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The quantum pressure correction to the excitation spectrum of the trapped superfluid Fermi gases in a BEC-BCS crossover*

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Using quantum hydrodynamic approaches, we study the quantum pressure correction to the collective excitation spectrum of the interacting trapped superfluid Fermi gases in the BEC-BCS crossover. Based on a phenomenological equation of state, we derive hydrodynamic equations of the system in the whole BEC-BCS crossover regime. Beyond the Thomas–Fermi approximation, expressions of the frequency corrections of collective modes for both spherical and axial symmetric traps excited in the BEC-BCS crossover are given explicitly. The corrections of the eigenfrequencies due to the quantum pressure and their dependence on the inverse interaction strength, anisotropic parameter and particle numbers of the condensate are discussed in detail.

Keywords: BEC-BCS crossover, hydrodynamic approaches, perturbation method
PACC: 6740D, 3120L

1. Introduction

The crossover from a Bose–Einstein condensate (BEC) to a Bardeen–Cooper–Schrieffer (BCS) superfluid has attracted considerable attention for decades.[1] Ultracold Fermi system provides an excellent opportunity for exploring the properties of the BEC-BCS crossover in a controllable way. The interaction of dilute ultracold fermion atoms, characterized by the s-wave scattering length, $a_{\rm sc}$, can be tuned by Feshbach resonance,[2] allowing one to manipulate the interaction strength over the range $-\infty < y = 1/(k_{\rm F}a_{\rm sc}) < \infty$, where $k_{\rm F} = (3\pi^2\rho)^{1/3}$ is the Fermi wave vector and $\rho(\boldsymbol{r},t)$ is the superfluid local particle density. Since the excitation spectra can be measured with high precision, both experimental[3,4] and theoretical[5−14] studies have paid much attention to the collective excitations of an interacting trapped superfluid Fermi gases.

It is known that the BEC-BCS crossover can be divided into three regimes. (i) BEC regime ($y > 1$): Bounded molecules (or called dimers) form by two fermions. These preformed fermionic pairs have a small size in space and show repulsive interaction.[5] The particular case $y \gg 1$ is called the deep BEC regime. (ii) BCS regime ($y < -1$): The fermions of the system form weakly bounded Cooper pairs. These condensed fermionic pairs have a large size in space and show attractive interaction.[5] In particular, $y \ll -1$ is called the deep BCS regime. (iii) BEC-BCS crossover regime ($-1 < y < 1$): The condensed fermionic pairs have the character of both BEC molecules and BCS Cooper pairs, or their mixing. Passing through the BEC-BCS crossover, a smooth transition occurs from BEC superfluid to BCS superfluid. Especially, the point $y = 0$ is called the unitarity limit, corresponding to $a_{\rm sc} = \pm\infty$. At this limit, the average spacing of particles $d_0 = (3\pi^2 N_0)^{1/3}/k_{\rm F}$ is still a typical length with N_0 the condensed particle number of the system.

Since no thermal excitation is present at $T = 0$, the system is a perfect superfluid and can be well described by the local particle density $\rho(\boldsymbol{r},t)$ and local velocity $\boldsymbol{v}(\boldsymbol{r},t)$ based on the time-dependent density functional theory.[7−11] It is well known that the interaction between the condensate fermionic pairs in a homogeneous system is characterized by a chemical potential $\mu(\rho)$ (or called equation of state), which can be obtained by many techniques. For fermions confined in a trapping potential $V(\boldsymbol{r})$, the density profile $\rho(\boldsymbol{r},t)$ changes slowly in space if the particle number of the system is large enough. Under such conditions

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a local density approximation can be applied to the equation of state and hence one can suggest a hydrodynamic approach to investigate the physical properties of the collective excitations at low temperatures.

On the one hand, theoretical studies of the collective excitations in the BEC-BCS crossover have conducted based on the hydrodynamic approach, and all eigenspectra and eigenfunctions of the collective modes in the harmonic trap have been obtained analytically under a Thomas–Fermi (TF) approximation.[7] However, in the usual experiments the trapped systems are always performed with a finite number of atoms where the TF approximation is not valid. Further attempts are necessary to go beyond TF limit for seeking the effects of the density fluctuation on kinetic energy enhancement. This is because the quantum press term is important especially for small numbers of atoms. The study of this effect by the numerical work has been reported in Ref.[10]. But to our best knowledge, no analytical work has been reported. On the other hand, a very intriguing question in the study of the BCS-BEC crossover is the presence or absence of the famous Lee–Huang–Yang terms[15,16] in the equation of state, corresponding to beyond-mean-field terms of composite bosons. The experimental result reported recently by Grimm's group[17] demonstrated clearly that mean-field theory is not valid and hence one should develop BCS-BEC crossover theory beyond the mean-field approximation. The generalized quantum hydrodynamic approach also takes into account the beyond-mean-field corrections. Further attempts are also necessary to construct the equation of state based on the asymptotic behaviour and quantum Monte Carlo simulation. In one word, for a better understanding of the competition among kinetic energy, interaction energy and external potential, we must go beyond the TF approximation and consider the quantum pressure term in the BEC-BCS crossover.

The goal of this paper is to study the quantum pressure correction to the excitation spectrum and to explain the effects of the inverse interaction strength, anisotropic parameter and particle number on the corrections for the trapped superfluid Fermi gases in the BEC-BCS crossover. The article is arranged as follows: in Section 2 we propose a phenomenological equation of state and obtain an analytical expression for the ground state by use of the variational method. In Section 3 we list the zeroth-order solution to the hydrodynamic equation and obtain the first-order correction for both sphere condensate ($\lambda = 1$) and spheroid condensate ($\lambda \neq 1$) by use of the standard perturbation method. We emphasize the importance of the quantum pressure term to improve the hydrodynamic approach. In Section 4, a conclusion is drawn.

2. Equation of state and ground state

By use of a Gibbs–Duhem relation, the chemical potential reads $\mu(\rho) = \dfrac{\partial[\rho\varepsilon(\rho)]}{\partial\rho}$, where $\varepsilon(\rho)$ is the ground state energy per particle at zero temperature, and can be written as $\varepsilon(\rho) = \dfrac{3}{5}\epsilon_{\rm F}\epsilon(y)$ with $\epsilon_{\rm F} = \dfrac{\hbar^2 k_{\rm F}^2}{2M} = \dfrac{\hbar^2}{2M}(3\pi^2\rho)^{2/3}$. Here $\epsilon_{\rm F}$ is the Fermi energy and M the mass of fermions. But $\epsilon(y)$ is still an unknown function in the general case. We first focus on the expression for $\epsilon(y)$, after some physical consideration.

