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# Wigner functions and tomograms of the even and odd binomial states*

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Using the coherent state representation of Wigner operator and the technique of integration within an ordered product (IWOP) of operators, the Wigner functions of the even and odd binomial states (EOBSs) are obtained. The physical meaning of the Wigner functions for the EOBSs is given by means of their marginal distributions. Moreover, the tomograms of the EOBSs are calculated by virtue of intermediate coordinate-momentum representation in quantum optics.

**Keywords:** even and odd binomial states, integration within an ordered product (IWOP) technique, Wigner function, marginal distribution

**PACC:** 4250, 0365

## 1. Introduction

It is well known that the Wigner function[1,2] in phase space plays an important role in modern quantum optics and quantum statistics. Since two marginal distributions of the Wigner function lead to measuring probabilities in coordinate space and momentum space separately. The Wigner function is most popularly used in various branches of physics.

In this paper we derive Wigner functions and tomograms of the even and odd binomial states (EOBSs). There are two reasons why we pay attention to this case: Firstly, the EOBSs are eigenstates of some nonlinear combinations of radiation field operators and exhibit strong nonclassical properties such as sub-Poissonian statistics, antibunching, and squeezing.[3] Also, the even binomial state (EBS) and the odd binomial state (OBS) display different nonclassical properties. Secondly, various photon states have been generated by the micromaser[4] and the Wigner functions of some cavity fields can be measured in the whole phase space by a scheme based on the interaction between cavity fields and atoms.[5,6] Therefore, it is very interesting to construct and measure the Wigner functions and the tomograms of the EOBSs by using some theoretical schemes.

The binomial state defined by Stoler[7]

$$|\eta, M\rangle = \sum_{n=0}^{\infty} \sqrt{\binom{M}{n} \eta^n (1-\eta)^{M-n}}\, |n\rangle\,, \quad 0 < \eta < 1, \tag{1}$$

which is an intermediate state between the coherent state and the number state $|n\rangle = a^{\dagger n}|0\rangle/\sqrt{n!}$.

In Ref.[8], the EBS and the OBS are defined as, respectively,

$$|\eta, M\rangle_{\rm e} = C_{\rm e} \sum_{n=0}^{\left[\frac{M}{2}\right]} \sqrt{\binom{M}{2n} \eta^{2n} (1-\eta)^{M-2n}}\, |2n\rangle\,, \tag{2}$$

and

$$|\eta, M\rangle_{\rm o} = C_{\rm o} \sum_{n=0}^{\left[\frac{M-1}{2}\right]} \sqrt{\binom{M}{2n+1} \eta^{2n+1} (1-\eta)^{M-(2n+1)}}\, |2n+1\rangle\,, \tag{3}$$

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where $0 < \eta < 1$, $n$=0, 1, 2, ..., $\infty$, and the normalization constants $C_{\rm e}$ and $C_{\rm o}$ are obtained from the normalization condition ${}_{\rm e,o}\langle \eta, M\,|\eta, M\rangle_{\rm e,o} = 1$, so that

$$C_{\rm e} = \sqrt{\frac{2}{1+(1-2\eta)^M}}, \tag{4}$$

and

$$C_{\rm o} = \sqrt{\frac{2}{1-(1-2\eta)^M}} \tag{5}$$

with the subscripts 'e' and 'o' denoting even state and odd state respectively. The states $|\eta, M\rangle_{\rm e}$ and $|\eta, M\rangle_{\rm o}$ approach to the even and odd coherent states under the limiting condition of parameters: $\eta \to 0$ and $M \to \infty$, such that the mean photon number $\bar{n} = \eta M$ keeps constant. While under another limiting condition $\eta \to 1$ and $0 < \eta < 1$, the states $|\eta, M\rangle_{\rm e}$ and $|\eta, M\rangle_{\rm o}$ become the even number state and the odd number state respectively. It is interesting to see that by changing two parameters: $\eta$ and $M$, in the EOBSs, we can obtain different quantum states.

