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The influences of dipole–dipole interaction and detuning on the sudden death of entanglement between two atoms in the Tavis–Cummings model*

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The influences of dipole–dipole interaction and detuning on the entanglement between two atoms with different initial tripartite entangled W-like states in the Tavis–Cummings model have been investigated by means of Wootters' concurrence, respectively. The results show that the entanglement between the two atoms can be enhanced via appropriately tuning the strength of dipole–dipole interaction of two atoms or the detunings between atom and cavity, and the so-called sudden death effect can be weakened simultaneously.

Keywords: concurrence, sudden death of entanglement, dipole–dipole interaction, detuning
PACC: 0365, 4250

1. Introduction

The view of entanglement has gone from an important consequence of quantum theory to a physical resource on which many kinds of quantum technologies are based. Hence, it is of crucial importance not only to understand entanglement dynamics, but also to manipulate entanglement states in a practical way. However, it is still a challenge to control the evolution of qubits for realizing quantum computation and quantum information processing.[1−6] For the past few years, entanglement dynamics has been studied qualitatively and quantitatively.[7−14] Yu and Eberly investigated the evolution of entanglement of a bipartite system in which the entanglement between two particles coupled with two independent cavities became completely vanishing in a finite time. This intriguing discovery is termed entanglement sudden death (ESD).[8,9] It has aroused great interest in the dynamics of entanglement in bipartite systems. Further investigations have been made by various groups.[15−29] The effects on the entanglement between particles, such as dipole–dipole interaction between the particles and the couplings of particles with the same cavity field, have been studied extensively. Recently, the entanglement character between two identical two-level atoms in a two-mode cavity field has been studied.[15] The authors discussed the influence of dipole–dipole interaction on entanglement between atoms. The result showed that the introduction of dipole–dipole interaction can weaken the sudden death effect. In previous works, the authors all make the cavity initially in the vacuum state and the two atoms in a pure entangled state. Different from those contexts, Yang *et al*[17] studied the entanglement dynamics of two atoms with initial tripartite entangled W-like state in the Tavis–Cummings model (TCM)[16] in which the dipole–dipole interaction between atoms was ignored.[17] Enlightened by the above works, we consider the influence of dipole–dipole interaction on the sudden death of two atoms with different initial tripartite entanglement states in TCM. The results show that the introduction of dipole–dipole interaction can give rise to a higher entanglement between the atoms and diminish the difference of entanglement evolvement between two different initial states of the system. It is possible to completely eliminate the sudden death effect for some initial states of the system. Furthermore, we study the influence of atom–cavity detuning on the evolution of entanglement between two atoms in the absence of the dipole–dipole interaction in TCM, and find that the sudden death effect

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can also be weakened through appropriately modulating the detunings between the frequencies of the atomic transition and the cavity.

2. The influence of dipole–dipole interaction on entanglement

We consider the TCM in which two identical two-level atoms interact with a single-mode cavity resonantly in the presence of dipole–dipole interaction of the two atoms. In the rotation wave approximation, this system is described by the interaction Hamiltonian in the interaction picture

$$\begin{aligned} H_{\rm I} =&\lambda \sum_{i={\rm A,B}} \left(a\sigma_i^\dagger + \sigma_i a^\dagger\right) \\ &+ \Omega\left(\sigma_{\rm A}^\dagger\sigma_{\rm B} + \sigma_{\rm B}^\dagger\sigma_{\rm A}\right), \end{aligned} \tag{1}$$

where $a^\dagger$ (a) denotes the creation (annihilation) operator of the cavity mode; $\sigma_i = |g_i\rangle\langle e_i|$ is the atomic operator with $|e_i\rangle$ and $|g_i\rangle$ being the excited and ground states of the ith atom; λ is the coupling constant of the interaction between each atom and the cavity; Ω is dipole–dipole coupling strength between the atoms.