On the one hand, for the analytical treatment, one assumes $\varepsilon(\rho) = c\rho^\gamma$ with c the constant dependent on y and γ. Here γ as a function of y[8] is $\gamma(y) = \left[\dfrac{2}{3}\epsilon(y) - \dfrac{2y}{5}\epsilon'(y) + \dfrac{y^2}{15}\epsilon''(y)\right] / \left[\epsilon(y) - \dfrac{y}{5}\epsilon'(y)\right]$. On the other hand, $\epsilon(y)$ have been obtained in the BEC, BCS and unitarity limits as power series of y:

$$\begin{cases} \epsilon_{\rm BCS} = 1 + \dfrac{10}{9\pi y} + \dfrac{4}{21\pi^2}(11 - 2\ln 2)\dfrac{1}{y^2} + O(y^{-3}) & \text{if } \ y \ll -1(\gamma \leq 2/3), \\ \epsilon_{\rm UL} = 0.42 - y - \dfrac{5}{3}y^2 + O(y^3) & \text{if } \ |y| \ll 1(\gamma \approx 2/3), \\ \epsilon_{\rm BEC} = \dfrac{1}{6\pi y}\left[1 + \dfrac{128}{15\sqrt{6\pi^3}}\left(\dfrac{3}{5y}\right)^{2/3} + O(y^{-2})\right] & \text{if } \ y \gg 1(\gamma \approx 1). \end{cases} \tag{1}$$

Here $\epsilon_{\rm BCS}$ and $\epsilon_{\rm BEC}$ are the microscopic many-body theoretical results and $\epsilon_{\rm UL}$ is the quantum Monte Carlo simulation result[10] for the asymptotic behaviour of $\epsilon(y)$ in the BEC, BCS, and unitarity limits. We now

propose a phenomenological expression of $\epsilon(y)$ for connecting above three limits smoothly:

$$\begin{cases} \epsilon = \alpha_1 - \dfrac{2(1-\alpha_1)}{\pi}\arctan\left[\dfrac{9(1-\alpha_1)}{5y}\dfrac{\beta_1 - y}{\beta_2 - y}\right] & \text{if } \ y \le 0, \\ \epsilon = \dfrac{2}{\pi}\arctan(1/y)\left[\alpha_1 - \alpha_2 \arctan\dfrac{\alpha_3 y^{3/2}}{\arctan(\alpha_4\sqrt{y})}\right] & \text{if } \ y \ge 0, \end{cases} \tag{2}$$

with six parameters $\alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4$ and $\beta_1,\ \beta_2$. After contrasting function (2) to the three limits (1) with their values and the first-order derivatives, we obtain the exact results for the six parameters: $\alpha_1 = 0.42$, $\alpha_2 = \dfrac{2}{\pi}(\alpha_1 - 1/12)$, $\alpha_3 = 15\sqrt{2}(5\pi)^{3/2}\dfrac{\alpha_1 - 1/12}{32}$, $\alpha_4 = \dfrac{\alpha_2\alpha_3}{1 - 2\alpha_1/\pi}$, $\beta_1 = \dfrac{6}{7}\dfrac{11 - 2\ln 2}{5\pi - 18(1-\alpha_1)^2}$ and $\beta_2 = \beta_1 - \dfrac{6}{25\pi}(11 - 2\ln 2)$. So we have the simple and analytical expression of $\varepsilon(\rho) = c\rho^\gamma = \dfrac{3}{5}\epsilon_{\rm F}\epsilon(y)$ with phenomenological formula of $\epsilon(y)$ and $\gamma(y)$ above in the whole BEC-BCS crossover regime.

We then turn to analyse the ground state based on the equation of state $\mu(\rho) = (\gamma + 1)c\rho^\gamma$. We write down the time-dependent nonlinear Schrödinger equation[8,10,11]

$$\mathrm{i}\hbar\dot{\Phi}(\boldsymbol{r},t) = \left[-\frac{\hbar^2}{2M}\nabla^2 + V(\boldsymbol{r}) + \mu(\rho(\boldsymbol{r},t))\right] \times \Phi(\boldsymbol{r},t). \tag{3}$$

By taking $\Phi(\boldsymbol{r},t) = \sqrt{\rho(\boldsymbol{r},t)}\mathrm{e}^{\mathrm{i}\phi(\boldsymbol{r},t)}$, Eq.(3) is transferred into the following hydrodynamic equations[10]

$$\dot{\rho}(\boldsymbol{r},t) + \nabla[\rho(\boldsymbol{r},t)\boldsymbol{v}(\boldsymbol{r},t)] = 0, \tag{4}$$

$$M\dot{\boldsymbol{v}}(\boldsymbol{r},t) + \nabla[T_{\rm qp} + V(\boldsymbol{r}) + \mu(\rho(\boldsymbol{r},t)) + \frac{1}{2M}|\boldsymbol{v}(\boldsymbol{r},t)|^2] = 0, \tag{5}$$

where $\boldsymbol{v}(\boldsymbol{r},t) = -\dfrac{\hbar}{M}\nabla\phi(\boldsymbol{r},t)$ is the superfluid velocity and $T_{\rm qp} = -\hbar^2\dfrac{\nabla^2\sqrt{\rho(\boldsymbol{r},t)}}{2M\sqrt{\rho(\boldsymbol{r},t)}}$ is the quantum pressure term. For the ground state of the system, one has $\boldsymbol{v}(\boldsymbol{r},t) = 0$ and $\rho(\boldsymbol{r},t) = \rho_0(\boldsymbol{r})$. So Eqs. (4) and (5) reduce to

$$-\frac{\hbar^2}{2M}\nabla^2\sqrt{\rho_0(\boldsymbol{r})} + [V(\boldsymbol{r}) + \mu(\rho_0(\boldsymbol{r}))]\sqrt{\rho_0(\boldsymbol{r})} = \mu_G\sqrt{\rho_0(\boldsymbol{r})}, \tag{6}$$

where $\mu_{\rm G}$ is the chemical potential in the ground state. The trapping potential in the usual experiments[4,5,12–14,18,19] is the harmonic one

$$V(\boldsymbol{r}) = \frac{1}{2}M\omega_\perp^2(s^2 + \lambda^2 z^2), \tag{7}$$

where $s^2 = x^2 + y^2$, $\omega_\perp$ is the harmonic oscillator frequency in the x and y directions, and $\lambda = \omega_z/\omega_\perp$ is the anisotropic parameter (ω_z is the frequency in the z direction). Since it is difficult to solve Eq.(6) for the ground state exactly, we adopt a variational solution

$$\rho_0(\boldsymbol{r}) = \rho_0(0)(1 - \bar{s}^2 - \lambda^2\bar{z}^2)^\beta, \tag{8}$$

where $\bar{s} = s/R_\perp$, $\bar{z} = z/R_z$ and $\beta = (1+q)/\gamma$. In Eq.(8) we have introduced three dimensionless variational parameters: $\tilde{R}_\perp = R_\perp/a_{\rm ho} \gg 1$ and $\tilde{R}_z = R_z/a_{\rm ho} \gg 1$ are respectively the radial and axial TF radius of the fermionic cloud in the ground state scaled in $a_{\rm ho} = \sqrt{\hbar/M\omega_\perp}$, the characteristic length of the harmonic oscillator, and q ($0 < q < \infty$) is the sign of departure of the system from the TF limit. The normalization condition gives $N_0 = \rho_0(0)2\pi R_\perp^2 R_z B\left(\dfrac{3}{2}, 1+\beta\right)$ where $B(v_1,\ v_2)$ is the Beta function with arguments v_1 and v_2.

