Y,Z,T\right)$ is order parameter of condensate, $(Y,\ Z)$ and X are the directions of strong transverse confinement and axial lattice. $N=\int \mathrm{d}r\left|\Psi\right|^2$ is the number of atoms, and $g=4\pi\hbar^2 a_{\mathrm{s}}/m$ is interatomic interaction strength with the atomic mass m and the s-wave scattering length $a_{\mathrm{s}}(a_{\mathrm{s}}>0$ represents the repulsively interatomic interaction). The combined potential $V(X,\ Y,\ Z)$ of a periodic potential and a harmonic oscillator trap is

$$V\left(X,R^2\right)=E_0\cos\left(\frac{2\pi X}{d}\right)+\frac{1}{2}m\left(\omega_X^2X^2+\omega_\perp^2R^2\right),\quad \omega_X\ll\omega_\perp, \quad (2)$$

where $R^2=Y^2+Z^2$, E_0 is the lattice depth. $d=\lambda_{\mathrm{L}}/2$ is the lattice constant, λ_{L} is the wavelength of laser beams, ω_X and $\omega_\perp$ are frequencies of the magnetic trap in the axial (X) and transverse (Y and Z) directions, respectively.

For the solitary excitations of the one-dimensional (1D) geometry (i.e., in 'cigar-shaped' traps), we may set $\Psi\left(X,Y,Z,T\right)=\phi(X,T)G(R)$. Substituting it into Eq.(1), and considering the strong confinement in the transverse direction, we can well describe the spatial structure of function $G(R)$ by a solution of two-dimensional radial symmetric quantum harmonic-oscillator equation, i.e., $-\left[\hbar^2/(2m)\right]\nabla_\perp^2G-\hbar\omega_\perp G+(1/2)\,m\omega_\perp^2R^2G=0$. The ground-state solution has the form $G(R)=C\exp\left[-m\omega_\perp R^2/\left(2\hbar\right)\right]$, where $C=\sqrt{m\omega_\perp/\left(\pi\hbar\right)}$ can be found from the normalization condition $\int_{-\infty}^{\infty}\left|G\right|^2R\mathrm{d}R=1$. So, Eq.(1) is reduced into

$$\mathrm{i}\hbar\frac{\partial\phi}{\partial T}=\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial^2X}+E_0\cos\left(\frac{2\pi X}{\mathrm{d}}\right)+\hbar\omega_\perp+\frac{1}{2}m\omega_X^2X^2+g\left|\phi\right|^2\right]\phi. \quad (3)$$

In order to obtain effectively 1D GP equation, we have multiplied Eq.(3) by G^* and integrated the resulting equation with respect to the transverse coordinate to eliminate the dependence on transverse plane.[11,33]

For convenience, we introduce the dimensionless variables $t=T\left(\pi^2\hbar/md^2\right)$, $x=\pi X/d$, and $\phi\left(X,T\right)=\sqrt{\pi/\left(4a_sd^2\right)}\,\psi\left(x,t\right)\exp\left(-i\omega_\perp T\right)$. So the corresponding dimensionless GP equation is

$$\mathrm{i}\frac{\partial\psi}{\partial t}=-\frac{1}{2}\frac{\partial^2\psi}{\partial x^2}+V_0\cos\left(2x\right)\psi+\frac{1}{2}\lambda^2x^2+\left|\psi\right|^2\psi, \quad (4)$$

where $\lambda=\left[d/\left(a_0\pi\right)\right]^2\Omega$ with $\Omega=\omega_X/\omega_\perp$ and $a_0=\sqrt{\hbar/\left(m\omega_\perp\right)}$ (transverse harmonic-oscillator length), $V_0=E_0/E_{\mathrm{rec}}$, here $E_{\mathrm{rec}}=\left(\pi\hbar\right)^2/\left(md^2\right)$ is the lattice recoil energy. Expressing the wavefunction in terms of modulus and phase, we set $\psi\left(x,t\right)=A\left(x,t\right)\exp\left[-\mathrm{i}\mu t+\mathrm{i}\varphi\left(x,t\right)\right]$. Substituting it into Eq.(4), and then separating the real and imaginary parts we obtain hydrodynamical models, i.e.,

$$\frac{\partial A}{\partial t}+\frac{\partial A}{\partial x}\frac{\partial\varphi}{\partial x}+\frac{1}{2}A\frac{\partial^2\varphi}{\partial x^2}=0, \quad (5)$$

$$-\frac{1}{2}\frac{\partial^2A}{\partial x^2}+\left[\frac{1}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2-\mu+\frac{1}{2}\lambda^2x^2+\frac{\partial\varphi}{\partial t}+V_0\cos\left(2x\right)\right]A+A^3=0. \quad (6)$$

