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Noise in a coupling electromagnetic detecting system for high frequency gravitational waves*

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This paper discusses the basic categories of noise in detecting high frequency gravitational waves in the microwave band (∼0.1–10 GHz), which contain shot noise from the laser and the thermal radiation photons, thermal noise from statistical fluctuation of the thermal photons and fluctuation of the temperature, radiation press noise on the fractal membrane, the noise caused by the scattering of the Gaussian Beam (GB) in the detecting tube and noise in the microwave radiometers. The analysis shows that a reasonable signal-to-noise ratio may be achieved for a detecting device with the fixed power of GB (10^5 W), only when the temperature of the environment is no more than T=1 K, and the optimal length of the microwave radiometers is about 0.3 m.

Keywords: high frequency gravitational waves, shot noise, radiation press noise, fractal membrane
PACC: 0430, 0480, 0620D

According to a set of definitions provided by Hawking and Israel (1979),[1] high frequency gravitational waves (HFGWs) have frequencies of 100 kHz to 100 MHz and have the most promise for terrestrial generation and practical, scientific and commercial applications. In fact, unlike usual low-frequency gravitational waves (GWs), the quintessential inflationary models and some string cosmology sceneries all predicted high frequency relic GWs in microwave band (ν ∼0.1–10 GHz).[2−5] In addition, thermal motion of plasma of stars, interaction of the electromagnetic waves (EMWs) with interstellar plasma and magnetic fields, evaporation of primordial black holes, etc., are possible means to generate GWs of the microwave band even higher frequencies.[6] The interactions between GWs and EMWs have been researched, such as the modulation on EMWs in a circular waveguide by HFGWs.[7] At present, a new detecting scheme in Ref.[8,9] developed a scheme for HFGWs detection is promising, in which a laser beam of high energy EMW (i.e., Gaussian Beam (GB)) passes through a cylindrical cavity in a strong static magnetic field, and fractal membrane in the middle of cavity axis. The interaction between the EM fields and the HFGWs in the cavity will change the propagation of some photons which is the signal photons. Here the effect of fractal membrane includes: (a) distinguishing between signal photons and background electromagnetic noise; (b) making the signal photons decay much slower than background photons after being reflected and transmitted by fractal membrane.[10,11] Those signal photons will pass through a rectangular tube upright to the cavity axis and are detected by a microwave radiometers in the tube. That is an improvement on the scheme discussed by Lee and Wan.[12]

Signal-to-noise-ratio (SNR) is important for every scheme of signal detection. Both the environment and equipment cause noise. GWs are very weak signals, so the noise should be controlled. Besides the noises in the work of Lee and Wan, other two kinds of noises for the fractal membrane are analysed in detail here.

Here a beam of circularly polarized monochromatic plane HFGWs propagates along the positive z-axis, the form of the HFGWs in our research can be simplified as

$$h_{\oplus} = A_{\oplus}\exp[\mathrm{i}(kz - \omega_{\mathrm{g}}t)],$$
$$h_{\otimes} = A_{\otimes}\exp[\mathrm{i}(kz - \omega_{\mathrm{g}}t)], \qquad (1)$$

where $A_{\oplus}$, $A_{\otimes}$ are the constant amplitude of HFGWs. In our work, the EMW system consists of GB in GHz

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band, and a static magnetic field on y-direction $B_y^{(0)}$. The general form of GB is

$$\psi = \frac{\psi_0}{\sqrt{1+(z/f)^2}} \times \exp\left(-\frac{r^2}{W^2}\right)\exp\left\{\mathrm{i}\left[(k_\mathrm{e}z-\omega_\mathrm{e}t) - \arctan\frac{z}{f}+\frac{k_\mathrm{e}r^2}{2R}+\delta\right]\right\}, \quad (2)$$

where $r^2 = x^2+y^2$, $k=2\pi/\lambda_\mathrm{e}$, $W=W_0[1+(z/f)^2]^{1/2}$, $R=z+f^2/z$. $f=\pi W_0^2/\lambda_\mathrm{e}$, ψ_0 is the amplitude of electric (or magnetic) field of the GB, W_0 is the minimum spot radius, R is the curvature radius of the wave fronts of the GB at z, ω_e is the angular frequency of EMW, $\omega_\mathrm{e}=\omega_\mathrm{g}$. λ_e is the wavelength of EMW, the z-axis is the symmetrical axis of the GB, and δ is a phase difference between a HFGW and GB. We choose the GB with the double transverse polarized electric modes. Among the three directions of background photon flux (BPF) , we focus on the x-axis,

$$\begin{aligned} n_x^{(0)} &= \frac{1}{\hbar\omega_\mathrm{e}}\left\langle\frac{1}{\mu_0}(\tilde{E}_y^{(0)}\tilde{B}_z^{(0)})\right\rangle \\ &= \frac{1}{2\mu_0\hbar\omega_\mathrm{e}}Re\left\{\psi_y^*\left[\frac{\mathrm{i}}{\omega_\mathrm{e}}\left(\frac{\partial\psi_x}{\partial y}-\frac{\partial\psi_y}{\partial x}\right)\right]\right\} \\ &= f_x^{(0)}\exp\left(-\frac{2r^2}{W^2}\right). \end{aligned} \quad (3)$$

