Vol 18 No 3, March 2009
1674-1056/2009/18(03)/0910-05
Chinese Physics B
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Scheme for realizing quantum computation and quantum information transfer with superconducting qubits coupling to a 1D transmission line resonator*

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(Received 20 September 2007; revised manuscript received 6 July 2008)

This paper proposes a simple scheme for realizing one-qubit and two-qubit quantum gates as well as multiqubit entanglement based on dc-SQUID charge qubits through the control of their coupling to a 1D transmission line resonator (TLR). The TLR behaves effectively as a quantum data-bus mode of a harmonic oscillator, which has several practical advantages including strong coupling strength, reproducibility, immunity to $1/f$ noise, and suppressed spontaneous emission. In this protocol, the data-bus does not need to stay adiabatically in its ground state, which results in not only fast quantum operation, but also high-fidelity quantum information processing. Also, it elaborates the transfer process with the 1D transmission line.

Keywords: dc-SQUID charge qubits, 1D transmission line resonator, quantum logic gates, quantum information transfer

PACC: 0367, 0365

In cavity quantum electrodynamics (QED),[1] the coupling effects of two atoms[2] inside a cavity have been theoretically studied to explore exotic quantum coherent phenomena,[3] e.g., coherent population trapping and dark states. However, the weak dipole–dipole interaction between atoms makes it difficult to experimentally demonstrate these phenomena.

Recent experiments using on-chip superconducting qubits[4−6] show the strong-coupling effects between: (i) two superconducting qubits;[5,6] (ii) a charge qubits and a superconducting transmission line resonator (TLR);[7,8] and (iii) a flux qubit and an LC oscillator.[9,10] The latter two are referred to as circuit QED. The demonstration of 2-bit conditional gate of charge qubits was reported most recently in Ref.[5]. These developments have paved the way towards the realization of the two important elements of universal quantum computation: the ability to implement arbitrary single-bit rotations and the controlled logic gates between two qubits. Various theoretical schemes have been proposed for quantum logic gate operations of superconducting qubits. Most protocols[11−17] involve the coupling of each superconducting qubit to an auxiliary data-bus and affect an effective coupling between two qubits through the elimination of dynamic variables of the data-bus.

The existing schemes[11−17] usually apply to weak coupling parameters, and thus lead to slow gates with long operation times. Most of them require the data-bus to be adiabatic, i.e. to stay in its ground state or a different pure state during gate operation. Recently, Wang *et al* proposed a theoretical protocol for fast entanglement of two charge-phase qubits through nonadiabatic coupling to a large junction,[10] they designed a basic storage unit consisting of three symmetrical dc-SQUIDs and proposed to couple them with a one-dimensional (1D) transmission line to physically realize a quantum storage network.[18] In this paper, we suggest a protocol which is capable of efficient and fast gate operations between two Josephson charge qubits, and elaborate the quantum information transfer process. As will be shown in detail below, our protocol is insensitive to the state of the data-bus, thus it is not restricted to the weak coupling limit as required by

*Project supported by Hunan Provincial Natural Science Foundation of China (Grant No 06JJ50014) and the Key Project Foundation of the Education Commission of Hunan Province of China (Grant No 06A055).
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the adiabatic condition. In fact, the resulting strong effective coupling leads to short gate operation times.

Our main idea is to construct a quantum network including N Cooper pair boxes (CPB) linked by a quantum data-bus of a harmonic oscillator with the creation (annihilation) operator $\widehat{a}\left(\widehat{a}^{+}\right)$.[8,19] Here, we select the 1D TLR as possible data-bus. It has been illustrated to have several practical advantages including strong coupling strength, reproducibility, immunity to $1/f$ noise, and suppressed spontaneous emission.[18,19] The quantum network setup is schematically shown in Fig.1.