3. Linear stability condition of soliton formation

Due to the strong confinement in the transverse direction, the system is similar to a waveguide, in which the excitation propagates in the elongated direction.[11,12] Therefore, we set $A=u_0\left(x\right)+\alpha\left(x,t\right)$ (without loss of generality, we assume that $u_0\left(x\right)$ characterizes the condensate background) with $\alpha\left(x,t\right)=\alpha_0\exp\left(\mathrm{i}\beta\right)+\mathrm{c.c}$ and $\varphi=\varphi_0\exp\left(\mathrm{i}\beta\right)+c.c.$ Here, c.c is complex conjugate and $\beta=kx-\omega t$ where k is wave number and ω indicates eigen-frequency. Considering that α_0 and φ_0 are small constants, we obtain

$$\mathrm{i}\omega\alpha_0=\mathrm{i}k\varphi_0\frac{\partial u_0}{\partial x}-\frac{1}{2}k^2u_0\varphi_0, \quad (7)$$

$$\mu u_0 = -\frac{1}{2}\frac{\partial^2}{\partial x^2}u_0 + V_0\cos(2x)\,u_0 + \frac{1}{2}\lambda^2 x^2 u_0 + u_0^3, \quad (8)$$

$$\begin{aligned}\mu\alpha_0 =& \frac{1}{2}k^2\alpha_0 - \mathrm{i}\omega\varphi_0 u_0 + V_0\cos(2x)\,\alpha_0 \\ &+ \frac{1}{2}\lambda^2 x^2\alpha_0 + 3u_0^2\alpha_0 \end{aligned} \quad (9)$$

from the linearization of Eqs.(5) and (6). Based on the experimental parameters, the ratio Ω of the confinement strengths in the axial (X) to the transverse (Y and Z) direction varies from 0.1[22,23] to $1/\sqrt{2}$.[34,35] It implies that the λ is a small insignificant parameter. In the case of linear case, Eq.(8) is just Hill's equation[36]

$$\frac{\mathrm{d}^2 u_0}{\mathrm{d}x^2} + [2\mu - 2V_0\cos(2x)]\,u_0 = 0. \quad (10)$$

In the presence of weakly optical lattice, the lattice strength can be regarded as a smaller value. So, we may use Wentzel–Kramer–Brillouin (WKB) method[37] to set $\mu = \mu_0 + \mu_1 V_0 + \mu_2 V_0^2 + \ldots = \sum_{k=0}^{\infty}\mu_k V_0^k$, and $u_0 = u_{00} + u_{01}V_0 + u_{02}V_0^2 + \ldots = \sum_{k=0}^{\infty} u_{0k}V_0^k$. Substituting them into Eq.(10), and then separating each order in terms of the power of V_0, we obtain

$$\frac{\mathrm{d}^2 u_{00}}{\mathrm{d}x^2} + 2\mu_0 u_{00} = 0, \quad (11)$$

$$\frac{\mathrm{d}^2 u_{01}}{\mathrm{d}x^2} + 2\mu_0 u_{01} = 2u_{00}\cos(2x) - 2\mu_1 u_{00}. \quad (12)$$

For convenience, we here set $2\mu_0 = n^2$. From Eq.(11), we find that u_{00} is linear superposition of functions $\cos(nx)$ and $\sin(nx)$. For the sake of simplicity, we only consider the case of $u_{00} = \cos(nx)$. Inserting it into Eq.(12), we have

$$\frac{\mathrm{d}^2 u_{01}}{\mathrm{d}x^2} + n^2 u_{01} + 2\mu_1\cos(nx) = 2\cos(nx)\cos(2x). \quad (13)$$

Multiplying Eq.(13) by u_{00} and then integrating from $x = 0$ to $x = 2\pi$, we obtain that $\mu_1 = 0.5$ at $n = 1$, and $\mu_1 = 0$ at $n \neq 1$. Solving the above equations, we obtain $u_{01} = -(1/8)\cos(3x)$ at $n = 1$; $u_{01} = (1/4)\{[\cos(n-2)x]/(n-1) - [\cos(n+2)x]/(n+1)\}$ at $n \neq 1$. Finally, the solution of Hill's equation (10) is given by

$$u_0(x) = \begin{cases} \cos x - \dfrac{1}{8}V_0\cos(3x) + O\left(V_0^2\right), & (n=1), \\ \cos(nx) - \dfrac{V_0}{4}\left[\dfrac{\cos(n-2)x}{n-1} - \dfrac{\cos(n+2)x}{n+1}\right] + O\left(V_0^2\right), & (n\neq 1). \end{cases} \quad (14)$$