Generally, $n_x^{(1)}$ varies with δ. Putting $\delta=\pi/2$ will be always possible by regulating the phase of GB. Therefore the final approximate expression of the perturbational photon flux (i.e., signal photons) is[9]

$$n_x^{(1)} \approx \frac{1}{\mu_0\hbar\omega_\mathrm{e}}\frac{A_\otimes B_y^{(0)}\psi_0}{[1+(z/f)^2]^{\frac{1}{2}}}\exp\left(-\frac{r^2}{W^2}\right). \quad (4)$$

Above results have been discussed in detail in the work of Li *et al.* In this paper, we just further analyse the noise in the system.

In our work, y, z are restricted in the regions $y\in(0,0.05m)$ and $z\in(0,0.3m)$ respectively. According to Eq.(3) and Appendix, the $n_x^{(0)}$ only depends on the x after the integration of y and z.

$$N_x^{(0)} = \int_0^{0.05}\mathrm{d}y\int_0^{0.3} n_x^{(0)}\mathrm{d}z. \quad (5)$$

Similarly, the $N_x^{(1)}$ can be expressed as:

$$N_x^{(1)} \approx \int_0^{0.05}\mathrm{d}y\int_0^{0.3}\frac{1}{\mu_0\hbar\omega_\mathrm{e}}\frac{A_\otimes B_y^{(0)}\psi_0}{[1+(z/f)^2]^{\frac{1}{2}}} \times\exp\left(-\frac{r^2}{W^2}\right)\mathrm{d}z. \quad (6)$$

In our experiment, the laser is the source of EMW, and we count photons on the x-axis by the microwave radiometers. Measuring optical power is equivalent to determine the number of photons arrived during a measurement interval. The geometric structure of the detecting device is shown in Fig.1.

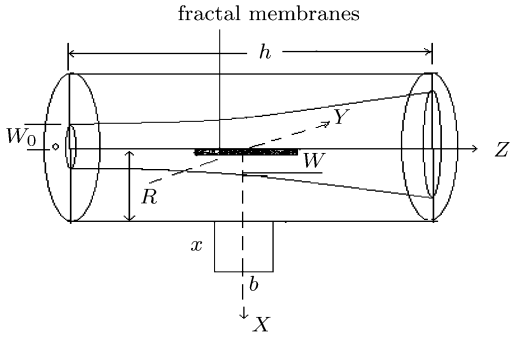


Fig.1. Geometric structure of the detecting device. The radius of the cylinder is R, its length is h. The radius of the beam at the beginning is W_0 and exhales to W with a propagation distance $\frac{h}{2}$. The tube upright to the cylinder has a microwave radiometers at x and has a rectangular cross-section with the side lengths a and b, respectively.

During a measurement interval, there would be photons drifted away the mean number $\overline{N}=\overline{n}\Delta t$, which is characterized by a probability called Poisson distribution,

$$P(N) = \frac{\overline{N}^N\mathrm{e}^{-\overline{N}}}{N!}. \quad (7)$$

The fluctuation of a single measurement of the photon arrival rate is thus given by[13]

$$\frac{\Delta n_{s,\bar{N}}}{\bar{N}} = \frac{\sqrt{\bar{n}\Delta t}}{\bar{n}\Delta t} = \frac{1}{\sqrt{\bar{n}\Delta t}} = \sqrt{2\pi\hbar c/(\lambda P_\mathrm{out}\Delta t)}. \quad (8)$$

Therefore the BPF enters into the photon detector as P_out, then

$$\frac{\Delta n_{s,\bar{N}}}{\bar{N}} = \sqrt{2\pi\hbar c/(\lambda N_x^{(0)}\hbar\omega_\mathrm{e}\Delta t)}, \quad (9)$$

$$\Delta n_{s,\bar{N}} = \sqrt{\lambda N_x^{(0)}\hbar\omega_\mathrm{e}\Delta t/2\pi\hbar c}. \quad (10)$$