We choose the atoms and the cavity to be prepared in an entangled W-like state at the initial time,

$$|\psi_0\rangle = a|eg0\rangle + b|ge0\rangle + c|gg1\rangle, \tag{2}$$

with $|a|^2 + |b|^2 + |c|^2 = 1$. There are many ways to prepare this form of state.[30,31]

Because the state of the system follows the Schrödinger equation, we can acquire the corresponding time evolution of the system at time t according to Eqs.(1) and (2)

$$|\psi(t)\rangle = x_1|eg0\rangle + x_2|ge0\rangle + x_3|gg1\rangle. \tag{3}$$

The corresponding coefficients are given by

$$\begin{aligned} x_1 =&\frac{\Lambda}{4\kappa}\left\{a\left[\epsilon\left(1-{\rm e}^{{\rm i}\kappa\theta}\right) + \kappa\left(1+{\rm e}^{{\rm i}\kappa\theta}+2\Gamma\right)\right]\right. \\ &+ b\left[\epsilon\left(1-{\rm e}^{{\rm i}\kappa\theta}\right)\right. \\ &\left.\left.+ \kappa\left(1+{\rm e}^{{\rm i}\kappa\theta}-2\Gamma\right)\right] + 4c\left(1-{\rm e}^{{\rm i}\kappa\theta}\right)\right\}, \\ x_2 =&\frac{\Lambda}{4\kappa}\left\{a\left[\epsilon\left(1-{\rm e}^{{\rm i}\kappa\theta}\right) + \kappa\left(1+{\rm e}^{{\rm i}\kappa\theta}-2\Gamma\right)\right]\right. \\ &+ b\left[\epsilon\left(1-{\rm e}^{{\rm i}\kappa\theta}\right)\right. \\ &\left.\left.+ \kappa\left(1+{\rm e}^{{\rm i}\kappa\theta}+2\Gamma\right)\right] + 4c\left(1-{\rm e}^{{\rm i}\kappa\theta}\right)\right\}, \\ x_3 =&\frac{\Lambda}{2\kappa^2}\left\{2\kappa(a+b)\left(1-{\rm e}^{{\rm i}\kappa\theta}\right) + c\left[\left(8+\epsilon\kappa+\epsilon^2\right){\rm e}^{{\rm i}\kappa\theta}\right.\right. \\ &\left.\left.+ \left(8-\epsilon\kappa+\epsilon^2\right)\right]\right\} \end{aligned} \tag{4}$$

with $\epsilon = \Omega/\lambda$, $\theta = \lambda t$, $\kappa = \sqrt{8+\epsilon^2}$, $\Gamma = {\rm e}^{{\rm i}(3\epsilon+\kappa)\theta/2}$ and $\Lambda = {\rm e}^{-{\rm i}(\epsilon+\kappa)\theta/2}$. The information about the entanglement of two atoms is contained in the reduced density matrix $\rho^{\rm AB}$ for the two atoms which can be obtained from Eq.(3) by tracing out the state of the cavity. The explicit matrix written in the standard basis $\{|ee\rangle, |eg\rangle, |ge\rangle, |gg\rangle\}$ is given by

$$\rho^{\rm AB} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & |x_1|^2 & x_1x_2^* & 0 \\ 0 & x_2x_1^* & |x_2|^2 & 0 \\ 0 & 0 & 0 & |x_3|^2 \end{pmatrix}, \tag{5}$$

where $*$ stands for the complex conjugate.

In order to quantify the degree of entanglement, we use Wootters' concurrence[32] which is widely applied for an arbitrary state of two qubits. The corresponding concurrence for this density matrix Eq.(5) is given by

$$C(\rho^{\rm AB}) = 2\ \max\{0, |x_1x_2|\}. \tag{6}$$

Figure 1 shows the entanglement evolution of two atoms with different initial states. Obviously, there is no sudden death phenomenon when the system is in this initial state $|\psi_0\rangle = a|eg0\rangle + b|geo\rangle + c|gg1\rangle$. Comparing the solid line with the dashed one, one can find that the evolution period gets shorter with increasing ϵ. It can be observed easily that the entanglement between two atoms can be strengthened evidently by introducing the dipole–dipole interaction. However, the maximum of concurrence does not always increase with the increase of ϵ, which can be seen clearly in Fig.2. It can arrive at the maximum value 1 for a certain ϵ at some evolution time points for different initial states. When $\epsilon = 4$ and 6, the concurrence for the two different initial states corresponding to Figs.1(a) and 1(b) can arrive at the maximum, respectively.