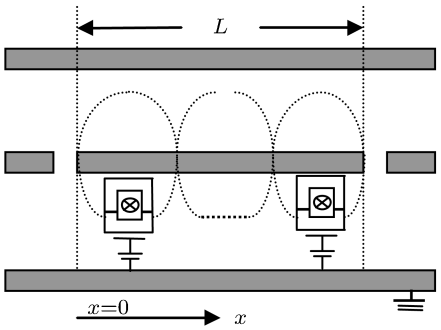


Fig.1. The scheme of our setup. A Josephson charge qubit array is placed in a 1D TLR. The charge qubits are placed between superconducting lines and located at the antinodes of a single-mode magnetic field of the TLR. The couplings between qubits and a single-mode quantized field are realized by the magnetic field through the qubit loops.

Each CPB is a dc-SQUID formed by a superconducting island connected to two Josephson junctions with the same capacitance C_{Jk}^{0} and Josephson energy E_{Jk}^{0}. The effective Josephson tunnelling energy can be tuned by the magnetic flux $\Phi_{\mathrm{e}}^{(k)}$ threading the dc-SQUID. With proper bias voltage V_{gk}, the CPB behaves as a qubit,[20] and in the spin-$\frac{1}{2}$ representation with charge state $|0\rangle$ and $|1\rangle$ ($|0\rangle$ and $|1\rangle$ denote 0 and 1 excess Cooper pair on the island respectively) the reduced two-state Hamiltonian is given by

$$H_{JJ}^{(k)}=E_{z}^{(k)}\sigma_{z}^{(k)}-E_{Jk}\cos\left(\frac{\pi\Phi_{\mathrm{e}}^{(k)}}{\Phi_{0}}\right)\sigma_{x}^{(k)}, \quad (1)$$

where $E_{Jk}=2E_{Jk}^{0}$ is the Josephson energy of the k-th loop, $E_{z}^{(k)}=4E_{ck}\left(n_{g}^{(k)}-\frac{1}{2}\right)$ with $n_{g}^{(k)}=C_{gk}V_{gk}/2e$, $E_{ck}=\frac{1}{2}\frac{e^{2}}{C_{gk}+2C_{Jk}^{0}}$ and Φ_{0} the flux quantum. The quasi-spin operators $\sigma_{z}^{(k)}=|0\rangle_{k\,k}\langle 0|-|1\rangle_{k\,k}\langle 1|$, $\sigma_{x}^{(k)}=|1\rangle_{k\,k}\langle 0|+|0\rangle_{k\,k}\langle 1|$.

In our setup, a 1D transmission line resonator (TLR) of length L is placed in parallel with this superconducting qubit array away from a distance d.[18,19] The quantized magnetic field at $x=\frac{nL}{2n_{0}}$ is

$$B_{y}\left(x=\frac{nL}{2n_{0}}\right)=\frac{1}{d}\sqrt{\frac{\hbar l\omega}{L}}\left(\widehat{a}+\widehat{a}^{+}\right), \quad (2)$$

where n_{0} is the mode number resonant with the qubits, n is an arbitrary integer, while the electric field is zero at these points. Here $\omega=\frac{n_{0}\pi}{L\sqrt{lc}}$, $l\,(c)$ is the inductance (capacitance) per unit length. The flux induced by the transmission line in a dc-SQUID with an enclosed area S is

$$\Phi_{x}^{(k)}=\frac{S}{d}\sqrt{\frac{\hbar l\omega}{L}}\left(\widehat{a}+\widehat{a}^{+}\right). \quad (3)$$

Here, we have assumed only a single mode of magnetic field with frequency ω, which is coupled with superconducting qubits array, and $\widehat{a}\left(\widehat{a}^{+}\right)$ is its annihilation (creation) operator.

