Vol 18 No 3, March 2009
1674-1056/2009/18(03)/0894-04

Chinese Physics B

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A scheme for implementing quantum game in cavity QED*

Cao Shuai(曹 帅)[a)b)], Fang Mao-Fa(方卯发)[b)†], Liu Jian-Bin(刘建斌)[a)], Wang Xin-Wen(汪新文)[b)d)e)], and Zheng Xiao-Juan(郑小娟)[b)c)]

[a)] *College of Sciences, South China Agricultural University, Guangzhou* 510642, *China*
[b)] *College of Physics and Information Science, Hunan Normal University, Changsha* 410081, *China*
[c)] *School of Physics Science and Technology, Central South University, Changsha* 410083, *China*
[d)] *Department of Physics, Hunan University of Science and Engineering, Yongzhou* 425100, *China*
[e)] *Department of Physics, Beijing Normal University, Beijing* 100875, *China*

(Received 10 June 2008; revised manuscript received 9 July 2008)

In this paper, we propose a scheme for implementing quantum game (QG) in cavity quantum electrodynamics(QED). In the scheme, the cavity is only virtually excited and thus the proposal is insensitive to the cavity fields states and cavity decay. So our proposal can be experimentally realized in the range of current cavity QED techniques.

Keywords: entanglement, quantum game, cavity QED
PACC: 0365, 4250

1. Introduction

Game theory has been widely used in economics, biology and social sciences to model decision made in conflict situation. Recent development in quantum computation and quantum information theory[1,2] allows to extend the scope of game theory to quantum world. Quantum games (QGs) were introduced in 1999 in the two seminal works of Meyer[3] and Eisert *et al.*[4] Then the properties in different entanglement degrees of QG are presented in Ref.[5], and the investigations on multi-player quantum games,[6] multi-player and multi-choice quantum games,[7,8] continuous-variable QGs[9] have been presented. Recently, decoherence in QGs has been investigated.[10−12] The reason why QGs have become so popular is their relation with quantum algorithms, quantum communication and quantum cryptography.

Since QGs are in fact small algorithms, finding efficient ways of implementing them in different physical system is important for the development of improving quantum computation schemes. The prisoner dilemma game has first been realized on a nuclear magnetic resonance quantum computer.[13] Then two proposals of optically realizing QG[14,15] are presented. However, in their optical proposal, the Kerr medium and the technology of single photon are all very difficult in practical experiment. Recently, QG in ion traps has been discussed.[16] In this scheme, the decoherence is also a big problem in practical experiment.

As is well known, cavity QED is one of the important candidates for quantum information processing. Recently, Zheng and Guo proposed two atoms interacting with a nonresonant cavity, which is only virtually excited, and thus the proposal is insensitive to the cavity fields states and cavity decay.[17] Then various schemes have been proposed for implementing several quantum algorithms for cavity QED, for example quantum Fourier transform,[18] Deutsch–Jozsa algorithm,[19] quantum dense coding,[20] Grover search algorithm,[21] quantum clock synchronization algorithm[22] and so on. However, QG in cavity QED is little investigated. In this paper, we propose a scheme for implementing two player QG in cavity QED. So far as we know, up to now, this is the first scheme for the implementation of QG in cavity QED, and therefore, by proposing an cavity QED implementation scheme for QG, we fill an important gap in the

*Project supported by the National Natural Science Foundation of China (Grant No 10374025), Hunan Provincial Natural Science Foundation of China (Grant No 07JJ3013) and the Foundation of Hunan Provincial Education Department of China (Grant No 06A038), the Principal Foundation of South China Agricultural University (Grant Nos 4900-K07275 and 4900-06168), Guangdong Provincial Natural Science Foundation of China (Grant No 07300793).
†Corresponding author. E-mail: mffang@hunnu.edu.cn
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list of possible physical realization of QG.

2. Quantum prisoner dilemma

We first briefly recall the scheme of the quantum game presented in Ref.[4]. There are two players and each player can choose two possible strategies: cooperate($\widehat{C}$) and defect($\hat{D}$). Classically the dominant strategy for both players is to defect(the Nash equilibrium) since no player can improve his payoff by unilaterally changing his own strategy, even though the Pareto optimal is for both players to cooperate. This is the dilemma. In quantum version, each player possesses a qubit $|C\rangle$ ($|D\rangle$). The qubits firstly are prepared in the initial state $|C\rangle\,|C\rangle$, then the entangling gate $\hat{J}$ acts on it (see Fig.1).

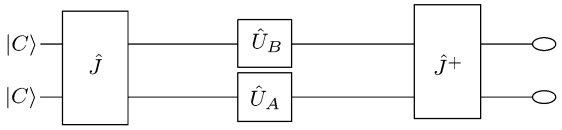


Fig.1. The setup of a two-player quantum game.