From Eqs.(3) and (8), the ground state energy functional reads

$$N_0\varepsilon_{\rm G} = \int\sqrt{\rho_0(\boldsymbol{r})}\left[-\frac{\hbar^2}{2M}\nabla^2 + \varepsilon(\rho) + \frac{1}{2}M\omega_\perp^2(s^2 + \lambda^2 z^2)\right]\sqrt{\rho_0(\boldsymbol{r})}\mathrm{d}^3\boldsymbol{r}. \tag{9}$$

Using the approximate relationship $a_{\rm sc} = \dfrac{a_{\rm ho}}{(24\lambda N_0)^{1/6}y}$ for the trapped system, we obtain the constant $c = \dfrac{3\hbar^2}{10M}\epsilon(y)(3\pi^2)^\gamma\left[\dfrac{a_{\rm ho}}{(24\lambda N_0)^{1/6}}\right]^{3\gamma-2}$ in the equation of state. Substituting $\varepsilon = c\rho^\gamma$ into Eq.(9), the ground state energy functional $\varepsilon_G(\tilde{R}_\perp,\ \tilde{R}_z,\ q)$ has the form

$$\varepsilon_{\rm G} = \frac{1}{2}\hbar\omega_\perp\left[\frac{B(3/2,2+q+\beta)}{B(3/2,1+\beta)}\frac{3\epsilon(y)(3\pi^2)^\gamma}{5(24\lambda N_0)^{\gamma/2-1/3}}\left(\frac{N_0}{2\pi\tilde{R}_\perp^2\tilde{R}_z B(3/2,1+\beta)}\right)^\gamma + \frac{2+\lambda\tilde{R}_\perp/\tilde{R}_z}{2\beta+5}\tilde{R}_\perp^2 + \frac{(1+\lambda^2/2)\beta(\beta+1/2)}{(\beta-1)\tilde{R}_\perp^2}\right]. \quad (10)$$

According to the variational principle, we can obtain the values of variational parameters $\tilde{R}_\perp$, $\tilde{R}_z$ and β by finding the minimum of the energy $\varepsilon_G(\tilde{R}_\perp,\ \tilde{R}_z,\ q)$, and they can describe well the properties of the ground state of the system.

3. The quantum pressure correction to the collective excitation spectrum

To study the linear excitations from the ground state, we write $\rho(\boldsymbol{r},t)=\rho_0(\boldsymbol{r})+\delta\rho(\boldsymbol{r},t)$ and $\boldsymbol{v}(\boldsymbol{r},t)=\delta\boldsymbol{v}(\boldsymbol{r},t)$, where $\delta\rho(\boldsymbol{r},t)$ and $\delta\boldsymbol{v}(\boldsymbol{r},t)$ are small quantities. Differentiating Eq.(4) with respect to time yields

$$\frac{\partial^2\delta\rho(\boldsymbol{r},t)}{\partial t^2}+\nabla\left[\frac{\delta\rho(\boldsymbol{r},t)}{\partial t}\delta\boldsymbol{v}(\boldsymbol{r},t) + \rho(\boldsymbol{r},t)\frac{\partial\delta\boldsymbol{v}(\boldsymbol{r},t)}{\partial t}\right]=0. \quad (11)$$

Using Eq.(6) and neglecting small quantities of all higher orders, Eq.(11) becomes

$$\frac{\partial^2\delta\rho(\boldsymbol{r},t)}{\partial t^2}-\frac{1}{M}\nabla\cdot[\rho_0(\boldsymbol{r})\nabla(\delta\mu+\delta T_{\rm qp})]=0, \quad (12)$$

where $\delta\mu=\mu-\mu_{\rm G}$ and $\delta T_{\rm qp}=T_{\rm qp}-T_{\rm qp}^{(0)}=\frac{-\hbar^2}{4M\sqrt{\rho_0}}\left[\frac{\delta\rho}{\rho_0}\nabla^2\sqrt{\rho_0}-\nabla^2\left(\frac{\delta\rho}{\sqrt{\rho_0}}\right)\right]$ are the deviations of the chemical potential and quantum pressure from those in the ground state, respectively. Let the density fluctuation have the oscillation solution $\delta\rho(\boldsymbol{r},t)=\delta\rho(\boldsymbol{r})\mathrm{e}^{\mathrm{i}\omega t}$, then Eq.(12) becomes

$$(\hat{H}_0+\hat{H}_1)\delta\rho=\omega^2\delta\rho, \quad (13)$$

where $\hat{H}_0=-\frac{1}{M}\nabla\cdot[\rho_0(\boldsymbol{r})\nabla]\frac{\partial\mu}{\partial\rho}$ and $\hat{H}_1\delta\rho=\frac{1}{M}\nabla\cdot(\rho_0\nabla\delta T_{\rm qp})$. In the TF limit (i.e., $N_0\to\infty$ when we can neglect the quantum pressure term, $\hat{H}_1\propto\delta T_{\rm qp}=0$), the exactly analytical solutions of eigenfunction $\delta\rho$ and eigenvalue $\omega_{\boldsymbol{n}}^2$ in Eq.(13) have been obtained explicitly.[7] Notice that the zero-order solutions of Eq.(13) are sensitive to the expression for the equation of state. Therefore the solutions are already beyond mean-field approximation when $c(\gamma,y)$ and $\gamma(y)$ include the quantum many-body effects. In the case of large but finite N_0,[20] $\hat{H}_1\delta\rho\ll\hat{H}_0\delta\rho$ and the perturbation method is valid. The first-order quantum pressure correction to the collective excitation spectrum is

$$\frac{\delta\omega_{\boldsymbol{n}}^2}{\omega_\perp^2}=\frac{\int\frac{\partial\mu}{\partial\rho}\mathrm{d}^3\boldsymbol{r}\delta\rho^*\nabla\cdot(\rho_0\nabla\delta T_{\rm qp})}{M\omega_\perp^2\int\frac{\partial\mu}{\partial\rho}\mathrm{d}^3\boldsymbol{r}|\delta\rho|^2}, \quad (14)$$

where $\frac{\partial\mu}{\partial\rho}=c\gamma(\gamma+1)(1-\bar{s}^2-\lambda^2\bar{z}^2)^{\beta(\gamma-1)}$ is the integral weight. The mode $\boldsymbol{n}=n_rlm$ is for the spherical symmetry trap $\lambda=1$ and $\boldsymbol{n}=n_zn_sm$ for the axial symmetry trap $\lambda\neq1$. By introducing dimensionless $\bar{\nabla}=R_\perp\nabla$, $\bar{r}=\boldsymbol{r}/R_\perp$, $\bar{\omega}=\omega/\omega_\perp$, $\bar{\rho}_0=\rho_0/\rho_0(0)$ and $\delta\bar{\rho}=\delta\rho/\rho_0(0)$, Eq.(14) becomes

$$\delta\bar{\omega}_{\boldsymbol{n}}^2=\frac{-1}{4\tilde{R}_\perp^4}\frac{\int\bar{\rho}_0^{\gamma-1}\mathrm{d}^3\bar{r}\delta\bar{\rho}^*\bar{\nabla}\bar{\rho}_0\cdot\bar{\nabla}\frac{1}{\sqrt{\rho_0}}\left[\frac{\delta\bar{\rho}}{\bar{\rho}_0}\bar{\nabla}^2\sqrt{\rho_0}-\bar{\nabla}^2\frac{\delta\bar{\rho}}{\sqrt{\rho_0}}\right]}{\int\bar{\rho}_0^{\gamma-1}\mathrm{d}^3\bar{r}|\delta\bar{\rho}|^2}. \quad (15)$$

From Eq.(15), we can obtain the first-order quantum pressure correction to the entire modes $\boldsymbol{n}$ with the known variational ground state solution $\bar{\rho}_0$ and corresponding exact zero-order excitation eigenfunctions $\delta\bar{\rho}_n$ and eigenvalues $\bar{\omega}_{\boldsymbol{n}}^2$. We next give the analytical expressions and numerical results for the corrections

of Eq.(15) in the spherical case $\lambda = 1$ and axial case $\lambda \neq 1$, separately.