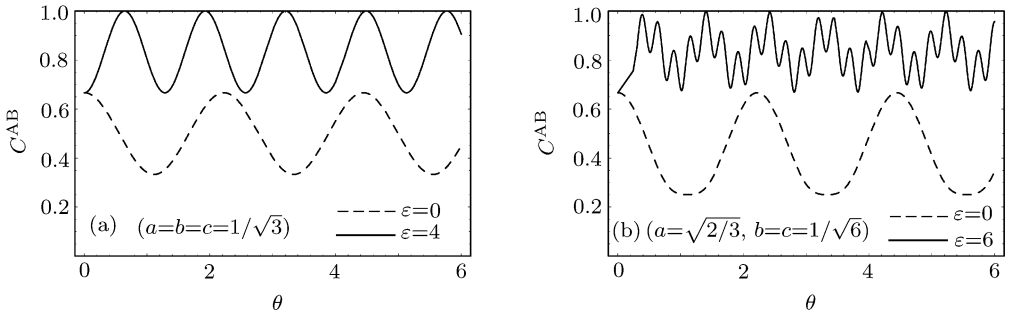


Fig.1. The concurrence for atom–atom entanglement with the initial systemic state $|\psi_0\rangle = a|eg0\rangle + b|geo\rangle + c|gg1\rangle$.

From Fig.2 three points can be concluded as below: (i) In the region of $\epsilon \leq 1$, $C^{\rm AB}_{\rm max}$ is unchanged. Namely, the dipole–dipole interaction does not contribute to the maximum of concurrence. (ii) The concurrence can arrive at 1 when $\epsilon = 4$. (iii) When $\epsilon > 1$, the $C^{\rm AB}_{\rm max}$ is always larger than the one with $\epsilon = 0$. When the system is in different initial conditions, we can also find the corresponding optimal values of ϵ in a similar way.

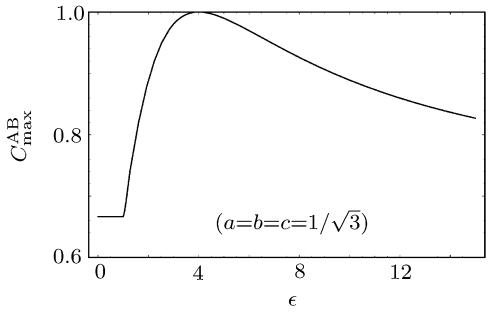


Fig.2. The variation of $C^{\rm AB}_{\rm max}$ with $\epsilon = \Omega/\lambda$.

Now we assume that the initial state for the total system is in another form of W-like state,

$$|\psi_0'\rangle = a|ge1\rangle + b|eg1\rangle + c|ee0\rangle \tag{7}$$

with $|a|^2 + |b|^2 + |c|^2 = 1$.

In this case, the state of the total system governed by the Hamiltonian (1) at time t can be expressed in the standard basis

$$|\psi(t)\rangle = x_1|ee0\rangle + x_2|eg1\rangle + x_3|ge1\rangle + x_4|gg2\rangle, \tag{8}$$

where the coefficients are now given by

$$\begin{aligned}
x_1 =& \frac{\nu}{6\eta}\big\{6(a+b)\big(1-\mathrm{e}^{\mathrm{i}\eta\theta}\big) + c\big[\epsilon\big(\mathrm{e}^{\mathrm{i}\eta\theta}-1\big) \\
&+ \eta\big(1+\mathrm{e}^{\mathrm{i}\eta\theta}+4\xi\big)\big]\big\}, \\
x_2 =& \frac{\nu}{4\eta}\big\{a\big[\epsilon\big(1-\mathrm{e}^{\mathrm{i}\eta\theta}\big) + \eta\big(1+\mathrm{e}^{\mathrm{i}\eta\theta}-2\Xi\big)\big] \\
&+ b\big[\epsilon\big(1-\mathrm{e}^{\mathrm{i}\eta\theta}\big) \\
&+ \eta\big(1+\mathrm{e}^{\mathrm{i}\eta\theta}+2\Xi\big)\big] + 4c\big(1-\mathrm{e}^{\mathrm{i}\eta\theta}\big)\big\}, \\
x_3 =& \frac{\nu}{4\eta}\big\{a\big[\epsilon\big(1-\mathrm{e}^{\mathrm{i}\eta\theta}\big) + \eta\big(1+\mathrm{e}^{\mathrm{i}\eta\theta}+2\Xi\big)\big] \\
&+ b\big[\epsilon\big(1-\mathrm{e}^{\mathrm{i}\eta\theta}\big) \\
&+ \eta\big(1+\mathrm{e}^{\mathrm{i}\eta\theta}-2\Xi\big)\big] + 4c\big(1-\mathrm{e}^{\mathrm{i}\eta\theta}\big)\big\}, \\
x_4 =& \frac{\nu}{3\sqrt{2}\eta}\big\{6(a+b)\big(1-\mathrm{e}^{\mathrm{i}\eta\theta}\big) + c\big[\epsilon\big(\mathrm{e}^{\mathrm{i}\eta\theta}-1\big) \\
&+ \eta\big(1+\mathrm{e}^{\mathrm{i}\eta\theta}-2\xi\big)\big]\big\}
\end{aligned} \tag{9}$$

