Vol 18 No 2, February 2009
1674-1056/2009/18(02)/0616-08

Chinese Physics B

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Solitons in optical lattices with nonlocal nonlinearity*

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(Received 7 April 2008; revised manuscript received 17 September 2008)

The propagation of spatial solitons is systematically investigated in nonlocal nonlinear media with an imprinted transverse periodic modulation of the refractive index. Based on the variational principle and the infinitesimal approximation of Maclaurin series expansion, we obtain an analytical solution of such nonlocal spatial solitons and an interesting result that the critical power for such solitons propagation is smaller than that in uniform nonlocal self-focusing media. It is found that there exist thresholds in modulation period and lattice depth for such solitons. A stable spatial soliton propagation is maintained with proper adjustment of the modulation period and the lattice depth.

Keywords: optical lattice solitons, nonlocal nonlinearity
PACC: 4265, 4265S

1. Introduction

During recent years, nonlocal solitons have opened new directions in nonlinear science both theoretically[1−11] and experimentally[12−18] because of their unique physical properties. After the pioneering work[19] of Snyder and Mitchell on a highly nonlocal model,[1] in which they treated soliton collision and interaction as linear harmonic oscillation (accessible solitons), nonlocal solitons have been studied extensively, such as for the exact soliton solution in weakly nonlocal media,[2] the attraction of nonlocal dark solitons,[3,17] modulation instability of nonlocal solitons,[4,5,13−15] and the review paper on nonlocal effects.[7] There has been discussion of vortex solitons[8,9] in nonlocal nonlinear media and discrete propagation of solitons in nematic liquid crystals,[16] which have a high degree of nonlocality arising from the reorientational nonlinearity, so the observation of accessible solitons comes to reality.[18] In Ref.[10] is shown the stabilization of vortex solitons in nonlocal nonlinear media. Nonlocality can also describe parametric wave interaction,[20−22] including parametric solitons,[20] X-waves[21] and modulational instability.[22] It was discovered that nonlocality (both strong and weak) also provides an elegant and accurate description of soliton pulse compression.[23] Incoherent solitons can also exist in instantaneous nonlocal nonlinear media, which is a new type of soliton family.[24] Nonlocal white-light solitons have been discussed in the case of logarithmic nonlinearity,[25] where the degree of the nonlocality is arbitrary. The propagation of partially coherent solitons,[26] elliptic incoherent solitons[27] and incoherently coupled two-colour Manakov vector solitons[28] in Kerr media with extremely large nonlocality have been discussed recently.

Since Fleischer *et al* [29] observed optical solitons in optically induced nonlinear photonic lattices, the evolution of waves and the formation of optical solitons in nonlinear waveguide lattices has prompted extensive research.[29−37] Recent progress in optical spatial solitons supported by reconfigurable optical lattice in photorefractive crystals,[38] nematic liquid crystals[13] and Bose–Einstein condensates[39] opened a direct way to explore the properties of solitons by varying the lattice depth and period. Many intense studies have been done by Kartashov *et al*: e.g., soliton dragging[32] and stable ring-profile vortex solitons.[33] Makris *et al* [40] discussed surface lattice solitons. It is also shown that dipole solitons can be supported by optical lattice[41] and defect superlattice

*Project supported in part by the National Natural Science Foundation of China (Grant Nos 60677030 and 60808002), the Specialized Research Fund for the Doctoral Program of Higher Education, China (Grant No 20060280007), the Science and Technology Commission of Shanghai Municipality, China (Grant No 06ZR14034). Ming Shen is also supported by the Australian Endeavor Research Fellowship scholarship, and appreciates the hospitality of the Laser Physics Center during his stay in Canberra.
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solitons.[42] Meanwhile, photorefractive media, liquid crystals and Bose–Einstein condensates may feature a nonlocal nonlinear response. This gives rise to, therefore, a principal question about the effect of the combination of periodic refractive index modulation with nonlocality of nonlinear response on soliton properties. Fratalocchi and Assanto[43] have discussed discrete light localization in the one-dimensional nonlinear lattice with arbitrary nonlocality in nematic liquid crystals. Kartashov *et al* have also done important research on nonlocal optical lattices,[44,45] such as the phenomenon of solitons self-bending[44] and solitons mobility.[45]