3.1. Correction to $0l$ and $1l$ modes in the spherical symmetry trap $\lambda = 1$

For the entire modes of $n_r l$ (the spectrum is independent of m), the zero-order spectra are $\bar{\omega}^2_{n_r l} = l + n_r\gamma(2n_r + 2l + 1) + 2n_r$. We have the expressions for $\bar{\rho}_0$ and $\delta\bar{\rho}$ as[7] $\bar{\rho}_0(\bar{r}) = (1 - \bar{r}^2)^\beta$ and $\delta\bar{\rho} = (1 - \bar{r}^2)^{\beta(1-\gamma)}(1 - \eta\bar{r}^2)\bar{r}^l Y_{lm}$ for the special modes, where $\eta = 0$ for $0l$ modes ($l = 1, 2, 3, \ldots$) and $\eta = 1 + \dfrac{\beta}{l + 3/2}$ for $1l$ modes ($l = 0, 1, 2, \ldots$). Substituting these expressions of $\bar{\rho}_0(\bar{r})$ and $\delta\bar{\rho}$ into Eq.(15), we obtain

$$\delta\bar{\omega}^2_{n_r l} = -\frac{P}{4\tilde{R}^4_\perp Q}, \tag{16}$$

with $n_r = 0, 1$. The detail of notations P and Q have been provided in the Appendix A. Since the Beta function $B(v_1, v_2)$ diverges at $v_2 = 0, -1, -2, \ldots$, the required $0 \leq q < 1/2$ ensures the perturbation theory to be valid. We may choose N_0 large enough in order to adjust q into this range. Figure 1 shows the quantum pressure corrections $\delta\bar{\omega}^2_{n_r l}$ v.s. the inverse interaction strength y for surface 02 mode [Fig.1(a)] and monopole 10 mode [Fig.1(b)]. In the figure, the solid lines, long-dashed lines and short-dashed lines are our calculated results for $N_0 = 2 \times 10^5, N_0 = 2 \times 10^6$ and $N_0 = 2 \times 10^7$, respectively. From Fig.1, we see that the quantum pressure correction enhances either with the decrease of N_0 or with the increase of y especially in the BEC side. The monopole mode has similar behaviour to the surface mode.

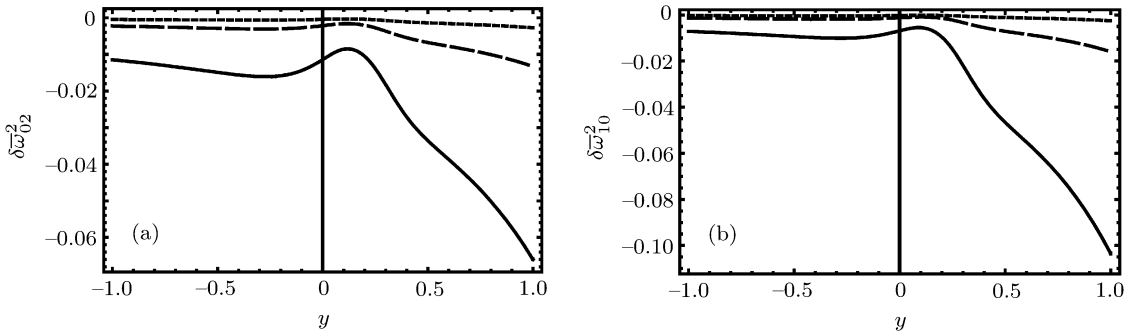


Fig.1. The quantum pressure corrections vs. the dimensionless interaction parameter y with $\lambda = 1$ for (a) 02 mode and (b) 10 mode. The solid lines, long-dashed lines and short-dashed lines are for $N_0 = 2 \times 10^5$, $N_0 = 2 \times 10^6$ and $N_0 = 2 \times 10^7$, respectively.

3.2. Correction to breathing modes in the axial symmetry trap $\lambda \neq 1$

For the usually measured breathing modes in experiment, the zero-order spectra are $\bar{\omega}^2_\pm = (1+\gamma)(1+|m|) + \left(1 + \dfrac{\gamma}{2}\right)\lambda^2 \pm \Lambda^{1/2}$ with $\Lambda = (1 + \gamma + \gamma|m|)^2 + [-2 - 3\gamma + \gamma^2 + (-2+\gamma)\gamma|m|]\lambda^2 + \left(1 + \dfrac{\gamma}{2}\right)\lambda^4$. We give the expressions for $\bar{\rho}_0$ and $\delta\bar{\rho}$ as[7] $\bar{\rho}_0 = (1 - \bar{s}^2 - \lambda^2\bar{z}^2)^\beta$ and $\delta\bar{\rho} = (1 + b_1\bar{z}^2 + b_2\bar{s}^2)(1 - \bar{s}^2 - \lambda^2\bar{z}^2)^{\beta - \beta\gamma}$, where the coefficients in $\delta\bar{\rho}$ are $b_1 = [2 + 2|m| + \dfrac{1}{\gamma}(2 + |m| - \bar{\omega}^2_\pm)]\dfrac{|m| - \bar{\omega}^2_\pm}{2 + |m| - \bar{\omega}^2_\pm}$ and $b_2 = \dfrac{-|m| + \bar{\omega}^2_\pm}{2 + |m| - \bar{\omega}^2_\pm}$. Substituting these expressions of $\bar{\rho}_0(\bar{r})$ and $\delta\bar{\rho}$ into Eq.(15), we obtain

$$\delta\bar{\omega}^2_\pm = -\frac{R}{4\lambda^2\tilde{R}^2_\perp\tilde{R}^2_z F}. \tag{17}$$

The detail of notations of R and F has been provided in Appendix B. Figure 2 shows the quantum pressure corrections $\delta\bar{\omega}^2_\pm$ v.s. the inverse interaction strength y for breathing mode $\bar{\omega}_+$ with $\lambda = 0.20$[Fig.2(a)] and for axial breathing model $\bar{\omega}_-$ with $\lambda = 0.05$ [Fig.2(b)]. In the figure, the solid lines, long-dashed lines and short-dashed lines are our calculating results for $N_0 = 2 \times 10^5, N_0 = 2 \times 10^6$ and $N_0 = 2 \times 10^7$, respectively. From Fig.2, we see that there are several interesting features in the quantum pressure corrections. (i) $\delta\bar{\omega}^2_\pm$ enhances either with the decrease of N_0 or with the increase of y especially in the BEC side. (ii) $\delta\bar{\omega}^2_\pm$ varies non-monotonically around $y \gtrsim 0$ for the smaller $N_0 \sim 10^5$, which is a reflection of the transition near the unitarity limit. (iii) The quantum pressure correction to the eigenfrequency is negative (positive) for the ω_+ mode (ω_- mode).

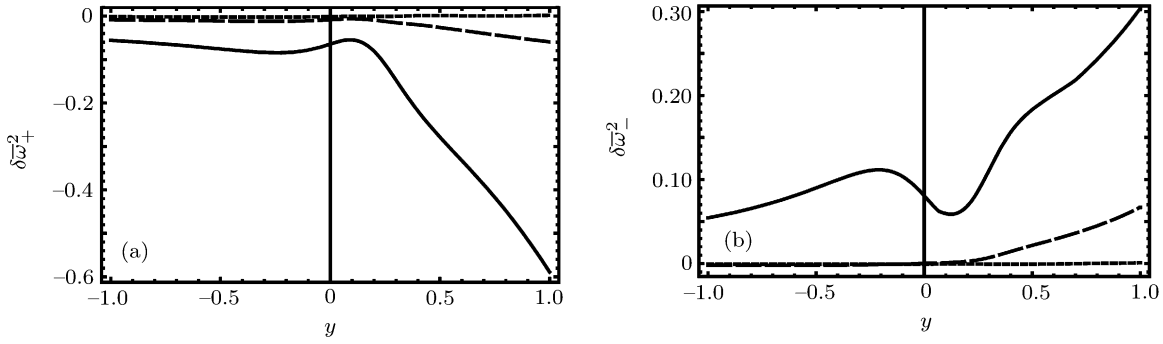


Fig.2. The quantum pressure corrections vs. the dimensionless interaction parameter y for (a) $\bar{\omega}_+$ mode ($\lambda = 0.20$) and (b) $\bar{\omega}_-$ mode ($\lambda = 0.05$). The solid lines, long-dashed lines and short-dashed lines are for $N_0 = 2 \times 10^5$, $N_0 = 2 \times 10^6$ and $N_0 = 2 \times 10^7$, respectively.