Utilizing Eqs.(7), (8) and (9), we obtain

$$\omega^2 = \left(\frac{1}{2u_0}\frac{\partial^2 u_0}{\partial x^2} + \frac{1}{2}k^2 + 2u_0^2\right)\left(\frac{1}{2}k^2 - \frac{\mathrm{i}k}{u_0}\frac{\partial u_0}{\partial x}\right). \quad (15)$$

To obtain the stable condition of the soliton formation, we set $\omega = \omega_\mathrm{r} + \mathrm{i}\omega_\mathrm{i}$, where ω_r and ω_i denote the real and imaginary parts, respectively. If $\omega_\mathrm{i} \neq 0$, it implies exponential growth of the mode and hence the state is dynamically instable.[33,38] If the eigenfrequency of the associated quasi-particle spectrum is real, it means that the soliton is stable.[33,38] From Eq.(15), we find that the stable condition of soliton formation satisfies $\frac{1}{u_0}\frac{\partial u_0}{\partial x} = 0$. Furthermore, in order to find out a general rule of the stable condition, we plot the value of function $\frac{1}{u_0}\frac{\partial u_0}{\partial x}$ with the lattice depth $V_0 = 0.01$ in Fig.1. In the case of $n = 1$ (see Fig.1(a)), one sees that the relation satisfying the stable condition of soliton formation is $x = m\pi$, where m is an integer. Meanwhile, from Figs.1(b) and 1(c), we

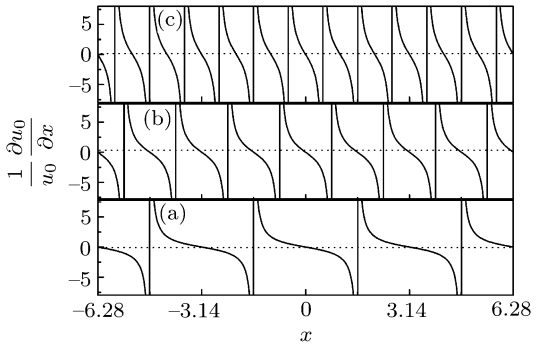


Fig.1. The value of function with the lattice strength $V_0 = 0.01$: (a) $n = 1$, (b) $n = 2$, and (c) $n = 3$. All the dotted lines denote the value of function being equal to zero.

find that the stable conditions of soliton formation are $x = \frac{m}{2}\pi$ and $x = \frac{m}{3}\pi$ when $n = 2$ and $n = 3$, respectively. So, we conclude that the general rule of the stable condition of soliton formation is $x = \frac{m}{n}\pi$. In what follows, unless otherwise stated, we will choose $x = 0$ as the initial state of matter wave to investigate the dynamical properties of solitary excitation.

4. Asymptotic expansion and VCKdV equation

Although an exact solution of nonlinear excitations of Eqs.(5) and (6) cannot be obtained straightforwardly, we may simplify the problem by considering the relative importance of the physical quantities appearing in the system. Then we can obtain an approximately analytical solution of the problem based on a perturbation expansion.[39,40] We here introduce a fast variable, $x_0 = x$, representing the direction of solitary excitation propagation, and two slow variables, $\xi = \varepsilon(x - V_g t)$ and $\tau = \varepsilon^3 t$, characterizing the slow variation of solitary excitation dynamics. Moreover, we assume that $A(x,t)$ and $\varphi(x,t)$ can be respectively expanded into a polynomial form, $A = u_0(x_0) + \varepsilon^2 a^{(0)}(\xi,\tau) + \varepsilon^4 a^{(1)}(\xi,\tau) + \ldots$ and $\varphi = \varepsilon\varphi^{(0)}(\xi,\tau) + \varepsilon^3\varphi^{(1)}(\xi,\tau) + \ldots$, where ε is a small parameter characterizing the relative amplitude of the solitary excitation. Inserting them into Eqs.(5) and (6), and then separating each order in terms of the power of ε, respectively, from Eq.(5), we have

$$V_g \frac{\partial a^{(0)}}{\partial \xi} - \frac{1}{2}u_0 \frac{\partial^2 \varphi^{(0)}}{\partial \xi^2} = 0, \tag{16}$$