In this paper, we discuss the self-trapping of spatial solitons in strongly nonlocal nonlinear media imprinted with a transverse periodic modulation of the refractive index. Following the variational principle,[46,47] we obtain the trends of the evolution for propagation of beamwidth, beam amplitude and beam curvature. Based on the infinitesimal approximation of Maclaurin expansion and the highly nonlocal limit, we obtain an exact analytical solution of such a nonlocal spatial soliton, whose radius is related to the total power, the lattice depth as well as the modulation period. We find that there exist thresholds of the modulation period and the lattice depth for the nonlocal spatial solitons, and that the critical input power for the nonlocal spatial solitons propagation is smaller than that in uniform nonlocal self-focusing media. It is also found that the larger the modulation period, the stronger the diffraction of the beam will be, and that as the lattice depth increases, the diffraction of the beam will be weakened. Evolution behaviours of the solitons are also described when the solitons undergo linear harmonic oscillation.

2. The propagation model

We consider a one-dimensional optical beam propagating in a strongly nonlocal Kerr-type nonlinearity with a transverse periodic modulation of the refractive index. In this case, the nonlinear propagation equation for the optical beam is given by

$$\mathrm{i}\frac{\partial\psi}{\partial z}+\frac{1}{2k}\frac{\partial^2\psi}{\partial x^2}+hp(x)\psi + \rho\psi\int_{-\infty}^{+\infty}R(x-\xi)|\psi(\xi,z)|^2\mathrm{d}\xi=0, \tag{1}$$

where $\psi(x,z)$ is a paraxial beam, k is the wave number, n_0 is the linear part of the refractive index, h is proportional to the depth of refractive index modulation, $p(x)=\cos(2\pi x/T)$ describes the transverse refractive index profile, T is the modulation period, $\rho=k\eta$, η is media constant ($\eta>0$ or $\eta<0$ corresponds respectively to self-focusing or defocusing materials). Here, $R(x)$ is the normalized nonlocal response function of the medium so that $\int_{-\infty}^{+\infty}R(x)\mathrm{d}x=1$. In strongly nonlocal media the nonlocal response function is much wider than the beam width, so the nonlocal response function $R(x-\xi)$ can be expanded in Taylor's series with respect to ξ about $x=\xi$ to the second order,[5,6] then we have $\int_{-\infty}^{+\infty}R(x-\xi)|\psi(\xi,z)|^2\mathrm{d}\xi = R_0P_0-\frac{1}{2}\gamma P_0x^2$, and Eq.(1) turns into

$$\mathrm{i}\frac{\partial\psi}{\partial z}+\frac{1}{2k}\frac{\partial^2\psi}{\partial x^2}+hp(x)\psi+\rho\psi\left(R_0P_0-\frac{1}{2}\gamma P_0x^2\right)=0. \tag{2}$$

Here $P_0=\int_{-\infty}^{+\infty}|\psi(x,z)|^2\mathrm{d}x$ is the total power of the beam, $R_0=R(x)|_{x=0}$ is the maximum of $R(x)$, $\gamma=-R^{(2)}(x)|_{x=0}=-[\partial^2R(x)/\partial x^2]|_{x=0}>0$ [$R^{(2)}(0)<0$ because R_0 is a maximum of $R(x)$].

Let us restate the problem of solving Eq.(2) as a variational problem corresponding to the Lagrangian density

$$\mathcal{L}=\frac{\mathrm{i}}{2}\left(\psi\frac{\partial\psi^*}{\partial z}-\psi^*\frac{\partial\psi}{\partial z}\right)+\frac{1}{2k}|\frac{\partial\psi}{\partial x}|^2-hp(x)|\psi|^2 -\frac{1}{2}\rho|\psi|^2\left(R_0P_0-\frac{1}{2}\gamma P_0x^2\right). \tag{3}$$

We introduce a Gaussian ansatz of the form

$$\psi(x,z)=A(z)\exp\left[-\frac{x^2}{2a^2(z)}+\mathrm{i}b(z)x^2\right], \tag{4}$$

where the parameter $A(z)$ is the complex amplitude, $a(z)$ is the beam width, and $b(z)$ is curvature of the beam. Inserting Eq.(4) into Eq.(3), an effective Lagrangian is obtained by calculating the average Lagrangian density

$$\begin{aligned}L=<\mathcal{L}>=\int_{-\infty}^{+\infty}\mathcal{L}\mathrm{d}x=\frac{\sqrt{\pi}}{2}\Bigg[&\mathrm{i}a\left(A\frac{\mathrm{d}A^*}{\mathrm{d}z}-A^*\frac{\mathrm{d}A}{\mathrm{d}z}\right)\\&+|A|^2a^3\frac{\mathrm{d}b}{\mathrm{d}z}+\frac{1}{2k}|A|^2\left(\frac{1}{a}+4b^2a^3\right)\\&-2h|A|^2a\exp\left(-\frac{\pi^2a^2}{T^2}\right)\\&-\rho|A|^2\left(aR_0P_0-\frac{1}{4}\gamma P_0a^3\right)\Bigg].\end{aligned} \tag{5}$$