Since the anisotropic parameter λ is tunable in the usual experiments, we discuss the effect of λ on the corrections $\delta\omega_\pm^2$. Figure 3 shows the quantum pressure corrections $\delta\omega_\pm^2$ v.s. the anisotropic ratio λ with $N_0 = 2 \times 10^7$ for ω_+ mode [Fig.3(a)] and for ω_- mode [Fig.3(b)]. Notice that the unit of $\delta\omega_\pm^2$ is $\omega_\perp^2$ for $\lambda \leq 1$ and $\omega_z^2 = \lambda^2\omega_\perp^2$ for $\lambda \geq 1$. This is because λ determines the shape of the condensate. For $\lambda < 1$ the condensate is a cigar type, $\omega_\perp (< \omega_z)$ dominates the dynamic process; while for $\lambda > 1$ the condensate is a disc type, $\omega_z (< \omega_\perp)$ delineates the dynamic process better. In the figure, the solid lines, long-dashed lines and short-dashed lines are our calculating results for $y = -1, y = 0$ and $y = 1$, respectively. From Fig.3, we see that the quantum pressure corrections have obvious effects when $y > 0$. The physical reason is that for $y \leq 0$, the correction is very small so the solid lines and long-dashed lines are essential indistinguishable. In the BEC side ($y \geq 1$) the curve shows the non-monotonic behaviour for ω_+ mode around $\lambda = 1$ and shows divergence for ω_- mode at $\lambda \simeq 1.5$. This is a reflection of the change of the symmetry of the system for $\omega_+(\lambda)$ at $\lambda = 1$, and there exists a condensate resonance for $\omega_-(\lambda)$ at $\lambda = 1.5$.

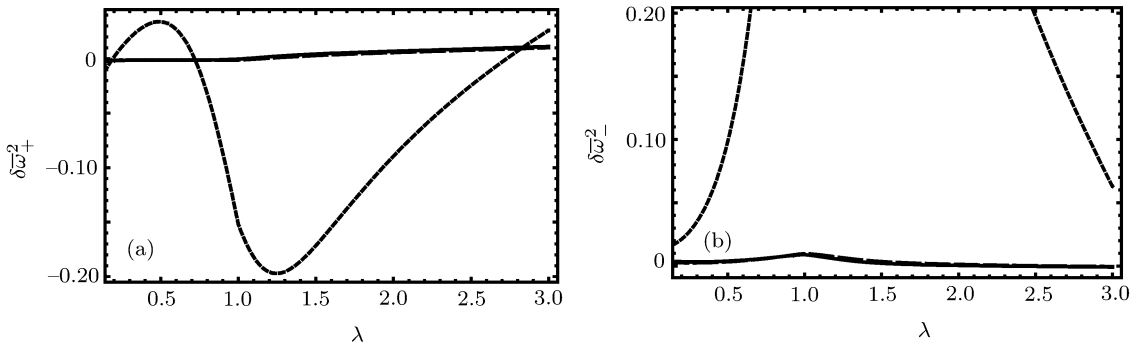


Fig.3. The quantum pressure corrections vs. the anisotropic parameter λ with $N_0 = 2 \times 10^7$ for (a) ω_+ mode and (b) ω_- mode. Notice that the unit of $\delta\omega_\pm^2$ is $\omega_\perp^2$ for $\lambda \leq 1$ and is $\omega_z^2 = \lambda^2\omega_\perp^2$ for $\lambda \geq 1$. The solid lines, long-dashed lines and short-dashed lines are for $y = -1, 0, 1$, respectively.

We have also calculated the relationship between the quantum pressure correction and particle number N_0. Figure 4 shows the quantum pressure corrections $\delta\omega_\pm^2$ v.s. the particle number N_0 with $\lambda = 0.20$ for ω_+ mode [Fig.4(a)] and ω_- mode [Fig.4(b)]. In the figure, the solid lines, long-dashed lines and short-dashed lines are our calculating results for $y = -1,\ y = 0$ and $y = 1$, respectively. From Fig.4, we also see that the quantum pressure corrections enhance either with the decrease of N_0 or with the increase of y on the BEC side. In summary, for the system particle number N_0 not large enough, the correction can not be neglected for an accurate calculation, especially on the BEC side of $y > 1$.

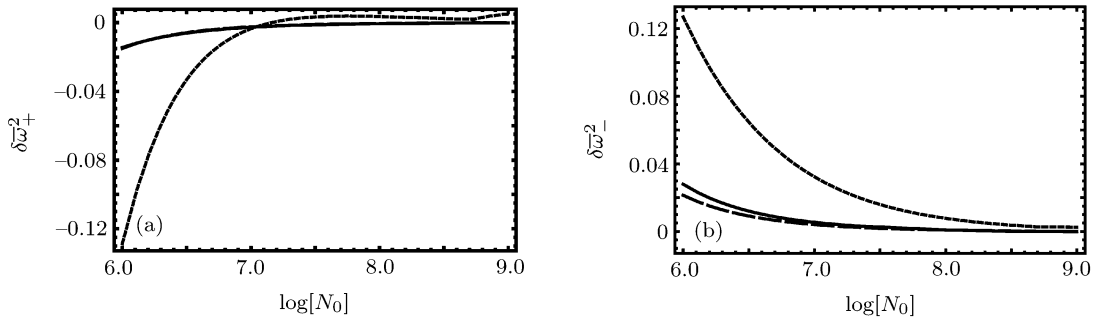


Fig.4. The quantum pressure corrections vs. the particle number N_0 with $\lambda = 0.20$ for (a) ω_+ mode and (b) ω_- mode. The solid lines, long-dashed lines and short-dashed lines are for $y = -1, 0, 1$, respectively.

4. Conclusion

We have investigated the quantum pressure correction to the collective excitation of the interacting harmonically trapped superfluid Fermi gases in the BEC-BCS crossover. Based on the phenomenological equation of state we have derived the hydrodynamic equations including the quantum pressure term in the TF kinetic energy. Beyond the TF approximation, we have calculated the quantum pressure corrections to the collective excitation spectrum and discuses the correction dependence on the inverse interaction strength, anisotropic parameter and particle numbers in the condensate in the whole BEC-BCS crossover regime. Our results agree well with the spectrum calculation for the finite particle numbers.[21] We have shown that the calculated correction to the hydrodynamic approximation is very important and has considerable improvement, even for the systems with rather large atom numbers. The prediction for the spectrum corrections' non-monotonic dependence on the anisotropic parameter (especially in the resonance region) needs to be confirmed in further experiments.