$$\begin{aligned} & V_g \frac{\partial a^{(1)}}{\partial \xi} - \frac{1}{2}u_0 \frac{\partial^2 \varphi^{(1)}}{\partial \xi^2} \\ =& \frac{\partial a^{(0)}}{\partial \xi}\frac{\partial \varphi^{(0)}}{\partial \xi} + \frac{1}{2}a^{(0)}\frac{\partial^2 \varphi^{(0)}}{\partial \xi^2} + \frac{\partial a^{(0)}}{\partial \tau}, \end{aligned} \tag{17}$$

and from Eq.(6), we have

$$-\frac{1}{2}\frac{\partial^2 u_0}{\partial x_0^2} - \mu u_0 + \frac{1}{2}\lambda^2 x^2 u_0 + V_0 \cos(2x_0)\, u_0 + u_0^3 = 0, \tag{18}$$

$$\begin{aligned} & -\mu a^{(0)} + \frac{1}{2}\lambda^2 x^2 a^{(0)} + V_0 \cos(2x_0)\, a^{(0)} + 3u_0^2 a^{(0)} \\ =& V_g u_0 \frac{\partial \varphi^{(0)}}{\partial \xi}, \end{aligned} \tag{19}$$

$$\begin{aligned} & -\mu a^{(1)} + \frac{1}{2}\lambda^2 x^2 a^{(1)} + V_0 \cos(2x_0)\, a^{(1)} + 3u_0^2 a^{(1)} \\ =& \frac{1}{2}\frac{\partial^2 a^{(0)}}{\partial \xi^2} - \frac{1}{2}u_0 \left(\frac{\partial \varphi^{(0)}}{\partial \xi}\right)^2 - 3u_0 \left(a^{(0)}\right)^2 \\ & + V_g u_0 \frac{\partial \varphi^{(1)}}{\partial \xi} + V_g a^{(0)} \frac{\partial \varphi^{(0)}}{\partial \xi} - u_0 \frac{\partial \varphi^{(0)}}{\partial \tau}. \end{aligned} \tag{20}$$

Note that the form of Eq.(18) is the same as that of Eq.(8). In fact, BEC in the experiments are dilute and very weakly interacting: $n|a_s|^3 << 1$, where n is the average density of the condensate.[22−24] So, Eq.(18) can be transformed into Hill's equation. The solution of Hill's equation is given by Eq.(14). Subsequently, by comparing Eq.(16) with Eq.(19), we find that $V_g^2 = u_0^2 + \frac{1}{4u_0}\frac{\partial^2 u_0}{\partial x_0^2}$. Similarly, from Eqs.(17) and (20), we have

$$\frac{\partial \varphi^{(0)}}{\partial \tau} + \frac{3}{8}\left(1 + \frac{u_0^2}{V_g^2}\right)\left(\frac{\partial \varphi^{(0)}}{\partial \xi}\right)^2 - \frac{1}{8V_g}\frac{\partial^3 \varphi^{(0)}}{\partial \xi^3} = 0. \tag{21}$$

Substituting Eq.(16) into Eq.(21), one obtains

$$\frac{\partial a^{(0)}}{\partial \tau} + \frac{3}{2}\left(\frac{V_g}{u_0} + \frac{u_0}{V_g}\right) a^{(0)} \frac{\partial a^{(0)}}{\partial \xi} - \frac{1}{8V_g}\frac{\partial^3 a^{(0)}}{\partial \xi^3} = 0. \tag{22}$$

Equations (21) and (22) should be called variable coefficient Korteweg-de Vries (VCKdV) equation. Making transformation $\lambda = \varepsilon^2 a^{(0)}$, $x_0 = x$, $\xi = \varepsilon X = \varepsilon(x - V_g t)$, and $\tau = \varepsilon^3 t$, from Eq.(22) we obtain

$$\frac{\partial \lambda}{\partial t} + \frac{3}{2}\left(\frac{V_g}{u_0} + \frac{u_0}{V_g}\right)\lambda \frac{\partial \lambda}{\partial X} - \frac{1}{8V_g}\frac{\partial^3 \lambda}{\partial X^3} = 0, \tag{23}$$

where $u_0(x) = u_0(x_0)$. The single-soliton solution of the Eq.(23) is given by

$$\lambda = -A_0 \mathrm{sech}^2 \left\{ \sqrt{V_g A_0 \left(\frac{V_g}{u_0} + \frac{u_0}{V_g}\right)} \left[x - \left(V_g - \frac{A_0 V_g}{2u_0} - \frac{u_0 A_0}{2V_g} \right) t - x_{00} \right] \right\}, \tag{24}$$