Based on the Euler–Lagrange equations

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{\partial L}{\partial q_j{}'}\right)-\frac{\partial L}{\partial q_j}=0, q_j\equiv\{A^*,A,a,b\}, \tag{6}$$

the effective Lagrangian satisfies the following equations:

$$\frac{\delta L}{\delta q_j}=0, q_j\equiv\{A^*,A,a,b\}. \tag{7}$$

Following Eq.(7), we obtain

$$\begin{aligned}\frac{\mathrm{d}(\mathrm{i}aA)}{\mathrm{d}z}=&-\mathrm{i}a\frac{\mathrm{d}A}{\mathrm{d}z}+Aa^3\frac{\mathrm{d}b}{\mathrm{d}z}+\frac{1}{2k}Aa^3\left(4b^2+\frac{1}{a^4}\right)\\&-2haA\exp\left(-\frac{\pi^2a^2}{T^2}\right)\\&-\rho A\left(aR_0P_0-\frac{1}{4}\gamma P_0a^3\right),\end{aligned} \tag{8}$$

$$\begin{aligned}-\frac{\mathrm{d}(\mathrm{i}aA^*)}{\mathrm{d}z}=&\mathrm{i}a\frac{\mathrm{d}A^*}{\mathrm{d}z}+A^*a^3\frac{\mathrm{d}b}{\mathrm{d}z}+\frac{1}{2k}A^*a^3\left(4b^2+\frac{1}{a^4}\right)\\&-2haA^*\exp\left(-\frac{\pi^2a^2}{T^2}\right)\\&-\rho A^*\left(aR_0P_0-\frac{1}{4}\gamma P_0a^3\right),\end{aligned} \tag{9}$$

$$\begin{aligned}&\mathrm{i}\left(A^*\frac{\mathrm{d}A}{\mathrm{d}z}-A\frac{\mathrm{d}A^*}{\mathrm{d}z}\right)\\=&3|A|^2a^2\frac{\mathrm{d}b}{\mathrm{d}z}+\frac{6}{k}|A|^2a^2b^2-\frac{|A|^2}{2ka^2}\\&-2h|A|^2\exp\left(-\frac{\pi^2a^2}{T^2}\right)\left(1-\frac{2\pi^2a^2}{T^2}\right)\\&-\rho|A|^2\left(R_0P_0-\frac{3}{4}\gamma P_0a^2\right),\end{aligned} \tag{10}$$

$$\frac{\mathrm{d}(|A|^2a^3)}{\mathrm{d}z}=\frac{4|A|^2a^3b}{k}. \tag{11}$$

Solving Eqs.(8) and (9), we have

$$\frac{\mathrm{d}(|A|^2a)}{\mathrm{d}z}=0. \tag{12}$$

Equation (12) shows that the beam power is conservative, from which we know that

$$a|A|^2=a_0|A_0|^2=\frac{P_0}{\sqrt{\pi}}, \tag{13}$$

where P_0 is the input power, $a_0=a(z)|_{z=0}$ is the initial beam width. Substitution of Eq.(12) into Eq.(11) yields

$$\frac{\mathrm{d}a}{\mathrm{d}z}=\frac{2ab}{k}. \tag{14}$$

Combining Eqs.(8), (9) and (10) with Eq.(14) gives the important ordinary differential equation for the beam width

$$\frac{\mathrm{d}^2a}{\mathrm{d}z^2}=\frac{1}{k^2a^3}-\frac{4\pi^2a}{kT^2}h\exp\left(-\frac{\pi^2a^2}{T^2}\right)-\frac{1}{2k}\gamma\rho P_0a. \tag{15}$$

Equation (15) is equivalent to Newtonian second law in classical mechanics for the motion of a one-dimensional particle acted by an equivalent force $F=1/(k^2a^3)-(4\pi^2a/kT^2)h\exp(-\pi^2a^2/T^2)-(1/2k)\gamma\rho P_0a$. Letting $F=0$ and $a(z)=a_0$, we obtain the critical input power for the nonlocal lattice soliton

$$P_{\mathrm{co}}=\frac{2}{\gamma\eta k^2a_0^4}-\frac{8\pi^2h}{\gamma\eta kT^2}\exp\left(-\frac{\pi^2a_0^2}{T^2}\right). \tag{16}$$