The initial state of the quantum game is $|\psi_i\rangle = \widehat{J}\,|CC\rangle$, here $\hat{J}$ is $\exp(i\gamma D\otimes D/2)$, with $\gamma\in[0,\pi/2]$ measuring the entanglement of the state $|\psi_i\rangle$:

$$|\psi_i\rangle = \cos\left(\frac{\gamma}{2}\right)|C\rangle|C\rangle + \mathrm{i}\sin\left(\frac{\gamma}{2}\right)|D\rangle|D\rangle. \quad (1)$$

Then the unitary $\hat{U}_A$ and $\hat{U}_B$ are chosen from 2-parameter set of unitary $2\otimes 2$ matrices :

$$\hat{U}\,(\theta,\phi) = \begin{pmatrix} \mathrm{e}^{\mathrm{i}\phi}\cos(\theta/2) & \sin(\theta/2) \\ -\sin(\theta/2) & \mathrm{e}^{-\mathrm{i}\phi}\cos(\theta/2) \end{pmatrix}, \quad (2)$$

for all $0\le\theta\le\pi$ and $0\le\phi\le\pi/2$; indeed, the strategies C and D can be associated with $U(0,0)$ and $U(\pi,0)$ respectively. Finally, the disentangling gate $\widehat{J}^+$ is carried out and the system is measured in the computational basis, giving rise to one of the four outcomes $|CC\rangle$, $|CD\rangle$, $|DC\rangle$, $|DD\rangle$. Alice's and Bob's payoffs are $\$_A = 3P_{CC} + 5P_{DC} + P_{DD}$ and $\$_B = 3P_{CC} + 5P_{CD} + P_{DD}$ respectively, where $P_{\sigma\sigma'} = \langle\sigma\sigma'|\,\rho\,|\sigma\sigma'\rangle$ is the probability of finding the state $|\sigma\sigma'\rangle$ (with $\sigma,\sigma'\in\{C,D\}$).

3. Scheme for implementing QG in cavity QED

Cavity QED has been extensively studied as a candidate system for realizing an efficient quantum information processor (QIP). In order to implement QG, we encode the qubits $|C\rangle$ ($|D\rangle$) in the form $|C\rangle = |g\rangle$, and $|D\rangle = |e\rangle$. Firstly, we consider how to realize CNOT gate and controlled phase gate(CZ gate).

Consider two identical two-level atoms simultaneously interacting with a single-mode cavity field. In the case $\delta\gg g$, there is no energy exchange between the atomic system and the cavity, where δ is the detuning between the atomic transition frequency ω_0 and the cavity frequency ω, and g is the atom–cavity coupling strength. Consider that the cavity field is in the vacuum state. In the interaction picture, the effective interaction Hamiltonian for the system is[17]

$$H = \lambda\left(\sum_{j=1}^{2}|e_j\rangle\langle e_j| + s_1^+ s_2^- + s_1^- s_2^+\right), \quad (3)$$

where $\lambda = g^2/\delta$, $s_j^+ = |e_j\rangle\langle g_j|$, $s_j^- = |g_j\rangle\langle e_j|, (j = 1,2)$, $|e_j\rangle$ and $|g_j\rangle$ are the excited and ground states of the jth atom, respectively.

Assume that two atoms are initially in the state $|e_1\rangle|g_2\rangle$ and the cavity field is in the vacuum state. As the two atoms interact simultaneously with a cavity, the state evolution of the system is

$$|\,\Psi\rangle = \mathrm{e}^{-\mathrm{i}\lambda t}[\cos(\lambda t)|e\rangle_1|g\rangle_2 - \mathrm{i}\sin(\lambda t)|g\rangle_1|e\rangle_2]. \quad (4)$$

We use ladder-type three-level atoms, whose states are denoted by $|g\rangle, |e\rangle, |i\rangle$. The transition frequency between the states $|e\rangle$ and $|i\rangle$ is highly detuned from the cavity frequency and thus the state $|i\rangle$ is not affected during the atom–cavity interaction. The quantum information is stored in the states $|g\rangle$ and $|e\rangle$. Assume that atom 1 acts as the control bit and atom 2 acts as the controlled bit. First, we let atom 2 cross two classical fields tuned to the transitions $|g\rangle\to|e\rangle$ and $|e\rangle\to|i\rangle$, respectively. Choose the amplitudes and phases of the classical fields appropriately so that this atom undergoes the transition: $|e_2\rangle \to \frac{1}{\sqrt{2}}(|e_2\rangle + |g_2\rangle) \to \frac{1}{\sqrt{2}}(|i_2\rangle + |g_2\rangle), |g_2\rangle \to \frac{1}{\sqrt{2}}(|g_2\rangle - |e_2\rangle) \to \frac{1}{\sqrt{2}}(|g_2\rangle + |i_2\rangle)$. Then we let atoms 1 and 2 simultaneously enter a single mode cavity. The interaction Hamiltonian has no effect on states $|g_1\rangle|g_2\rangle$ and $|g_1\rangle|i_2\rangle$, but induces a phase factor of Eq.(4). After an interaction time $\frac{\pi}{\lambda}$, we obtain $|e_1\rangle|i_2\rangle\to -|e_1\rangle|i_2\rangle$, $|e_1\rangle|g_2\rangle\to|e_1\rangle|g_2\rangle$. Then if atom 2 crosses one classical field tuned to the transi-