where A_0 is a positive constant, x_{00} is a constant denoting the initial position of the soliton on the pedestal

background. Exact to the first order, the condensate-state wavefunction takes the form

$$\psi = \left\{ u_0 - A_0 \mathrm{sech}^2 \left[\sqrt{V_\mathrm{g} A_0 \left(\frac{V_\mathrm{g}}{u_0} + \frac{u_0}{V_\mathrm{g}} \right)} \left(x - V_\mathrm{g} t + \frac{A_0 V_\mathrm{g}}{2u_0} t + \frac{u_0 A_0}{2V_\mathrm{g}} t - x_{00} \right) \right] \right\} \times \exp\left[\mathrm{i} \left(-\mu t + \varphi \right) \right]. \tag{25}$$

The phase function reads

$$\varphi = \frac{-2V_\mathrm{g}\sqrt{A_0}}{\sqrt{u_0 V_\mathrm{g}^2 + u_0^3}} \tanh \left\{ \sqrt{V_\mathrm{g} A_0 \left(\frac{V_\mathrm{g}}{u_0} + \frac{u_0}{V_\mathrm{g}} \right)} \left[x - V_\mathrm{g} t + \frac{1}{2} \left(\frac{V_\mathrm{g}}{u_0} + \frac{u_0}{V_\mathrm{g}} \right) A_0 t - x_{00} \right] \right\}. \tag{26}$$

Equation (25) is just the soliton solution of the BEC confined in the combined potential consisting of a weakly periodic potential and a harmonic magnetic traps.

5. Numerical results and discussions

To observe the effect of weakly periodic potential on the solitary excitation, we further give numerical calculation of the soliton dynamical properties.

First, we plot the density distribution of the condensates with different lattice depth at the initial stage in Fig.2. We see that there exists a sinus in its amplitude function at $x = 0$, which can be associated with dark soliton. By comparing different lattice depth $V_0 = 0$ (solid line) and $V_0 = 0.1$ (dashed line), we find that the condensates still exhibit the dark soliton, however the amplitude of the dark soliton decreases due to the modulating effect of the weakly periodic potential.

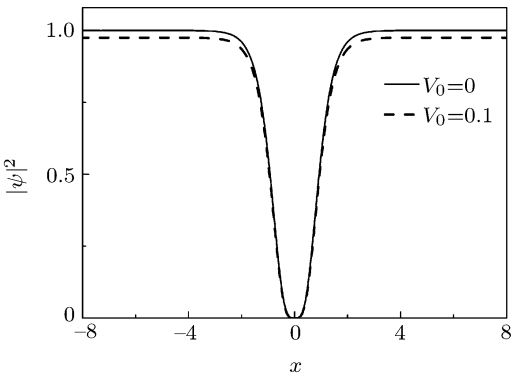


Fig.2. The density distribution of the condensate with lattice depth $V_0 = 0$ (solid line) and 0.1 (dashed line) at the initial time. Other parameters used are $A_0 = 1$, $\mu = 0.5$, and $x_{00} = 0$.

Subsequently, we probe the stability problem of the dark soliton in the weakly modulated periodic potential, which is shown in Fig.3. One can see that the dark soliton propagates leftward without attenuation and changes in shapes (including its height and width) as the time going on. It illustrates that the dark soliton is a stable propagating solitary wave, which is the result from the interplay between the nonlinearity and atomic dispersion caused by inter-site tunnelling.[31]

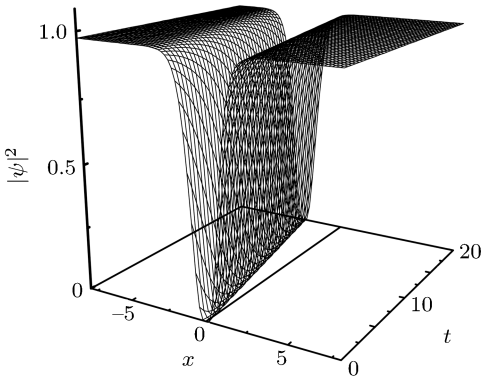


Fig.3. The space–time evolution of the density of the condensate with weakly periodic potential $V_0 = 0.1$. The parameters used are the same as those in Fig.2.