In the Lamb–Dicke limit, the Hamiltonian of the interacting k-th qubit and resonator system is

$$\begin{aligned}H^{(k)}=&E_{z}^{(k)}\sigma_{z}^{(k)}+f_{k}\sigma_{x}^{(k)}+g_{k}\left(\widehat{a}+\widehat{a}^{+}\right)\sigma_{x}^{(k)}\\&+\hbar\omega\widehat{a}^{+}\widehat{a}, \quad (4)\end{aligned}$$

where

$$f_{k}=-E_{Jk}\cos\frac{\pi\Phi_{\mathrm{e}}^{(k)}}{\Phi_{0}},$$

$$g_{k}=\frac{\pi S}{\Phi_{0}d}\sqrt{\frac{\hbar\omega}{L}}E_{Jk}\sin\frac{\pi\Phi_{\mathrm{e}}^{(k)}}{\Phi_{0}}.$$

And the Hamiltonian of whole system (the qubit array and the TLR) is

$$\begin{aligned}H=&\sum\nolimits_{k=1}^{N}\left[E_{z}^{(k)}\sigma_{z}^{(k)}+f_{k}\sigma_{x}^{(k)}+g_{k}\left(\widehat{a}+\widehat{a}^{+}\right)\sigma_{x}^{(k)}\right]\\&+\hbar\omega\widehat{a}^{+}\widehat{a}. \quad (5)\end{aligned}$$

Quantum logic gate operation and multi-qubit entanglement

When two dc-SQUID are placed in the TLR, the evolution of the system governs by the following Hamiltonian:[10]

$$\begin{aligned}H_{\mathrm{gate}}=&\sum_{k=1}^{2}\left[E_{z}^{(k)}\sigma_{z}^{(k)}+f_{k}\sigma_{x}^{(k)}+g_{k}\left(\widehat{a}+\widehat{a}^{+}\right)\sigma_{x}^{(k)}\right]\\&+\hbar\omega\widehat{a}^{+}\widehat{a}. \quad (6)\end{aligned}$$

Set $n_{g}^{(k)}=\frac{1}{2}$, then the Hamiltonian reduces to

$$H_{\mathrm{CNOT}}=H_{1}+H_{2}, \quad (7)$$

where

$$H_1 = \sum_{k=1}^{2} f_k \sigma_x^{(k)}, \tag{8}$$

$$H_2 = \hbar\omega \widehat{a}^+ \widehat{a} + \sum_{k=1}^{2} \left[g_k \left(\widehat{a} + \widehat{a}^+ \right) \sigma_x^{(k)} \right]. \tag{9}$$

This Hamiltonian can be solved exactly since the interaction term $g_k\left(\widehat{a}+\widehat{a}^+\right)\sigma_x^{(k)}$ commutes with the term $f_k\sigma_x^{(k)}$. With the Wei–Norman algebraic method,[21] we find that the exact time evolution of the Hamiltonian (7) is[10,22]

$$U(t) = U_1[t; f_k] \otimes U_2[t; g_k], \tag{10}$$

where

$$U_1[t, f_k] = \exp\left(-\mathrm{i} \frac{\sum\limits_{k=1,2} f_k \sigma_x^{(k)}}{\hbar} t \right), \tag{11}$$

$$\begin{aligned} U_2[t; g_k] =& \exp\left[-\mathrm{i}C(t) - \mathrm{i}A(t)\, \sigma_x^1 \sigma_x^2 \right] \\ & \times \prod_{k=1,2} \exp\left[-\mathrm{i}B_k(t)\, \hat{a} \sigma_x^{(k)} \right] \\ & \times \exp\left[-\mathrm{i}B_k^*(t)\, \hat{a}^+ \sigma_x^{(k)} \right], \end{aligned} \tag{12}$$

and the time-dependent parameters are given by

$$\begin{aligned} B_k(t) &= \frac{g_k}{-\mathrm{i}\hbar\omega} \left(\mathrm{e}^{-\mathrm{i}\omega t} - 1 \right), \\ A(t) &= \frac{2 g_1 g_2}{\omega} \left(\frac{1}{\mathrm{i}\hbar^2 \omega} \left(\mathrm{e}^{\mathrm{i}\omega t} - 1 \right) - \frac{t}{\hbar^2} \right), \\ C(t) &= \frac{g_1^2 + g_2^2}{\omega} \left(\frac{1}{\mathrm{i}\hbar^2 \omega} \left(\mathrm{e}^{\mathrm{i}\omega t} - 1 \right) - \frac{t}{\hbar^2} \right). \end{aligned} \tag{13}$$