Under the condition $T\to\infty$, i.e., the uniform nonlinear media, the second term of P_{co} is negligible, so $P_{\mathrm{co}}=2/(\gamma\eta k^2a_0^4)$. This indicates that the critical input power for the soliton propagation in uniform self-focusing media with nonlocal nonlinearity is larger than the critical power in the optical lattice with an imprinted transverse periodic modulation of the refractive index. We know that the periodic modulation of the refractive index plays an important role in compressing the beam in self-focusing media.

From Eq.(14), we obtain that the curvature of the beam is $b(z)=(k/2)\mathrm{d}(\ln a)/\mathrm{d}z$. Assuming that the amplitude has the form $A(z)=|A(z)|\exp[\mathrm{i}\phi(z)]$, we can obtain the parameter $\phi(z)$ by solving Eqs.(10) and (13)

$$\begin{aligned}\frac{\mathrm{d}\phi}{\mathrm{d}z}=&-\frac{1}{2ka^2}+h\exp\left(-\frac{\pi^2a^2}{T^2}\right)\left(1+\frac{\pi^2a^2}{T^2}\right)\\&+\frac{1}{2}\rho R_0P_0.\end{aligned} \tag{17}$$

Since $\pi^2a^2/T^2\ll1$ in this paper, we use the approximation

$$\exp\left(-\frac{\pi^2a^2}{T^2}\right)=1-\frac{\pi^2a^2}{T^2}. \tag{18}$$

Integrating Eq.(15), by using the above approximation we obtain the width of the beam which satisfies the following first-order ordinary differential equation

$$\left(\frac{\mathrm{d}a}{\mathrm{d}z}\right)^2+\frac{1}{k^2a^2}+\left(\frac{4h\pi^2}{kT^2}+\frac{1}{2k}\gamma\rho P_0\right)a^2-\frac{4h}{k}+c=0. \tag{19}$$

From the initial condition $a(z)|_{z=0}=a_0$, assuming $[\mathrm{d}a(z)/\mathrm{d}z]|_{z=0}=0$, the integration constant is $c=-\frac{1}{k^2a_0^2}-\left(\frac{4h\pi^2}{kT^2}+\frac{1}{2k}\gamma\rho P_0\right)a_0^2+\frac{4h}{k}$. So Eq.(19) turns into

$$\left(\frac{\mathrm{d}y}{\mathrm{d}z}\right)^2+V(y)=0, \tag{20}$$

where $y(z)=a(z)/a_0$ is the normalized beamwidth. $V(y)$ is in the following form:

$$V(y)=\mu\left(\frac{1}{y^2}-1\right)+\upsilon(y^2-1), \tag{21}$$

where $\mu=1/(k^2{a_0}^4)$, $\upsilon=4h\pi^2/(kT^2)+\gamma\rho P_0/(2k)$. $V(y)$ denotes the effective asymmetric potential of the

nonlocal solitons, which is illustrated in Fig.1. The parameters are $n_0 = 2.3$, $k = 2.89 \times 10^7$, $a_0 = 9\,\mu$m, $\gamma = 2.5 \times 10^8$, $T = 6 \times 10^{-4}$ for Fig.1(a), and $h = 9\times10^4$, $P_0 = 60$ for Fig.1(b). We also assume that the spatial nonlocal response function of the media is in the Gaussian form[6]

$$R(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{x^2}{2\sigma^2}\right), \tag{22}$$

which implies that $\gamma = -R^{(2)}(x)|_{x=0} = 1/(\sqrt{2\pi}\sigma^3)$, here σ is the width of the response function which is much larger than the beam width in strongly nonlocal media. From Fig.1 we see, as the incident power P_0 increases, the minimum and the width of the potential decreases synchronously. While as the modulation period T increases, the minimum and the width of the potential increases synchronously.