Appendix A: The notations of P and Q for the modes of $0l$ and $1l$

In Eq.(16), P and Q have the following form:

$$P = C_1 B\left(l+\frac{3}{2}, x_1\right) + C_2 B\left(l+\frac{5}{2}, x_1\right) + C_3 B\left(l+\frac{7}{2}, x_1\right) + C_4 B\left(l+\frac{9}{2}, x_1\right) + C_5 B\left(l+\frac{11}{2}, x_1\right), \quad \text{(A1)}$$

$$Q = B\left(l+\frac{3}{2}, x_2\right) - 2\eta B\left(l+\frac{5}{2}, x_2\right) + \eta^2 B\left(l+\frac{7}{2}, x_2\right) \quad \text{(A2)}$$

with $x_1 = -\beta\gamma + \beta - 3$ and $x_2 = -\beta\gamma + \beta + 1$. Here the coefficients $C_1 - C_5$ in P are

$$C_1 = (-3\beta - 3l\beta + 3\beta\gamma + 2l\beta\gamma - 3\eta - 2l\eta)(-l\beta + 3\beta\gamma + 2l\beta\gamma - 3\eta - 2l\eta),$$

$$\begin{aligned} C_2 =& (-3\beta - 3l\beta + 3\beta\gamma + 2l\beta\gamma - 3\eta - 2l\eta)(l\beta - \beta\gamma - 2l\beta\gamma - 2\beta^2\gamma + 2\beta^2\gamma^2 + 6\eta + 4l\eta \\ &+ 2\beta\eta + l\beta\eta - 7\beta\gamma\eta - 2l\beta\gamma\eta) + (-l\beta + 3\beta\gamma + 2l\beta\gamma - 3\eta - 2l\eta)(\beta + 3l\beta + 4\beta^2 - \beta\gamma \\ &- 2l\beta\gamma - 6\beta^2\gamma + 2\beta^2\gamma^2 + 6\eta + 4l\eta + 9\beta\eta + 3l\beta\eta - 7\beta\gamma\eta - 2l\beta\gamma\eta), \end{aligned}$$

$$\begin{aligned} C_3 =& (l\beta - \beta\gamma - 2l\beta\gamma - 2\beta^2\gamma + 2\beta^2\gamma^2 + 6\eta + 4l\eta + 2\beta\eta + l\beta\eta - 7\beta\gamma\eta - 2l\beta\gamma\eta) \\ &\times (\beta + 3l\beta + 4\beta^2 - \beta\gamma - 2l\beta\gamma - 6\beta^2\gamma + 2\beta^2\gamma^2 + 6\eta + 4l\eta + 9\beta\eta + 3l\beta\eta - 7\beta\gamma\eta - 2l\beta\gamma\eta) \\ &+ (-3\beta - 3l\beta + 3\beta\gamma + 2l\beta\gamma - 3\eta - 2l\eta)(-3\eta - 2l\eta - 2\beta\eta - l\beta\eta + 5\beta\gamma\eta \\ &+ 2l\beta\gamma\eta + 2\beta^2\gamma\eta - 2\beta^2\gamma^2\eta) + (-l\beta + 3\beta\gamma + 2l\beta\gamma - 3\eta - 2l\eta) \\ &\times (-3\eta - 2l\eta - 3\beta\eta - 3l\beta\eta - 4\beta^2\eta + \beta\gamma\eta + 2l\beta\gamma\eta + 6\beta^2\gamma\eta - 2\beta^2\gamma^2\eta), \end{aligned}$$

$$
\begin{aligned}
C_4 =&(\beta+3l\beta+4\beta^2-\beta\gamma-2l\beta\gamma-6\beta^2\gamma+2\beta^2\gamma^2+6\eta+4l\eta+9\beta\eta+3l\beta\eta-7\beta\gamma\eta-2l\beta\gamma\eta)\\
&\times(-3\eta-2l\eta-2\beta\eta-l\beta\eta+5\beta\gamma\eta+2l\beta\gamma\eta+2\beta^2\gamma\eta-2\beta^2\gamma^2\eta)\\
&+(l\beta-\beta\gamma-2l\beta\gamma-2\beta^2\gamma+2\beta^2\gamma^2+6\eta+4l\eta+2\beta\eta+l\beta\eta-7\beta\gamma\eta-2l\beta\gamma\eta)\\
&\times(-3\eta-2l\eta-3\beta\eta-3l\beta\eta-4\beta^2\eta+\beta\gamma\eta+2l\beta\gamma\eta+6\beta^2\gamma\eta-2\beta^2\gamma^2\eta),
\end{aligned}
$$

$$
\begin{aligned}
C_5 =&(-3\eta-2l\eta-2\beta\eta-l\beta\eta+5\beta\gamma\eta+2l\beta\gamma\eta+2\beta^2\gamma\eta-2\beta^2\gamma^2\eta)\\
&(-3\eta-2l\eta-3\beta\eta-3l\beta\eta-4\beta^2\eta+\beta\gamma\eta+2l\beta\gamma\eta+6\beta^2\gamma\eta-2\beta^2\gamma^2\eta).
\end{aligned}
$$

Appendix B: The notations of F and R for the breathing modes

In Eq.(17), after a lengthy and careful calculation F and R have the following form:

$$
\begin{aligned}
F =&B(1,x_3)\left[B\left(\frac{1}{2},x_3+1\right)+\frac{2b_1}{\lambda_1^2}B\left(\frac{3}{2},x_3+1\right)+\frac{b_1^2}{\lambda_1^4}B\left(\frac{5}{2},x_3+1\right)\right]\\
&+2b_2B(2,x_3)\left[B\left(\frac{1}{2},x_3+1\right)+\left(\frac{b_1}{\lambda_1^2}-1\right)B\left(\frac{3}{2},x_3+1\right)-\frac{b_1}{\lambda_1^2}B\left(\frac{5}{2},x_3+2\right)\right]\\
&+b_2^2B(3,x_3)\left[B\left(\frac{1}{2},x_3+1\right)-2B\left(\frac{3}{2},x_3+1\right)+B\left(\frac{5}{2},x_3+1\right)\right],
\end{aligned}
\tag{B1}
$$