If we use a particular time length $t_n = \dfrac{2m\pi}{\omega}$ for integer $m = 0,\ \pm1,\ \pm2,\ \ldots$, the time evolution operator of the whole system can be simplified to

$$\begin{aligned} U(t_m) =& \exp\left[\mathrm{i}2m\pi \left(g_1^2 + g_2^2 \right) / (\hbar\omega)^2 \right] \\ & \times \exp\left[\mathrm{i} \frac{4m\pi g_1 g_2}{(\hbar\omega)^2} \sigma_x^{(1)} \sigma_x^{(2)} \right] \\ & \times \exp\left[-\mathrm{i} \frac{2m\pi}{\hbar\omega} \left(f_1 \sigma_x^{(1)} + f_2 \sigma_x^{(2)} \right) \right]. \end{aligned} \tag{14}$$

This evolution operator plays a key role in the realization of one- or two-qubit quantum gates and multiqubit entanglement. Clearly, the operator is independent of variables of the resonator mode, then, the data-bus(1D TLR) does not require to stay adiabatically in its ground state, which results in not only fast quantum operation, but also high-fidelity quantum information processing.

We shall show how to realize one- and two-qubit quantum gate, and multiqubit entanglement. In order to do this, we discuss problems on the logic basis $\{|e\rangle_k = (|0\rangle_k + |1\rangle_k)\sqrt{2},\ |g\rangle_k = (|0\rangle_k - |1\rangle_k)/\sqrt{2}\}$, and for simplicity, we let $g_1 = g_2 = g,\ f_1 = f_2 = f$ through manipulating the external flux $\Phi_{\mathrm{e}}^{(k)}$ of the k-th loop, and introduce collective spin operator $J_x = \sum\limits_{k=1}^{2} \sigma_x^{(k)}$. In this case, the time evolution operator (14) reduces to

$$\begin{aligned} U'(t_m) =& \exp\left[\mathrm{i}2m\pi g^2 J_x^2 / (\hbar\omega)^2 \right] \\ & \times \exp\left[-\mathrm{i}2m\pi f J_x / (\hbar\omega) \right]. \end{aligned} \tag{15}$$

First, we show how to realize the single-qubit rotation operation on one charge qubit. In the case of one charge qubit in the 1D TLR, according to the evolution operator (15), we can obtain the rotation around the z axis:

$$R_z^{(k)} = \mathrm{e}^{\mathrm{i}2m\pi g^2/(\hbar\omega)^2} \begin{pmatrix} \mathrm{e}^{-\mathrm{i}2m\pi f/(\hbar\omega)} & 0 \\ 0 & \mathrm{e}^{\mathrm{i}2m\pi f/(\hbar\omega)} \end{pmatrix}. \tag{16}$$

Second, we show how to realize two-qubit quantum phase gate (QPG). In order to do this, we place two dc-SQUID in the 1D TLR. From Eq.(14) we easily obtain the following operation:

$$\begin{cases} |e\rangle_1 |e\rangle_2 \\ |e\rangle_1 |g\rangle_2 \\ |g\rangle_1 |e\rangle_2 \\ |g\rangle_1 |g\rangle_2 \end{cases} \xrightarrow{U(t_n)} \begin{cases} \mathrm{e}^{\mathrm{i}\left[8m\pi \frac{g^2}{(\hbar\omega)^2} - 4m\pi \frac{f}{\hbar\omega}\right]} |e\rangle_1 |e\rangle_2, \\ |e\rangle_1 |g\rangle_2, \\ |g\rangle_1 |e\rangle_2, \\ \mathrm{e}^{\mathrm{i}\left[8m\pi \frac{g^2}{(\hbar\omega)^2} + 4m\pi \frac{f}{\hbar\omega}\right]} |g\rangle_1 |g\rangle_2. \end{cases} \tag{17}$$

Obviously, this operation has achieved an universal controlled phase-shift gate. It is easy to check that the operation can implement a π-QPG by selecting $8m\pi \dfrac{g^2}{(\hbar\omega)^2} = \dfrac{3\pi}{2}$ and $4m\pi \dfrac{f}{\hbar\omega} = \dfrac{\pi}{2}$.