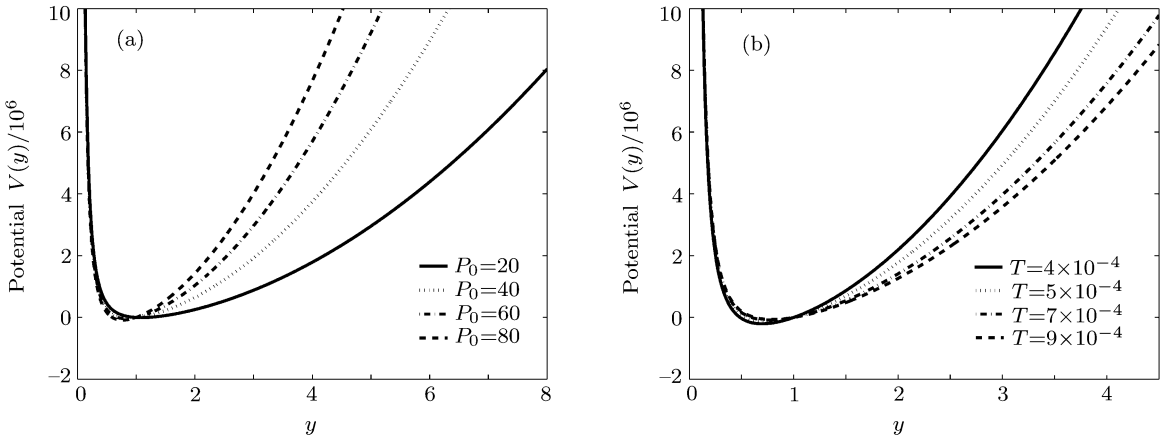


Fig.1. Potential $V(y)$ in a strongly nonlocal nonlinear media for different values of incident power P_0 (a) and the modulation period T (b).

3. Solitons in nonlocal nonlinear media with the periodic modulation of the refractive index

Differentiation of Eq.(21) with respect to y yields

$$V'(y) = -\frac{2\mu}{y^3} + 2\upsilon y. \tag{23}$$

By assuming $y = 1$, the width of solitons in optical lattices with nonlocal nonlinearity can be obtained as

$${a_0}^4 = \frac{2T^2}{8hk\pi^2 + \gamma\rho k P_0 T^2}, \tag{24}$$

so the critical input power for the soliton propagation is

$$P_{\rm c} = \frac{2}{\gamma\rho k a_0^4} - \frac{8\pi^2 h}{\gamma\rho T^2}. \tag{25}$$

This is the condition for soliton propagation in optical lattices with strongly nonlocal nonlinearity, which is approximately equivalent to Eq.(16) when $\pi^2 a^2/T^2 \ll 1$. It is indicated that the solitons will be stable when the input power obeys a restricted value which is related to the beam width, nonlocal response function of the media, the modulation period and the lattice depth.

In Fig.2(a) is plotted the radius of nonlocal solitons versus the incident power for different modulation periods. Figure 2(b) shows the radius of nonlocal soliton versus the modulation period for different incident powers. Both Figs.2(a) and 2(b) show that the soliton width will decrease when the incident power increases. When the incident power increases, the self-trapping effect will be stronger than the diffraction, leading to a decrease in the beamwidth. The incident power effectively increases the strength of the nonlinearity. As the modulation period increases, the solitons width will increase. This is because the diffraction will be strengthened when the modulation period is large, which leads to an increase in the beamwidth.

In the limit of uniform media, i.e., $T = \infty$, we obtain the radius of nonlocal lattice solitons in strongly nonlocal media

$$a_0^4 = \frac{2}{\gamma\rho k P_0}. \tag{26}$$

So the critical power of uniform nonlocal solitons is $P_{\rm c}' = n_0/(k^2\gamma a_0^4)$, which correctly reduces to the solution previously obtained by Snyder and Mitchell.[1]

Assuming that $P_0 = P_{\rm c}$, we can gain the following two equations from Eq.(25):

$${T_{\rm c}}^2 = 8\pi^2 hk a_0^4/(2 - \gamma\rho k a_0^4 P_0), \tag{27}$$

and

$$h_{\rm c} = (2 - \gamma\rho k a_0^4 P_0)T^2/8\pi^2 k a_0^4. \qquad (28)$$

It is shown that as P_0 decreases, the critical value of the modulation period will decrease, but the lattice depth will increase in Eqs.(27) and (28). For a smaller incident power, it needs a smaller modulation period or a larger lattice depth to make sure that the solitons maintain their width during propagation. And it is clearly shown that the nonlinearity of the medium depends on the incident power in strongly nonlocal media in Fig.2. When the incident power decreases, the self-trapping effect will be weaker than the diffraction, leading to an increase in the beamwidth. So, as the modulation period decreases with increasing lattice depth, the ability of compressing the beam increases. It is self-evident that the incident power $P_0 > 0$, so the modulation period must satisfy the inequality $T_{\rm c} > 2\sqrt{hk}\pi a_0{}^2$, depending on the lattice depth. There is an indication that nonlocal lattice solitons exist above a threshold of the modulation period. This means that as the incident power decreases, which strengthens the diffraction, a smaller modulation period is needed to balance the diffraction. In the same manner, for a certain modulation period, there is also a threshold of lattice depth. Next, we will study the linear oscillation of the nonlocal lattice solitons.