tion $|i\rangle \rightarrow |e\rangle$, we obtain the transformation

$$\begin{aligned} &|g_1\rangle|g_2\rangle \rightarrow |g_1\rangle|g_2\rangle, \\ &|g_1\rangle|e_2\rangle \rightarrow |g_1\rangle|e_2\rangle, \\ &|e_1\rangle|g_2\rangle \rightarrow |e_1\rangle|g_2\rangle, \\ &|e_1\rangle|e_2\rangle \rightarrow -|e_1\rangle|e_2\rangle. \end{aligned} \tag{5}$$

This is CS gate operation. If atom 2 crosses two classical field tuned to the transition $|e\rangle \rightarrow |i\rangle$ and $|g\rangle \rightarrow |e\rangle$, respectively. Choose the amplitudes and phases of the classical fields appropriately so that this atom undergoes the transition $|g_2\rangle \rightarrow \frac{1}{\sqrt{2}}(|g_2\rangle + |e_2\rangle)$, $|i_2\rangle \rightarrow |e_2\rangle \rightarrow \frac{1}{\sqrt{2}}(|e_2\rangle - |g_2\rangle)$. Thus we obtain CNOT gate:

$$\begin{aligned} &|g_1\rangle|g_2\rangle \rightarrow |g_1\rangle|g_2\rangle, \\ &|g_1\rangle|e_2\rangle \rightarrow |g_1\rangle|e_2\rangle, \\ &|e_1\rangle|g_2\rangle \rightarrow |e_1\rangle|e_2\rangle, \\ &|e_1\rangle|e_2\rangle \rightarrow |e_1\rangle|g_2\rangle. \end{aligned} \tag{6}$$

The unitary transformations of the form $U(\theta, \phi)$, which is the QG player's strategy, can be simply implemented by passing the atom through a classical laser field. The Hamiltonian, in the rotating-wave approximation and interaction picture, is given by

$$H = \frac{\hbar\Omega}{2}(\mathrm{e}^{-\mathrm{i}\phi}|e\rangle\langle g| + \mathrm{e}^{\mathrm{i}\phi}|g\rangle\langle e|), \tag{7}$$

where Ω is the Rabi frequency. The corresponding time evaluating unitary operator is given by

$$\begin{aligned} U_I(t) =& \exp(-\mathrm{i}Ht) \\ =& \begin{pmatrix} \cos\dfrac{\Omega_R t}{2} & \mathrm{i}\sin\dfrac{\Omega_R t}{2}\mathrm{e}^{-\mathrm{i}\phi} \\ \mathrm{i}\sin\dfrac{\Omega_R t}{2}\mathrm{e}^{-\mathrm{i}\phi} & \cos\dfrac{\Omega_R t}{2} \end{pmatrix}. \end{aligned} \tag{8}$$

If we set $\phi = \dfrac{3\pi}{2}$, then

$$U_I(t) = \begin{pmatrix} \cos\dfrac{\Omega_R t}{2} & \sin\dfrac{\Omega_R t}{2} \\ -\sin\dfrac{\Omega_R t}{2} & \cos\dfrac{\Omega_R t}{2} \end{pmatrix},$$

which is $R_y(-\theta)$ operation. If we set $\phi = 0$, then we have

$$U_I(t) = \begin{pmatrix} \cos\dfrac{\Omega_R t}{2} & \mathrm{i}\sin\dfrac{\Omega_R t}{2} \\ \mathrm{i}\sin\dfrac{\Omega_R t}{2} & \cos\dfrac{\Omega_R t}{2} \end{pmatrix},$$

which is $R_x(-\theta)$ operation $\left(\theta \text{ is } \dfrac{\Omega_R t}{2}\right)$.