Third, we shall demonstrate that it is possible to generate entanglement of N charge qubits in the GHZ state $|\Psi\rangle = \dfrac{1}{\sqrt{2}} \left(\mathrm{e}^{\mathrm{i}\varphi_g} |gg\ldots g\rangle + \mathrm{e}^{\mathrm{i}\varphi_{\mathrm{e}}} |ee\ldots e\rangle \right)$. To generate the GHZ state, the N charge qubits being in their ground state $|gg\ldots g\rangle$ is required. For the case of N being even, if we select $\Phi_{\mathrm{e}}^{(k)} = \Phi_0/2$, then the evolution operator (15) reduces to: $U'(t_m) = \exp\left[\mathrm{i}2m\pi g^2 J_x^2 / (\hbar\omega)^2 \right]$. Using this evolution, we find that several schemes[22–25] are presented for generating the GHZ state $|\Psi\rangle$ with $\varphi_g = -\pi/4$ and $\varphi_{\mathrm{e}} = \pi/4 + N\pi/2$ by choosing $2m\pi g^2/(\hbar\omega)^2 = \pi/8$. For the case of N being odd, our scheme makes the generation of the GHZ state with the evolution

operator $U'(t_m) = \exp\left[\mathrm{i}\pi\left(2J_x + J_x^2\right)/8\right]$ by setting $2m\pi g^2/(\hbar\omega)^2 = \pi/8$ and $2m\pi f/(\hbar\omega) = -\pi/4$.

Alternatively we can remove the influence of the TLR mode on the two-qubit logic operation by using two operations in succession, a concept presented recently for a two-qubit gate on trapped ions[26] and superconducting circuit.[10] The two steps are described as follows.

First, we evolve the system with the Hamiltonian (7) for duration $\tau/2$. The evolution operator is

$$U\left(\frac{\tau}{2}\right) = U_1\left[\frac{\tau}{2}; f_k\right] \otimes U_2\left[\frac{\tau}{2}; g_k\right]. \tag{18}$$

Second, at time $t = \tau/2$, we instantly reverse the direction of the magnetic field such that there is a sudden change of flux from $\Phi_{\mathrm{e}}^{(k)}$ to $-\Phi_{\mathrm{e}}^{(k)}$, this leads to $g_k' = -g_k$ and $f_k' = f_k$. The system is then driven by a new Hamiltonian $H' = H_{\mathrm{CNOT}}(-g_k, f_k)$, with which we evolve for another $\tau/2$. The evolution operator becomes

$$U'\left(\frac{\tau}{2}\right) = U_1\left[\frac{\tau}{2}; f_k\right] \otimes U_2\left[\frac{\tau}{2}; -g_k\right]. \tag{19}$$

The combined dynamics from the above two steps is now described by the following evolution operator

$$\begin{aligned}\tilde{U}(\tau) =& U'\left(\frac{\tau}{2}\right) U\left(\frac{\tau}{2}\right)\\ =& \exp\left[-\mathrm{i}M(\tau) - \mathrm{i}N(\tau)\sigma_x^1\sigma_x^2\right]\\ & \times \exp\left(-\mathrm{i}\frac{\sum\limits_{k=1,2} f_k\sigma_x^k}{\hbar}\tau\right),\end{aligned} \tag{20}$$

where

$$\begin{aligned}M(\tau) =& C'\left(\frac{\tau}{2}\right) + C\left(\frac{\tau}{2}\right)\\ =& \frac{g_1^2 + g_2^2}{\omega}\left(\frac{2\sin\left(\frac{\omega\tau}{2}\right)}{\hbar^2\omega} - \frac{\tau}{\hbar^2}\right),\end{aligned} \tag{21}$$

and

$$\begin{aligned}N(\tau) =& A'\left(\frac{\tau}{2}\right) + A\left(\frac{\tau}{2}\right)\\ =& \frac{2g_1g_2}{\omega}\left(\frac{2\sin\left(\frac{\omega\tau}{2}\right)}{\hbar^2\omega} - \frac{\tau}{\hbar^2}\right).\end{aligned} \tag{22}$$

Again we see that after the above two successive operations, we realize an effective controlled rotation of the two qubits.