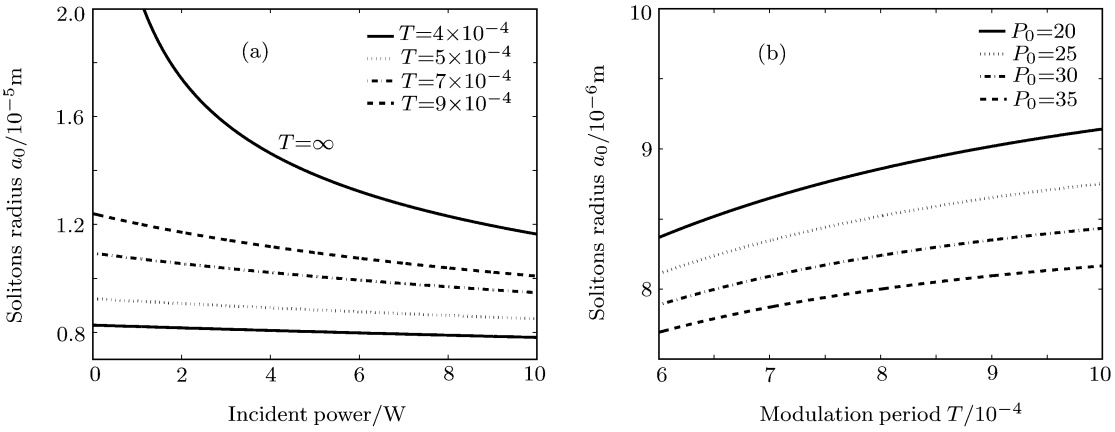


Fig.2. The width of nonlocal spatial soliton as a function of the incident power (a) and the modulation period (b). The initial parameters are $n_0 = 2.3$, $k = 2.89 \times 10^7$, $\gamma = 2.5 \times 10^8$, and $h = 6 \times 10^4$.

Rewrite Eq.(20) as

$$\left(\frac{{\rm d}y}{{\rm d}z}\right)^2 = \frac{\alpha^2(y^2-1)(\varepsilon - y^2)}{y^2}, \qquad (29)$$

where $\alpha^2 = \upsilon$, $\varepsilon = (1 + mP_{\rm c})/(1 + mP_0)$, and $m = \gamma\rho T^2/(8\pi^2 h)$. Equation (29) is analytically integrable. Solving Eq.(29) yields the simple form of solitons in strongly nonlocal media

$$a^2(z) = a_0^2\left[\cos^2(\alpha z) + \varepsilon \sin^2(\alpha z)\right]. \qquad (30)$$

When $\varepsilon = 1$ ($P_0 = P_c$), *i.e.*, $a(z) = a_0$, the optical lattice solitons maintain their width during the propagation and thus the solitons are stable. When the incident power do not satisfy Eq.(25), i.e., $\varepsilon \neq 1$, the solitons will undergo a linear harmonic oscillation.[1]

It is indeed the case, as illustrated in Fig.3, where we show the nonstationary propagation of optical solitons in strongly nonlocal media with periodic modulations of the refractive index when the critical power is not equal to the incident power ($\varepsilon \neq 1$).

From Fig.3(a) we can see that the beam will oscillate periodically with propagation distance between unity and somewhat larger values when the incident power is smaller than the critical power ($\varepsilon > 1$). The beamwidth will increase at first and decrease when it reaches the maximum. The amplitudes and periods of the oscillation are larger for lower incident power and smaller for higher incident power. In Fig.3(b) we show the linear harmonic oscillation of the normalized solitons width when the incident power is larger than the critical power ($0 < \varepsilon < 1$). The normalized beamwidth will oscillate between unity and somewhat smaller values. As the incident power increases, the oscillation amplitudes increase but the minima of the beamwidth decrease. The oscillation periods of beamwidth will decrease synchronously. From Fig. 3 we know that when the incident power increases, the beamwidth will decrease for both cases of $\varepsilon > 1$ and $0 < \varepsilon < 1$.