Finally, we discuss the effect of the weakly modulated periodic potential on dynamical characteristics of the soliton of the condensates. Figure 4 presents the amplitude of the soliton at initial stage under consideration for different level periodic potential at (a) $x = 0$ and (b) $x = 5$. From Fig.4(a), we find that the minimal amplitude of the soliton increases with the increasing lattice depth. Figure 4(b) represents that the smooth maximum amplitude of the soliton decreases with the increasing lattice depth. As a matter of fact, the difference between the two curves in Figs.4(a) and 4(b) is just the amplitude of the soliton. So, we conclude that a black soliton of the BEC

transforms to a grey soliton (according to the terminology in optical fibres) due to the weakly periodic potential modulating effect. Meanwhile, we depict the velocity of the soliton versus the lattice depth in Fig.5.

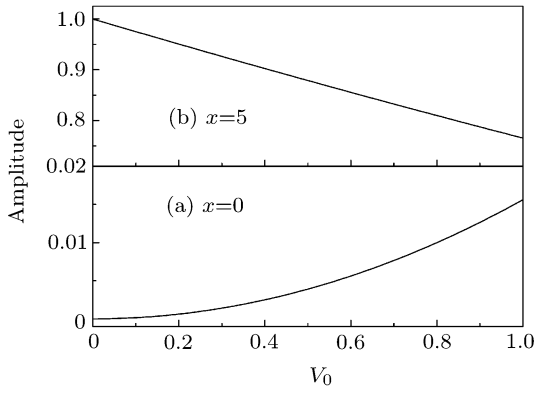


Fig.4. The amplitude of the soliton versus lattice parameter V_0, for space (a) $x = 0$ and (b) $x = 5$. The parameters used are the same as those in Fig.2.

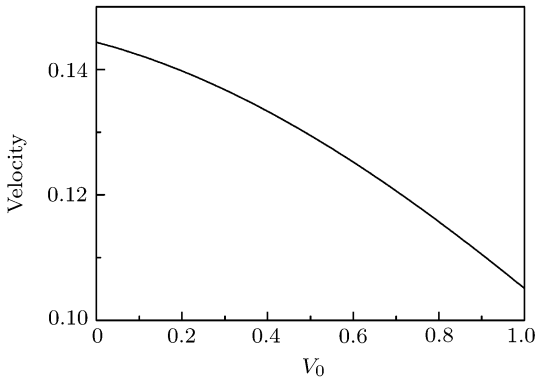


Fig.5. The velocity of the soliton as a function of lattice parameter V_0. The parameters used are the same as those in Fig.2.

One may see that the velocity of the soliton decreases due to the weakly periodic potential modulating effect. This might be due to damping effect of barrier of the periodic potential when the soliton propagates in BEC. In addition, the relation between the width of the soliton and the lattice depth is shown in Fig.6. When the lattice depth increases, the width of the soliton slowly decreases from the lattice depth $V_0 = 0$, goes through a minimum, and then slowly increases. Its minimum position (the critical value) is about at $V_0 = 0.3$. Based on above discussion, we find that the motion of solitons in the BEC trapped in harmonic magnetic potential can be effectively managed by means of weakly periodic traps.

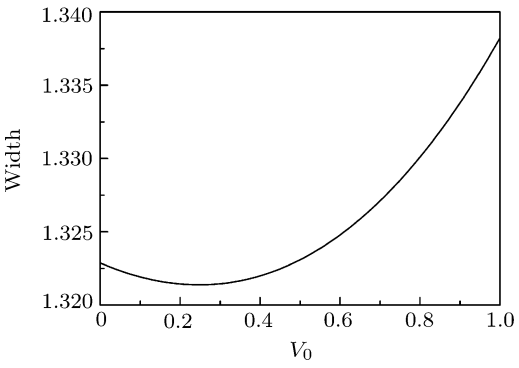


Fig.6. The width of the soliton versus lattice parameter V_0. The parameters used are the same as those in Fig.2.

6. Conclusions

In summary, we have developed multiple-scale method combined with WKB expansion to analytically study the modulating effect of the weakly periodic potential on the dynamical properties of the BEC trapped in harmonic magnetic potential. In the linear case, we have derived the relation of the stability condition of the soliton formation in the BEC. For the weak nonlinearity, the amplitude and phase of wavefunction of the condensates was governed by VCKdV equation. Our numerical calculation showed that the weakly periodic potential had an important effect on the dark soliton dynamical characteristics of the condensates. A black soliton of the BEC in harmonic magnetic traps might be transformed to a grey soliton due to the weakly periodic potential modulating effect. Meanwhile the velocity of the soliton decreases, and its width firstly decreases to go through a minimum position and then increases with the increasing lattice potential.

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