A key requirement for our protocol is the precise control of the operation time, especially in the second approach that needs a sudden switch of the magnetic field during two-qubit operation. However, the second scheme does not need to change the magnitude of the magnetic field but just to switch its direction.

Information transfer between the qubits

The Hamiltonian for the k-th qubit with $\Phi_{\mathrm{e}}^{(k)} = \Phi_0/2$ becomes

$$\begin{aligned}H^{(k)}(f_k = 0) =& E_z^{(k)}\sigma_z^{(k)} + g_k\left(\widehat{a} + \widehat{a}^{+}\right)\sigma_x^{(k)}\\ & + \hbar\omega\widehat{a}^{+}\widehat{a}.\end{aligned} \tag{23}$$

Under the condition $\left|E_z^{(k)} - \omega\right| / \left(E_z^{(k)} + \omega\right) << 1$, we can apply the rotating-wave approximation, and obtain the following Hamiltonian:

$$\begin{aligned}H_{\mathrm{RWA}}^{(k)}(f_k = 0) =& E_z^{(k)}\sigma_z^{(k)} + \hbar\omega\widehat{a}^{+}\widehat{a}\\ & + g_k\widehat{a}\sigma_+^{(k)} + g_k^{*}\widehat{a}^{+}\sigma_-^{(k)}.\end{aligned} \tag{24}$$

This is a typical Jaynes–Cummings model. Using this Hamiltonian, the scheme[19] was present for the quantum information transfer. To implement the quantum information transfer, we prepare first the transmission line in its ground state $|0\rangle_{\mathrm{TLR}}$, tune $n_g^{(k)}$ to have $E_z^{(k)} = \omega$ for a period $\pi/(2g_k)$ such that the state of the k-th qubit is stored in the transmission line. Then let this qubit be largely detuned with the TLR while make the frequency of another qubit to satisfy $E_z^{(k')} = \omega$ for another $t = \pi/(2g_{k'})$. This process can be explicitly illustrated as

$$\begin{aligned}&\left(\alpha|1\rangle^{(k)} + \beta|0\rangle^{(k)}\right) \otimes |0\rangle_{\mathrm{TLR}} \otimes |0\rangle^{(k')}\\ &\rightarrow |0\rangle^{(k)} \otimes \left(\beta\mathrm{e}^{\mathrm{i}\xi}|0\rangle_{\mathrm{TLR}} - \mathrm{i}\alpha\mathrm{e}^{-\mathrm{i}\xi}|1\rangle_{\mathrm{TLR}}\right) \otimes |0\rangle^{(k')}\\ &\rightarrow |0\rangle^{(k)} \otimes |0\rangle_{\mathrm{TLR}} \otimes \left(\alpha|1\rangle^{(k')} + \beta|0\rangle^{(k')}\right).\end{aligned}$$

In this way the information is transferred from the kth qubit to the k'th qubit.

We now consider the implementation of our protocol in realistic experiments. To see the experimental feasibility, we now examine the used conditions and approximations based on the possible experimental parameters. For example, based on the parameters in Ref.[8], if we take $L \sim 0.2\,\mathrm{mm}$, $S \sim 200\,\mu\mathrm{m}^2$, $d \sim 1\,\mu\mathrm{m}$ and $n_0 = 5$, then $g_k \sim 0.1E_{Jk}$.[19] With these parameters and the cavity QED parameters for the 1D superconducting system, the Lamb–Dicke approximation is justified. Besides, the operation time is estimated to be $\sim 10^{-10}\,\mathrm{s}$ for one CNOT operation process and $\sim 10^{-9}\,\mathrm{s}$ for one information transferring process, being much shorter than the coherence time for charge qubits at the degeneracy point ($\sim$800 ns currently). Therefore this process can be completed before the quantum decoherence happens.