In order to realize the operation of $R_Z(\theta)$, we can let the atom cross three classical laser fields. Let the atom interact with the first classical field, choosing the amplitude and phase of the classical field appropriately, $\Omega t = \pi/2, \phi = 3\pi/2$. Then, let the atom cross the second classical field, choosing $\Omega t = \theta, \phi = 0$. Finally, let the atom cross the third classical field, choosing $\Omega t = \pi/2, \phi = \pi/2$. After completing the above three steps, we obtain the transitions: $U_I(t) = \begin{pmatrix} \mathrm{e}^{\mathrm{i}\theta} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}\theta} \end{pmatrix}$, which is the rotation around z by an angle θ. So the unitary operator of the player's strategy can be realized by using the following identity:

$$\hat{U}(\theta, \phi) = R_Z(\phi)R_Y(-\theta)R_Z(\phi), \tag{9}$$

which can be easily realized in cavity QED.

The entangling $\hat{J}$ gate is the most important operation in QG since it introduces quantum entanglement. The transformation to realize the $\hat{J}$ gate can be expressed as the product of controlled-Z gate(CZ gate) transformation and controlled-NOT gate transformation and unitary transformation, which is shown in Fig.2.

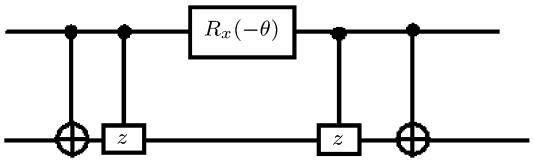


Fig.2. A circuit notation for $\hat{J}$ gate.

CZ gate and CNOT gate have been discussed as above. As a direct test of the entangling $\hat{J}$ gate, let us consider the setup. First, the state $|CC\rangle$ passes through, when the first CZ gate and CNOT gate are applied, and the state is unchanged. Then $R_x(-\theta)$ operation acts on the first qubit. Finally, CZ gate and CNOT gate are applied again. So the state evolves into a superposition state: $\cos(\gamma/2)|C\rangle|C\rangle + \mathrm{i}\sin(\gamma/2)|D\rangle|D\rangle$, where the parameter γ measures the entanglement in the game's initial state. With the realization of $\widehat{J}^+$ gate, the only difference from Fig.2 is the $R_x(\theta)$, which is a rotation around x by an angle θ.

4. Conclusion

In summary, we have proposed a scheme to realize QG in cavity QED. During the operation, the cavity is only virtually excited and thus the proposal is insensitive to the cavity field states and cavity decay. So the proposal can be experimentally realized in the range of current cavity QED techniques.

References

[1] Neilsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (New York: Cambridge University Press)

[2] Wu Y and Deng L 2005 *Phys. Rev.* A **71** 053806

[3] Meyer D 1999 *Phys. Rev. Lett.* **82** 1052

[4] Eisert J, Wilkens M and Lewenstein M 1999 *Phys. Rev. Lett.* **83** 3077

[5] Du J F, Li H, Xu X D, Zhou X Y and Han R D 2001 *Phys. Lett.* A **289** 9

[6] Benjamn S C and Hayden P M 2001 *Phys. Rev.* A **64** 030301

[7] Du J F, Li H, Xu X D, Zhou X Y and Han R D 2002 *Phys. Lett.* A **302** 229

[8] Du J F, Li H, Xu X D, Zhou X Y and Han R D 2002 *Chin. Phys. Lett.* **19** 1221

[9] Li H, Du J F and Massar S 2002 *Phys. Lett.* A **306** 73

[10] Chen K L, Ang H, Kiang D, Kwek L C and Lo C F 2003 *Phys. Lett.* A **316** 317

[11] Flitney A P and Abbott D 2005 *J. Phys.* A: *Math. Gen.* **38** 449

[12] Cao S and Fang M F 2006 *Chin. Phys.* **15** 60

[13] Du J F, Li H, Xu X, Shi M, Wu J and Han R 2002 *Phys. Rev. Lett.* **88** 137902

[14] Zhou L and Kuang L M 2003 *Phys. Lett.* A **315** 426

[15] Lu J, Zhou L and Kuang L M 2004 *Phys. Lett.* A **330** 48

[16] Buluta I M, Fujiwara , Hasegawa S 2006 *Phys. Lett.* A **330** 48

[17] Zheng S B and Guo G C 2000 *Phys. Rev. Lett.* **85** 2392

[18] Scully M O and Zubairy M Z 2002 *Phys. Rev.* A **65** 052324

[19] Collins D, Kim K W and Holton W C 1998 *Phys. Rev.* A **58** 1633
Zheng S B 2004 *Phys. Rev.* A **70** 034301

[20] Bennett C H and Wiesner S J 1992 *Phys. Rev. Lett.* **69** 2881
He J, Ye L and Ni Z X 2008 *Chin. Phys.* B **17** 1597

[21] Grover L K 1997 *Phys. Rev. Lett.* **79** 325
Deng Z J, Feng M and Gao K L 2005 *Phys. Rev.* A 2005 **73** 014302

[22] Chuang I L 2000 *Phys. Rev. Lett.* **85** 2006
Wu Q Q and Kuang L M 2006 *Chin. Phys.* **15** 2593