$$
\begin{aligned}
R =&m_0B(1,x_4+4)B\left(\frac{1}{2},x_4+4\right)+m_1B(1,x_4+3)B\left(\frac{1}{2},x_4+3\right)+m_2B(1,x_4+3)B\left(\frac{3}{2},x_4+3\right)\\
&+m_3B(2,x_4+3)\left[B\left(\frac{1}{2},x_4+3\right)-B\left(\frac{3}{2},x_4+3\right)\right]+m_4B(1,x_4+2)B\left(\frac{1}{2},x_4+2\right)\\
&+m_5B(1,x_4+2)B\left(\frac{3}{2},x_4+2\right)+m_6B(1,x_4+2)B\left(\frac{5}{2},x_4+2\right)+m_7B(2,x_4+2)B\left(\frac{1}{2},x_4+2\right)\\
&+m_8B(2,x_4+2)B\left(\frac{3}{2},x_4+2\right)+m_9B(2,x_4+2)B\left(\frac{5}{2},x_4+2\right)+m_{10}B(3,x_4+2)B\left(\frac{1}{2},x_4+2\right)\\
&+m_{11}B(3,x_4+2)B\left(\frac{3}{2},x_4+2\right)+m_{12}B(3,x_4+2)B\left(\frac{5}{2},x_4+2\right)+m_{13}B(1,x_4+1)B\left(\frac{3}{2},x_4+1\right)\\
&+m_{14}B(1,x_4+1)B\left(\frac{5}{2},x_4+1\right)+m_{15}B(1,x_4+1)B\left(\frac{7}{2},x_4+1\right)+m_{16}B(2,x_4+1)B\left(\frac{1}{2},x_4+1\right)\\
&+m_{17}B(2,x_4+1)B\left(\frac{3}{2},x_4+1\right)+m_{18}B(2,x_4+1)B\left(\frac{5}{2},x_4+1\right)+m_{19}B(2,x_4+1)B\left(\frac{7}{2},x_4+1\right)\\
&+m_{20}B(3,x_4+1)B\left(\frac{1}{2},x_4+1\right)+m_{21}B(3,x_4+1)B\left(\frac{3}{2},x_4+1\right)+m_{22}B(3,x_4+1)B\left(\frac{5}{2},x_4+1\right)\\
&+m_{23}B(3,x_4+1)B\left(\frac{7}{2},x_4+1\right)+m_{24}B(4,x_4+1)B\left(\frac{1}{2},x_4+1\right)\\
&+m_{25}B(4,x_4+1)\left[B\left(\frac{3}{2},x_4+1\right)-B\left(\frac{5}{2},x_4+1\right)+\frac{1}{3}B\left(\frac{7}{2},x_4+1\right)\right]\\
&+m_{26}B(1,x_4)B\left(\frac{5}{2},x_4\right)+m_{27}B(1,x_4)B\left(\frac{7}{2},x_4\right)+m_{28}B(1,x_4)B\left(\frac{9}{2},x_4\right)\\
&+m_{29}B(2,x_4)B\left(\frac{3}{2},x_4\right)+m_{30}B(2,x_4)B\left(\frac{5}{2},x_4\right)+m_{31}B(2,x_4)B\left(\frac{7}{2},x_4\right)+m_{32}B(2,x_4)B\left(\frac{9}{2},x_4\right)\\
&+m_{33}B(3,x_4)B\left(\frac{1}{2},x_4\right)+m_{34}B(3,x_4)B\left(\frac{3}{2},x_4\right)+m_{35}B(3,x_4)B\left(\frac{5}{2},x_4\right)+m_{36}B(3,x_4)B\left(\frac{7}{2},x_4\right)\\
&+m_{37}B(3,x_4)B\left(\frac{9}{2},x_4\right)+m_{38}B(4,x_4)B\left(\frac{1}{2},x_4\right)+m_{39}B(4,x_4)B\left(\frac{3}{2},x_4\right)+m_{40}B(4,x_4)B\left(\frac{5}{2},x_4\right)
\end{aligned}
$$

$$+ m_{41}B(4,x_4)B\Big(\frac{7}{2},x_4\Big) + m_{42}B(4,x_4)B\Big(\frac{9}{2},x_4\Big)$$
$$+ m_{43}B(5,x_4)\Big[B\Big(\frac{1}{2},x_4\Big) + 6B\Big(\frac{5}{2},x_4\Big) + B\Big(\frac{9}{2},x_4\Big) - 4B\Big(\frac{3}{2},x_4\Big) - 4B\Big(\frac{7}{2},x_4\Big)\Big], \tag{B2}$$

with $x_3 = -\beta\gamma + \beta + 1$, $\lambda_1 = \tilde{R}_\perp/\tilde{R}_z$ and $x_4 = -\beta\gamma + \beta - 3$. Here by taking $a_0 = \gamma(2+\lambda^2)$, $a_1 = [(2+5\lambda^2)\gamma - 2\lambda^2]b_1$, $a_2 = b_2[(6+\lambda^2)\gamma - 2]$, $w = 4\gamma(\beta\gamma - \beta + 1)$, and $\Gamma = 1 + (-1 + \beta - \beta\gamma)(\beta - \beta\gamma)$, the coefficients $m_0 - m_{43}$ in R are

$$m_0 = (b_1 + 2b^2)^2,$$

$$m_1 = (b_1 + 2b_2)\beta[a_0 + (2+\lambda_1^2)(\gamma - 1)],$$

$$m_2 = (b_1 + 2b_2)\Big[-4b_1 + \frac{a_1\beta}{\lambda_1^2} + b_1\beta\Big(5 + \frac{2}{\lambda_1^2}\Big)(\gamma - 1)\Big],$$

$$m_3 = (b_1 + 2b_2)[a_2\beta - 4b_2 + b_2\beta(6+\lambda_1^2)(\gamma - 1)],$$

$$m_4 = a_0\beta^2(2+\lambda_1^2)(\gamma - 1),$$

$$m_5 = -4a_0b_1\beta + \beta^2\Big(a_1 + 5a_0b_1 + \frac{2a_1}{\lambda_1^2} + \frac{2a_0b_1}{\lambda_1^2}\Big)(\gamma - 1) + (b_1 + 2b_2)\Big(\frac{1}{2}w\beta + 2\Gamma\Big)\lambda_1^2,$$

$$m_6 = \frac{a_1b_1\beta}{\lambda_1^2}[\beta\Big(\frac{2}{\lambda_1^2} + 5\gamma\Big)(\gamma - 1\Big) - 4] + (b_1 + 2b_2)b_1\Big(\frac{1}{2}w\beta + 2\Gamma\Big),$$

$$m_7 = a_2\beta^2(2+\lambda_1^2)(\gamma - 1) + a_0b_2\beta[(6+\lambda_1^2)\beta(\gamma - 1) - 4] + (b_1 + 2b_2)\Big(\frac{1}{2}w\beta + 2\Gamma\Big),$$

$$m_8 = -4a_2b_1\beta + 4a_0b_2\beta + 5a_2b_1\beta^2(\gamma^2 - 1) - a_1b_2\beta\Big(\beta + \frac{4}{\lambda_1^2}\Big) + \beta^2\Big[-a_0b_2(6+\lambda_1^2) + a_2(2-\lambda_1^2)$$
$$+ \frac{2}{\lambda_1^2}(a_2b_1 + 3a_1b_2)\Big](\gamma - 1) + (b_1 + 2b_2)\Big(-1 + \frac{b_1}{\lambda_1^2} + b_2\lambda_1^2\Big)\Big(\frac{1}{2}w\beta + 4\Gamma\Big),$$

$$m_9 = a_2b_1\beta\Big[4 - \Big(5\beta + \frac{2}{\lambda_1^2}\Big)(\gamma - 1)\Big] + \frac{a_1b_2\beta}{\lambda_1^2}[4 + \beta(6-\lambda_1^2)(\gamma - 1)](b_1 + 2b_2)\Big(\frac{b_1}{\lambda_1^2} + b_2\lambda_1^2\Big)\Big(\frac{1}{2}w\beta + 2\Gamma\Big),$$

$$m_{10} = a_2b_2\beta[-4 + \beta(\lambda_1^2 + 6)(\gamma - 1)] + b_2(b_1 + 2b_2)\Big(\frac{1}{2}w\beta + 2\Gamma\Big),$$

$$m_{11} = 2a_2b_2\beta[4 - \beta(6+\lambda_1^2)(\gamma - 1)] - b_2(b_1 + 2b_2)(w\beta + 4\Gamma),$$

$$m_{12} = a_2b_2\beta[-4 + \beta(6+\lambda_1^2)(\gamma - 1)] + b_2(b_1 + 2b_2)\Big(\frac{1}{2}w\beta + 2\Gamma\Big),$$

$$m_{13} = w\beta^2\lambda_1^2\Big(1 + \frac{1}{2}\lambda_1^2\Big)(\gamma - 1) + 2a_0\beta\Gamma\lambda_1^2,$$

$$m_{14} = b_1w\beta^2(2+3\lambda_1^2)(\gamma - 1) - 2b_1w\beta\lambda_1^2 + 2\beta\Gamma(a_1 + a_0b_1),$$

$$m_{15} = -2b_1^2w\beta + b_1^2w\beta^2\Big(\frac{5}{2} + \frac{1}{\lambda_1^2}\Big)(\gamma - 1) + \frac{2a_1b_1\beta}{\lambda_1^2}\Gamma\Big],$$