with $\eta = \sqrt{24+\epsilon^2}$, $\xi = \mathrm{e}^{\mathrm{i}(\epsilon+\eta)\theta/2}$, $\Xi = \mathrm{e}^{\mathrm{i}(3\epsilon+\eta)\theta/2}$ and $\nu = \mathrm{e}^{-\mathrm{i}(\epsilon+\eta)\theta/2}$. In the basis of $\{|ee\rangle, |eg\rangle, |ge\rangle, |gg\rangle\}$, we can write the corresponding reduced density matrix

$$\rho^{\rm AB} = \begin{pmatrix} |x_1|^2 & 0 & 0 & 0 \\ 0 & |x_2|^2 & x_2x_3^* & 0 \\ 0 & x_3x_2^* & |x_3|^2 & 0 \\ 0 & 0 & 0 & |x_4|^2 \end{pmatrix}. \tag{10}$$

The concurrence of this mixed state can be obtained in the same way

$$C(\rho^{\rm AB}) = 2\ \max\{0, |x_2x_3| - |x_1x_4|\}. \tag{11}$$

It can be seen that the state $\rho^{\rm AB}$ is separable if $|x_2x_3| - |x_1x_4| \leq 0$. We plot a figure of concurrence versus θ in order to see the more interesting consequences clearly. Other than the previous case, we could find that the entanglement can fall abruptly to zero and remain to be in this state for a period of time before entanglement resurrects, as shown in Fig.3. Hence, the ESD is closely related to the initial state of the system. The length of the time interval for zero entanglement is dependent on the concrete form of the initial state. In contrast to the dashed lines, the solid lines show that the averages of $C^{\rm AB}$ are much higher and the evolution periods get shorter. Another exciting and important point is that the introduction of dipole–dipole interaction can completely eliminate the sudden death effect when the coefficients are chosen at $a = b = c = 1/\sqrt{3}$ and $a = \sqrt{2/3}$, $b = c = 1/\sqrt{6}$.

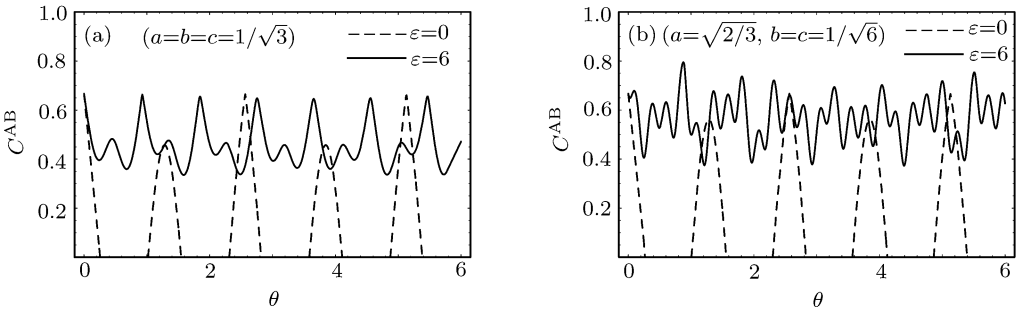


Fig.3. The concurrence for atom–atom entanglement with the initial systemic state $|\psi_0'\rangle = a|ge1\rangle + b|eg1\rangle + c|ee0\rangle$.

However, the ESD is not always absent for any ϵ, as shown in Fig.4. In the range of $\epsilon \leq 3$, the ESD is still present. The minimum of concurrence remains vanishing in this range. When $\epsilon > 3$, the value of $C^{\rm AB}_{\rm min}$ increases with the increase of ϵ and arrives at the maximum with ϵ roughly equal to 5. And then it can be seen that this maximum keeps nearly stationary with the increase of ϵ.