In addition, together with the signal photons, the thermal radiation photons transform into photoelectrons and form the shot noise. During the process, the shot noise coming from thermal radiation can be expressed as

$$\Delta n_{s,th} = \frac{1}{4}\sqrt{\frac{2\omega_\mathrm{e}^2 B_1}{Vc\pi}\frac{1}{\mathrm{e}^{\frac{\hbar\omega_\mathrm{e}}{2kT\pi}}-1}}. \quad (11)$$

The total fluctuation of the shot noise can be written as

$$\begin{aligned}\Delta n_{\rm s} =&\Delta n_{s,{\rm th}} + \Delta n_{S,\bar{N}}\\ =&\frac{1}{4}\sqrt{\frac{2\omega_{\rm e}^2 B_1}{Vc\pi}\frac{1}{{\rm e}^{\frac{\hbar\omega_{\rm e}}{2kT\pi}}-1}}\\ &+\sqrt{\lambda N_x^{(0)}\hbar\omega_{\rm e}\Delta t/2\pi\hbar c}.\end{aligned} \tag{12}$$

The power of the shot noise is:

$$P_{\rm s} = \Delta n_{\rm s} S\hbar\omega_{\rm e}, \tag{13}$$

where the S is the cross-section of the tube. From the work of Li *et al*,[8,9] we obtain $S = ab = 0.01\,{\rm m}^2$, Δt is a measurement interval $\sim 10^3\,{\rm s}$, the frequency of GB and GWs are $\omega_{\rm g} = \omega_{\rm e} = 2\pi\times 10^{10}\,{\rm Hz/rad}$, the frequency band is B_1=10 Hz. The volume of cavity is $V = 0.2\,{\rm m}^3$.

The thermal radiation causes the thermal noise from statistical fluctuation of the thermal photons density and a fluctuation of the temperature. It is written as

$$\begin{aligned}\Delta n_f =&\sqrt{\frac{\omega_{\rm e}^2 B_1}{8\pi Vc}\frac{{\rm e}^{\frac{\hbar\omega_{\rm e}}{2kT\pi}}}{({\rm e}^{\frac{\hbar\omega_{\rm e}}{2kT\pi}}-1)^2}+\frac{h\omega_{\rm e}^3 B_1}{4kT^2c^2\pi^2}}\\ &\times\frac{{\rm e}^{\frac{\hbar\omega_{\rm e}}{2kT\pi}}}{({\rm e}^{\frac{\hbar\omega_{\rm e}}{2kT\pi}}-1)^2}\Delta T.\end{aligned} \tag{14}$$

The power of the noises caused by the thermal radiation is

$$P_f = \Delta n_f S\hbar\omega_{\rm e}. \tag{15}$$

In this equation, the fluctuation of the temperature $\Delta T = 1.86\,{\rm mK}$. The calculated noise is about $10^{-31}\,{\rm W}$.

In this case, the photons $N_x^{(0)}$ radiate on the fractal membrane which is on the $y-z$ plane. It is well known that interaction between the laser and mirror will cause noisy force. It is very similar to the action of the GB on the fractal membrane generating noisy force. As light beams, they are bound to exert force on the fractal membrane. This noisy force is applied to each mass, and leads to the motion of membrane on x-direction.[7] The fluctuation of fractal membrane position on x-direction is

$$\Delta x = \frac{8\pi}{m\omega_{\rm e}^2}\sqrt{\frac{\hbar P_{in}}{8\pi^3 c\lambda}}, \tag{16}$$

where

$$P_{in} = N_x^{(0)}\hbar\omega_{\rm e}. \tag{17}$$

The fluctuation of fractal membrane position on x-direction causes the drift of photon number in the radiometer. The result is

$$\begin{aligned}\Delta n_{\rm rad} =&\frac{8\pi}{\mu_0\hbar\omega_{\rm e}^3 m}\Bigg\{\int_0^{0.03}{\rm d}y\int_0^1\frac{AB_y^{(0)}\psi_0}{[1+(z/f)^2]^{1/2}}\\ &\times\sqrt{\frac{\hbar P_{\rm in}}{8\pi^3 c\lambda}}\exp\left(-\frac{y^2}{W^2}\right){\rm d}z\Bigg\}\\ &\times\frac{2x}{W_0^2}\exp\left(-\frac{x^2}{W_0^2}\right).\end{aligned} \tag{18}$$

The power of the above noise is

$$P_{\rm rad} = \Delta n_{\rm rad} S\hbar\omega_{\rm e}. \tag{19}$$

The amplitude of HFGWs is $A\sim 10^{-30}$, the background of static magnetic field is $B_y^{(0)} = 3T$. $\psi_0 \sim 1.26\times 10^3{\rm V\cdot m^{-1}}$ is the amplitude of GB at $z = 0$. $m = 0.02\,{\rm kg}$ is the mass of fractal membrane.