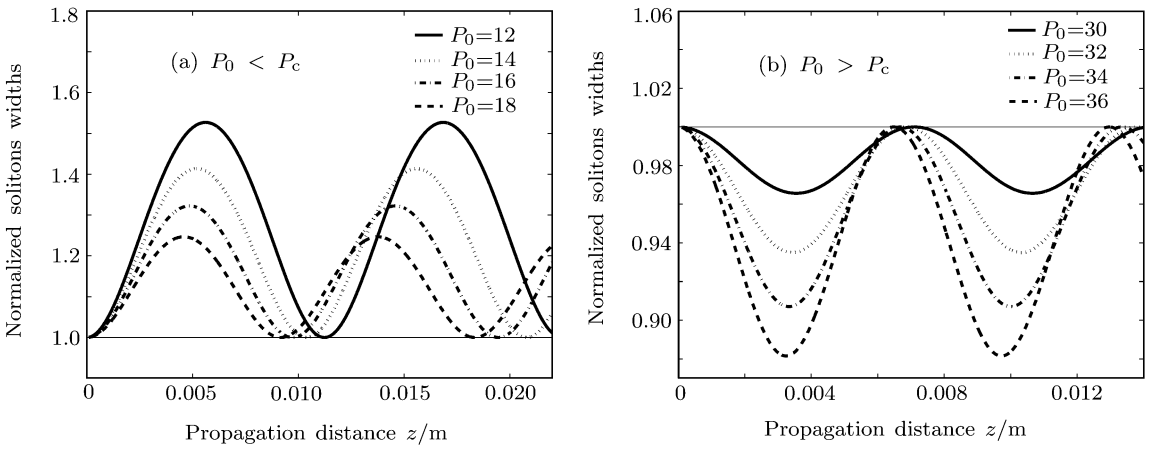


Fig.3. Linear harmonic oscillations of the solitons width when (a) $P_0 < P_c$ ($\varepsilon > 1$), (b) $P_0 > P_c$ ($0 < \varepsilon < 1$). The initial parameters are $n_0 = 2.3$, $k = 2.89 \times 10^7$, $a_0 = 9\,\mu$m, $\gamma = 2.5 \times 10^8$ ($\sigma = 133a_0$), and the critical power is $P_c = 27.98$ (Watt).

In Figs.4(a) and 4(b) we show that the beam will oscillate periodically with the given incident power $P_0 = 27.35$ and the lattice depth $h = 900$. In this case, there exists a critical value of the modulation period $T_c = 5.5 \times 10^{-4}$, which makes the solitons maintain their width during the propagation. When $T < T_c$, it