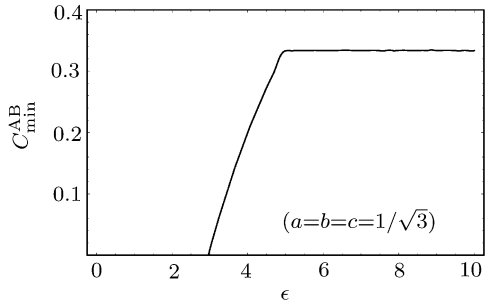


Fig.4. The variation of $C^{\rm AB}_{\rm min}$ with $\epsilon = \Omega/\lambda$.

3. The influence of detuning on entanglement

In the previous section, we discussed the dynamics of entanglement between the two atoms interacting resonantly with the cavity in the Tavis–Cummings model. The ESD effect can be weakened by introducing the dipole–dipole interaction. Next we will solely study that the influence of detuning on the evolution of entanglement between two atoms in the absence of the dipole–dipole interaction. A simple Hamiltonian can be written as

$$H_{\rm I} = \lambda \sum_{i={\rm A,B}} \left(a\sigma_i^{\dagger} {\rm e}^{{\rm i}\Delta t} + \sigma_i a^{\dagger} {\rm e}^{-{\rm i}\Delta t}\right), \qquad (12)$$

where $\Delta = \omega - \nu$, ω and ν are the frequencies of atomic transition and cavity, respectively.

Here we choose the same initial state given by Eq.(2). According to the previous calculation means, we can obtain the corresponding coefficients of Eq.(3),

$$\begin{aligned} x_1 =& \frac{1}{4\alpha\delta}\big\{(a+b)\big[\beta\alpha^2(\delta-\mu)+\beta(\delta+\mu)\big] \\ &+ 2\alpha\delta(a-b) + 4c\beta(1-\alpha^2)\big\}, \\ x_2 =& \frac{1}{4\alpha\delta}\big\{(a+b)\big[\beta\alpha^2(\delta-\mu)+\beta(\delta+\mu)\big] \\ &- 2\alpha\delta(a-b) + 4c\beta(1-\alpha^2)\big\}, \\ x_3 =& \frac{1}{2\alpha\beta\delta}\big\{\big(1-\alpha^2\big)[2(a+b)-\mu c] \\ &+ c\delta\big(1+\alpha^2\big)\big\} \end{aligned} \qquad (13)$$

with $\mu = \Delta/\lambda$, $\theta = \lambda t$, $\delta = \sqrt{8+\mu^2}$, $\alpha = {\rm e}^{{\rm i}\delta\theta/2}$, $\beta = {\rm e}^{{\rm i}\mu\theta/2}$.

In order to quantify the degree of entanglement, we adopt the same way as the previous scheme to calculate the concurrence $C^{\rm AB}$, as shown in Fig.5 which is very similar to Fig.1. Namely, the modulation of detunings makes a similar contribution to the evolution of entanglement as dipole–dipole interaction. It also can cause the entanglement between two atoms to enhance and arrive at the maximal value 1 for a certain μ at some evolution time points for different initial states.

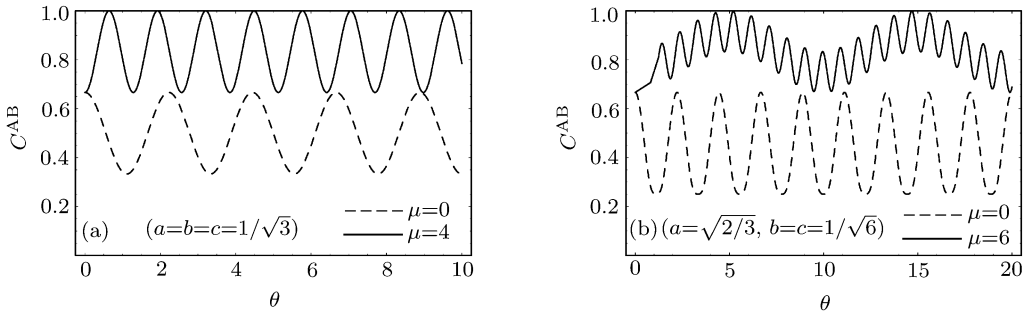


Fig.5. The concurrence for atom–atom entanglement with the initial systemic state $|\psi_0\rangle = a|eg0\rangle + b|geo\rangle + c|gg1\rangle$.