In conclusion, we have presented an efficient scheme to implement one-qubit rotation operation, a quantum controlled phase gate as well as quantum information transfer process by dc-SQUID charge qubits coupling to a 1D TLR. The 1D transmission line has several practical advantages including strong coupling strength, reproducibility, immunity to $1/f$ noise, and suppressed spontaneous emission. From the experimental point of view, our protocol seems feasible and advantageous as it only needs the control of one system parameter, that is the external applied magnetic field or controlled gate voltage $v_{\rm g}$.

References

[1] Berman P R (Ed) 1994 *Cavity Quantum Electrodynamics* (Boston: Academic Press)

[2] Seminara F and Leonardi C 1990 *Phys. Rev.* A **42** 5695
Peng J and Li G 1993 *Phys. Rev.* **47** 4212
Goldstein E V and Meystre P 1997 *Phys. Rev.* **56** 5135

[3] Scully M O and Zubairy M S 1997 *Quantum Optics* (Cambridge: Cambridge University Press)

[4] You J Q and Nori F 2005 *Phys. Today* **58** 42

[5] Pashkin Y A, Yamamoto T, Astafiev O, Nakamura Y, Averin D and Tsai J S 2003 *Nature* **421** 823

[6] Berkley A J, Xu H, Ramos R C, Gubrud M A, Strauch F W, Johnson P R, Anderson J R, Dragt A J, Lobb C J and Wellstood F C 2003 *Science* **300** 1548
McDermott R, Simmonds R W, Steffen M, Cooper K B, Cicak K, Osborn K D, Oh S, Pappas D P and Martinis J M 2005 *Science* **307** 1299

[7] You J Q and Nori F 2003 *Phys. Rev.* B **68** 064509

[8] Wallraff A, Schuster D I, Blais A, Frunzio L, Huang R S, Majer J, Kumar S, Girvin S M and Schoelkopf R J 2004 *Nature* **431** 162
Wallraff A, Schuster D I, Blais A, Frunzio L, Majer J, Devoret M H, Girvin S M and Schoelkopf R J 2005 *Phys. Rev. Lett.* **95** 060501

[9] Chiorescu I, Bertet P, Semba K, Nakamura Y, Harmans C J P M and Mooij J E 2004 *Nature* **431** 159

[10] Wang Y D, Zhang P, Zhou D L and Sun C P 2004 *Phys. Rev.* B **70** 224515

[11] Makhlin Y, SchÖn and Shnirman A 2001 *Rev. Mod. Phys.* **73** 357

[12] Averin D V and Bruder C 2003 *Phys. Rev. Lett.* **91** 057003

[13] Yang C P, Chu Shih-I and Han S 2003 *Phys. Rev.* A **67** 042311

[14] Song K H, Zhou Z W and Guo G C 2005 *Phys. Rev.* A **71** 052310

[15] Kim M D and Hong J 2003 *Phys. Rev.* B **68** 134513

[16] Blais A, van den Brink A M and Zagoskin A M 2003 *Phys. Rev. Lett.* **90** 127901

[17] You J Q, Tsai T S and Nori F 2002 *Phys. Rev. Lett.* **89** 197902

[18] Blais A, Huang R S, Wallraff A, Girvin S M and Schoelkopf R J 2004 *Phys. Rev.* A **69** 062320

[19] Wang Y D, Wang Z D and Sun C P 2005 *Phys. Rev.* B **72** 172507

[20] Nakamura Y, Pashkin Y A and Tsai J S 1999 *Nature* **398** 786

[21] Wei J and Norman E 1963 *J. Math. Phys.* **4** 575

[22] Zou X B, Pahlke K and Mathis W 2002 *Phys. Rev.* A **66** 044307

[23] Zhu S L, Wang Z D and Zanardi P 2005 *Phys. Rev. Lett.* **94** 100502

[24] Molmer K and Sorensen A 1999 *Phys. Rev. Lett.* **82** 1835

[25] Zheng S B 2002 *Phys. Rev.* A **70** 060303 (R)

[26] Zheng S B 2003 *Phys. Rev. Lett.* **90** 217901