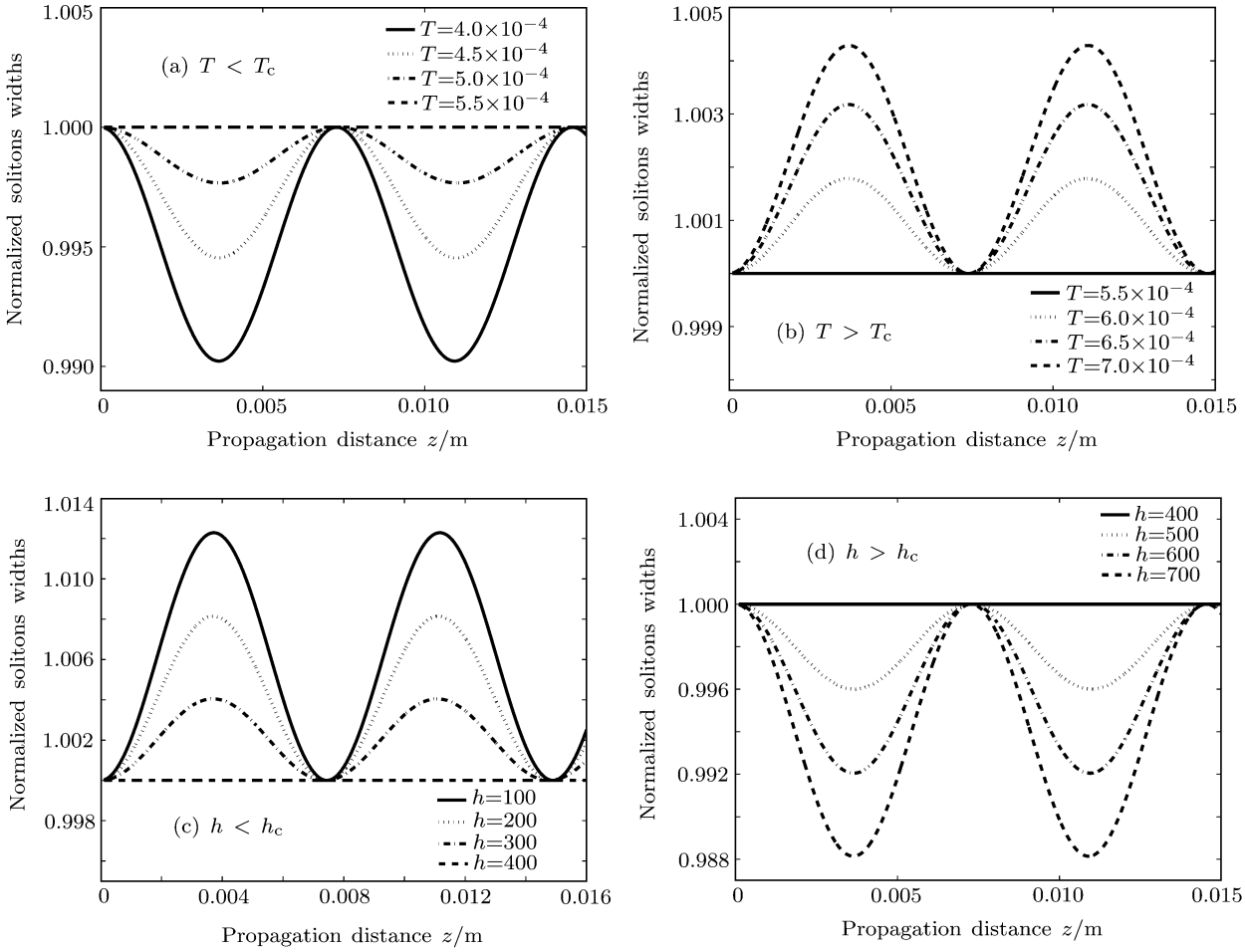


Fig.4. Linear harmonic oscillations of the solitons width with different modulation periods in (a) and (b) when $P_0 = 27.35$, $h = 900$, and with different modulation depths in (c) and (d) when $P_0 = 27.08$, $T = 3.05 \times 10^{-4}$. The initial parameters are $n_0 = 2.3$, $k = 2.89 \times 10^7$, $a_0 = 9\mu m$, $\gamma = 2.5 \times 10^8$ ($\sigma = 133a_0$).

can be seen that the beamwidth will decrease at first and increase when it reaches the minimum in Fig.4(a). For $T > T_c$, it is shown that the beamwidth will increase first and decrease when it reaches the maximum in Fig.4(b). Both in Figs.4(a) and 4(b), as the modulation period increases, the width of solitons increases (also see Fig.2). Figures 4(c) and 4(d) show that the beam will oscillate periodically at a certain input power $P_0 = 27.08$ and modulation period $T = 3.05\times10^{-4}$. In this case, a critical value of the lattice depth $h_c = 400$ guarantees a stable spatial soliton propagation. When $h < h_c$, the beamwidth will increase first and decrease when it reaches the maximum in Fig.4(c), which is different from the case $T < T_c$ in Fig.4(a). In Fig.4(d) the beamwidth will decrease at first and increase when it reaches the minimum for $h > h_c$. With increasing lattice depth, the width of solitons decreases as in Figs.4(c) and 4(d). We can draw the conclusion, based on the discussion above, that the larger the modulation period, the stronger the diffraction of the beam will be, and that as the lattice depth increases, the diffraction of the beam will be weakened.

4. Conclusion

In summary, we have studied the propagation of spatial solitons in a strongly nonlocal media with a Kerr nonlinearity, accompanied with a transverse periodic modulation of the refractive index. We obtain an exact analytical solution of such nonlocal spatial solitons. For such solitons to exist, the incident power should be equal to the critical value, which is related to the nonlocality, the lattice depth and the modulation period. When the incident power is not equal to the critical value, the solitons width will undergo linear harmonic oscillation. We find that there exist threshold values of the modulation period and the lattice depth for such nonlocal solitons, and that the critical incident power for the solitons propagating in the nonlocal lattice is smaller than that in uniform self-focusing media with nonlocal nonlinearity. It is also found that the larger the modulation period, the stronger the diffraction of the beam will be, and that as the lattice depth increases, the diffraction of the beam will be weakened.

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