$$m_{16} = 2a_0\beta\Gamma + c\beta^2\Big(1 + \frac{1}{2}\lambda_1^2\Big)(\gamma - 1),$$

$$m_{17} = 2\beta\Gamma\Big(-a_0 + \frac{a_1}{\lambda_1^2} + \frac{a_0b_1}{\lambda_1^2} + a_2\lambda_1^2 + a_0b_2\lambda_1^2\Big) - 2w\beta(b_1 + b_2\lambda_1^2)$$
$$+ w\beta^2\Big[\frac{5}{2}b_1 + \frac{b_1}{\lambda_1^2} + 3b_2\lambda_1^2 + \frac{1}{2}b_2\lambda_1^4 + \Big(-1 + \frac{b_1}{\lambda_1^2} + b_2\lambda_1^2\Big)(1+\lambda_1^2)\Big](\gamma - 1),$$

$$\begin{aligned}m_{18} =&2\beta\Gamma\bigg(a_2b_1+a_1b_2+\frac{a_1b_1}{\lambda_1^4}-\frac{a_1}{\lambda_1^2}-\frac{a_0b_1}{\lambda_1^2}-a_2\lambda_1^2-a_0b_2\lambda_1^2\bigg)\\&+w\beta b_2(b_1-\lambda_1^2)\bigg[\beta\bigg(3+\frac{1}{2}\lambda_1^2\bigg)(\gamma-1)-2\bigg]-2w\beta b_1\bigg(-1+\frac{b_1}{\lambda_1^2}+b_2\lambda_1^2\bigg)\\&+w\beta^2\bigg[\bigg(\frac{5}{2}+\frac{1}{\lambda_1^2}\bigg)\bigg(\frac{b_1}{\lambda_1^2}+b_2\lambda_1^2\bigg)b_1-3b_1-\frac{2b_1}{\lambda_1^2}-b_2\lambda_1^2-\frac{1}{2}b_2\lambda_1^4\bigg](\gamma-1),\end{aligned}$$

$$\begin{aligned}m_{19} =&-2\beta\Gamma\bigg(a_2b_1+a_1b_2+\frac{a_1b_1}{\lambda_1^4}\bigg)+2w\beta b_1\bigg(\frac{b_1}{\lambda_1^2}+b_2+b_2\lambda_1^2\bigg)\\&+w\beta^2b_1\bigg[\bigg(\frac{5}{2}-\frac{1}{\lambda_1^2}\bigg)\frac{b_1}{\lambda_1^2}+2(\lambda_1^2-2)b_2\bigg](\gamma-1),\end{aligned}$$

$$m_{20}=2\beta(a_2+a_0b_2)\Gamma+w\beta b_2[-2+\beta(4+\lambda_1^2)(\gamma-1)],$$

$$\begin{aligned}m_{21} =&2\Gamma\beta\bigg(-2a_2-2a_0b_2+\frac{a_2b_1}{\lambda_1^2}+\frac{a_1b_2}{\lambda_1^2}+a_2b_2\lambda_1^2\bigg)\\&+w\beta b_2\bigg\{2-2b_1-\beta\bigg(5-\frac{5}{2}b_1-\frac{b_1}{\lambda_1^2}+\frac{3}{2}\lambda_1^2\bigg)(\gamma-1)\\&+\bigg(1-\frac{b_1}{\lambda_1^2}-b_2\lambda_1^2\bigg)\bigg[2-\beta\bigg(3+\frac{1}{2}\beta\lambda_1^2\bigg)(\gamma-1)\bigg]\bigg\},\end{aligned}$$

$$\begin{aligned}m_{22} =&2\Gamma\beta(a_2+a_0b_2-\frac{2a_2b_1}{\lambda_1^2}-\frac{2a_1b_2}{\lambda_1^2}-2a_2b_2\lambda_1^2)\\&+w\beta b_2\bigg\{4b_1-2+\beta\bigg(2-5b_1-\frac{2b_1}{\lambda_1^2}\bigg)(\gamma-1)+\bigg(\frac{b_1}{\lambda_1^2}+b_2\lambda_1^2\bigg)[4-\beta(6+\lambda_1^2)(\gamma-1)]\bigg\},\end{aligned}$$

$$m_{23}=2\Gamma\beta\bigg(\frac{a_2b_1}{\lambda_1^2}+\frac{a_1b_2}{\lambda_1^2}+a_2b_2\lambda_1^2\bigg)-2w\beta b_1b_2\bigg[1+\frac{1}{\lambda_1^2}-\beta\bigg(\frac{3}{2}+\frac{2}{\lambda_1^2}\bigg)(\gamma-1)\bigg],$$

$$m_{24}=-2b_2^2w\beta+b_2^2w\beta^2\bigg(3+\frac{1}{2}\lambda_1^2\bigg)(\gamma-1)+2a_2b_2\beta\Gamma,$$

$$m_{25}=6b_2^2w\beta-6a_2b_2\beta\Gamma-3b_2^2w\beta^2\bigg(3+\frac{1}{2}\lambda_1^2\bigg)(\gamma-1),$$

$$m_{26}=w\beta\Gamma\lambda_1^4,$$

$$m_{27}=2b_1w\beta\Gamma\lambda_1^2,$$

$$m_{28}=b_1^2w\beta\Gamma,$$

$$m_{29}=2w\beta\Gamma\lambda_1^2,$$

$$m_{30}=2w\beta\Gamma(2b_1-\lambda_1^2+b_2\lambda_1^2),$$

$$m_{31}=2w\beta\Gamma\bigg(-2b_1+\frac{b_1^2}{\lambda_1^2}-b_2\lambda_1^4\bigg),$$

$$m_{32}=-2w\beta\Gamma b_1\bigg(\frac{b_1}{\lambda_1^2}+b_2\lambda_1^2\bigg),$$

$$m_{33}=w\beta\Gamma,$$

$$m_{34}=2w\beta\Gamma(-1+\frac{b_1}{\lambda_1^2}+2b_2\lambda^2),$$

$$m_{35}=w\beta\Gamma\bigg(1+4b_1b_2+\frac{b_1^2}{\lambda_1^4}-\frac{4b_1}{\lambda_1^2}-8b_2\lambda_1^2+b_2^2\lambda_1^4\bigg),$$

$$m_{36}=2w\beta^2\Gamma\bigg(-4b_1b_2-\frac{b_1^2}{\lambda_1^4}+\frac{b_1}{\lambda_1^2}+2b_2\lambda_1^2-b_2^2\lambda_1^4\bigg),$$

$$m_{37}=w\beta^2\Gamma\bigg(4b_2+\frac{b_1}{\lambda_1^4}\bigg)b_1,$$

$$m_{38}=2b_2w\beta^2\Gamma,$$

$$m_{39}=2w\beta^2\Gamma\bigg(-3+\frac{b_1}{\lambda_1^2}+b_2\lambda_1^2\bigg)b_2,$$

$$m_{40}=6w\beta^2\Gamma\bigg(1-\frac{b_1}{\lambda_1^2}-b_2\lambda_1^2\bigg)b_2,$$

$$m_{41}=2w\beta^2\Gamma\bigg(-1+\frac{3b_1}{\lambda_1^2}+3b_2\lambda_1^2\bigg)b_2,$$

$$m_{42}=-2w\beta^2\Gamma\bigg(\frac{b_1}{\lambda_1^2}+b_2\lambda_1^2\bigg)b_2,$$

$$m_{43}=b_2^2w\beta^2\Gamma.$$

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