The noises caused by the scattering of the GB $P_{\rm scat}$ in the detecting tube and noise in the microwave radiometers $P_{\rm m}$ were both discussed in Ref.[12]. Here we make a brief analysis and revise some parameters.

$$P_{\rm scat} = \frac{P_{\rm GB}}{4\pi^2}\arctan\frac{a}{2R}\eta^{\frac{hx}{2b(W-W_0)}}\left({\rm e}^{-\frac{\omega_{\rm e}^2 W_0^2 R^2}{c^2h^2+\frac{1}{2}\omega_{\rm e}^2 W_0^4}}-{\rm e}^{-\frac{2\omega_{\rm e}^2 W_0^2(hR+b(W-W_0))}{c^2h^2+\frac{1}{2}h\omega_{\rm e}^2 W_0^4}}\right), \tag{20}$$

where $a = 0.1\,{\rm m}$, $b = 0.1\,{\rm m}$, $\eta = 0.9$, $h = 6\,{\rm m}$ is the length of cavity. Here we only care for the influence of x on the power of scattering noise.

Thermal noise in the microwave radiometer will affect the detecting results. To obtain the main conclusion, we can make a brief analysis. These noises can be represented by the sensitivity ΔT. The power of the noise is

$$P_{\rm m} = k\Delta T B_1. \tag{21}$$

On the precondition of a narrow bandwidth $B_1 = 10\,\mathrm{Hz}$, such noise is neglectful in comparison with the noise mentioned above.[12] The detail analysis has been carried out in Refs.[14, 15].

The definition of SNR is

$$\sigma_{\mathrm{SNR}} = \sqrt{\frac{P_{\mathrm{signal}}}{P_{\mathrm{noise}}}}. \tag{22}$$

Since in our experiment

$$P_{\mathrm{signal}} = N_x^{(1)} \hbar \omega_{\mathrm{e}} S,$$

$$P_{\mathrm{noise}} = P_{\mathrm{s}} + P_{\mathrm{f}} + P_{\mathrm{rad}} + P_{\mathrm{scat}} + P_{\mathrm{m}}, \tag{23}$$

then substituting the above results into Eq.(22), we obtain the dependence of SNR on the environment temperature, and the length of tube.

As a conclusion, the tube should be designed carefully to reach a higher SNR. According to our analysis shown in Table 1, its length should be no more than 0.3 m, and the environment temperature $T \leq 1\,\mathrm{K}$. This is a more reasonable work temperature than that proposed by Li *et al* $T = 1\,\mathrm{mk}$ and Lee *et al* $T \leq 0.6\,\mathrm{K}$ for their device. Recently, for the rapid development of high-energy laser technology,[16,17] it is practical for HFGWs generation in laboratory.[18] Moreover the accuracy of photons counter can be improved greatly by the research of high accuracy photon flux detector.[19,20]

Table 1. Value of σ_{SNR} employed.

T/K	x/m	σ_{SNR}
0.1	0	133
0.3	0.06	79
0.5	0.12	40
0.7	0.18	15
0.9	0.24	4
1.1	0.30	1
1.3	0.36	0.01

Relic GWs in such microwave band is universal. But for their stochastic phase,[21,22] it is much more difficult to realize numerical analysis. There will be another paper to discuss it.

Appendix A

Since the integral

$$\int \exp\left[\left(\frac{\mathrm{i}k_{\mathrm{e}}}{2R} + \frac{1}{W^2}\right)y^2\right]\mathrm{d}y = \begin{cases} \dfrac{\sqrt{2\pi}}{\sqrt{-\dfrac{\mathrm{i}k_{\mathrm{e}}W^2 + 2R}{RW^2}}}, & c\mathrm{sgn}\left(\dfrac{\mathrm{i}k_{\mathrm{e}}}{2R} + \dfrac{1}{W^2}\right) = 1, \\ 0, & \text{otherwise}, \end{cases} \tag{A1}$$

where

$$c\mathrm{sgn}(z) = \begin{cases} 1 & \mathrm{Im}(z) > 0 \ \text{ and } \ \mathrm{Re}(z) = 0, \text{ or } \mathrm{Re}(z) > 0, \\ 1 & z = 0, \\ -1 & \text{otherwise}. \end{cases} \tag{A2}$$

hence

$$\int \exp\left[\left(\frac{\mathrm{i}k_{\mathrm{e}}}{2R} + \frac{1}{W^2}\right)y^2\right]\mathrm{d}y = \sqrt{2\pi}\left[\left(\frac{k}{R}\right)^2 + \left(\frac{2}{W^2}\right)^2\right]\exp\left(\mathrm{i}\frac{\arctan\left(\dfrac{kW^2}{2R}\right)}{2}\right). \tag{A3}$$

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