When we choose another form of initial state given by Eq.(7). The corresponding concurrence $C^{\rm AB}$ with $\lambda = 1$ can be given, as shown in Fig.6. In this case, if the frequency of atomic transition is larger than the one of cavity, i.e., $\Delta > 0$, we find that the effect is not expected by plotting the figures, on the contrary the sudden death of entanglement between two atoms is well weakened and even restrained when $\Delta < 0$, as shown in Fig.6. We can see when the detuning is given by $\Delta = -6$, the evolutions of the entanglement between two atoms exhibit periodical oscillations and the amplitudes show obvious increase, implying that the modulation of detunings makes the entanglement between the atoms stronger and the period of entanglement vanishing shorter. The sudden death effect can be completely eliminated for $a = \sqrt{2/3}$, $b = c = 1/\sqrt{6}$, as shown in Fig.6(b).

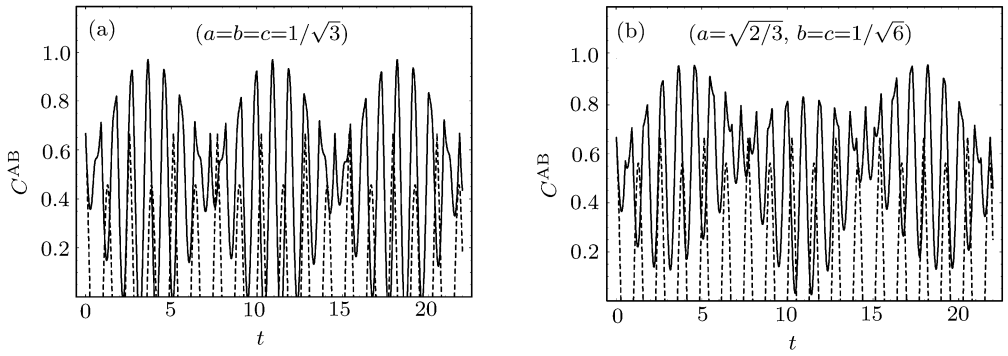


Fig.6. The concurrence for atom-atom entanglement with the initial systemic state $|\psi_0'\rangle = a|ge1\rangle + b|eg1\rangle + c|ee0\rangle$ when $\lambda = 1$. Dashed line: $\Delta = 0$; solid line: $\Delta = -6$.

4. Discussion and conclusion

How should we understand these interesting phenomena? We try to discuss this problem based on our model. We first review the Hamiltonian equations (1) and (12) at $\Omega = 0$ and $\Delta = 0$, respectively. It can be seen that the dynamics of the two independent two-level atoms is determined by the interaction Hamiltonian. It is possible that the energy transfer leads to the ESD effect. The complete disentanglement of the two atoms is possibly due to the information loss of atomic dynamics to the cavity mode. The entanglement sudden birth can be expected such that the loss information will come back to the atoms after a finite period since the cavity is lossless in our model. In our own opinion, it may also suggest some kind of 'entanglement conservation'. This is an open issue that is unexplored and we think that conservation of entanglement may not exist in the sense of dynamical conservation laws.

In the absence of dipole–dipole interaction, two atoms interact with the cavity individually in the TCM. Thus, the initial entangled state is destroyed in the process of interacting between the atoms and cavity. However, after introducing dipole–dipole interaction, the relationship of the two atoms gets stronger so

that the initial entanglement between the two atoms is strengthened and may arrive at the maximal value of unity. Moreover, the investigation shows that atom–cavity detuning interaction can cause the entanglement to be enhanced in the same way but a rigorous interpretation in physics is still ambiguous, which will be discussed in further research.

In summary, we have investigated the entanglement character between two atoms in different forms of W-like state in the TCM. The results demonstrate that for the two different initial states of the system, the entanglement evolution appears dramatically different. Therefore, the ESD is very sensitive to the initial condition of the system. Otherwise, it is worth to say that the introduction of dipole–dipole interaction and detuning interaction can strengthen the entanglement between two atoms and weaken the sudden death effect and even completely eliminate the ESD in some instances.

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