## 2. Wigner functions of the EOBSs

With the study of the Wigner function one can characterize the non-classical nature of a state. In the coherent state representation, the Wigner operator $\Delta(\alpha, \alpha^*)$ is[9]

$$\Delta(\alpha, \alpha^*) = \int \frac{{\rm d}^2 z}{\pi} \,|\alpha + z\rangle\,\langle \alpha - z|\, {\rm e}^{\alpha z^* - \alpha^* z}, \tag{6}$$

where $|\alpha + z\rangle$ is the coherent state and $\alpha$ is a complex number. Using the technique of integration within an ordered product (IWOP) of operators,[9] the Wigner operator $\Delta(\alpha, \alpha^*)$ reduces to the following form:

$$\Delta(\alpha, \alpha^*) = \frac{2}{\pi} D(2\alpha)(-1)^N, \tag{7}$$

where $N = a^\dagger a$, and $D(2\alpha) = \exp[2(\alpha a^\dagger - \alpha^* a)]$ is the displaced operator.

For a pure state $|\psi\rangle$, we can express it as

$$|\psi\rangle = \sum_{n=0}^{\infty} f_n \,|n\rangle\,. \tag{8}$$

The Wigner function of the state $|\psi\rangle$ is given by

$$\begin{aligned} W(\alpha, \alpha^*) &= \langle\psi|\,\Delta(\alpha, \alpha^*)\,|\psi\rangle \\ &= \frac{2}{\pi} \sum_{m,n=0}^{\infty} f_m^* f_n (-1)^m \chi_{mn}(2\alpha), \end{aligned} \tag{9}$$

where

$$\chi_{mn}(2\alpha) = \begin{cases} \sqrt{n!/m!}(2\alpha)^{m-n} \exp(-2\,|\alpha|^2) {\rm L}_n^{m-n}(4\,|\alpha|^2), & m \geq n, \\ \sqrt{m!/n!}(-2\alpha^*)^{n-m} \exp(-2\,|\alpha|^2) {\rm L}_m^{n-m}(4\,|\alpha|^2), & m < n, \end{cases} \tag{10}$$

where ${\rm L}_n^m(x)$ denotes an associated Laguerre polynomial[10]

$$ {\rm L}_n^m(x) = \sum_{l=0}^{n} \binom{n+m}{n-l} \frac{(-x)^l}{l!}. \tag{11}$$

Through the calculation, we find that the Wigner function $W_{\rm e}(\alpha, \alpha^*)$ of the state $|\eta, M\rangle_{\rm e}$ can be written as a sum of the mixture part $W_{\rm e}^M(\alpha, \alpha^*)$ and quantum interference part $W_{\rm e}^I(\alpha, \alpha^*)$,[11] i.e.

$$W_{\rm e}(\alpha, \alpha^*) = W_{\rm e}^M(\alpha, \alpha^*) + W_{\rm e}^I(\alpha, \alpha^*), \tag{12}$$

where the mixture part $W_{\rm e}^M(\alpha, \alpha^*)$ is found to be

$$W_{\rm e}^M(\alpha, \alpha^*) = \frac{4\exp(-2\,|\alpha|^2)}{\pi[1+(1-2\eta)^M]} \sum_{n=0}^{\left[\frac{M}{2}\right]} (-1)^{2m} \binom{M}{2n} \eta^{2n} (1-\eta)^{M-2n} {\rm L}_{2n}(4\,|\alpha|^2), \tag{13a}$$

and the quantum interference part $W_{\rm e}^I(\alpha, \alpha^*)$ has the following form:

$$W_{\rm e}^I(\alpha, \alpha^*) = \frac{4\exp(-2\,|\alpha|^2)}{\pi[1+(1-2\eta)^M]} \sum_{\substack{m,n=0 \\ m \neq n}}^{\left[\frac{M}{2}\right]} (-1)^{2m} \sqrt{\binom{M}{2n}\binom{M}{2m}} \eta^{m+n} (1-\eta)^{M-n-m}$$

$$\times\left[\sqrt{\frac{(2n)!}{(2m)!}}(2\alpha)^{2(m-n)}\mathrm{L}_{2n}^{2(m-n)}(4\,|\alpha|^2)+\sqrt{\frac{(2m)!}{(2n)!}}(-2\alpha^*)^{2(n-m)}\mathrm{L}_{2m}^{2(n-m)}(4\,|\alpha|^2)\right]. \tag{13b}$$

Similarly, using expressions (3)–(11) we have

$$W_{\mathrm{o}}^{M}(\alpha,\alpha^*)=\frac{4\exp(-2\,|\alpha|^2)}{\pi[1-(1-2\eta)^M]}\sum_{n=0}^{\left[\frac{M-1}{2}\right]}(-1)^{2m+1}\binom{M}{2n+1}\eta^{2n+1}(1-\eta)^{M-(2n+1)}\mathrm{L}_{2n+1}(4\,|\alpha|^2), \tag{14a}$$

and

$$W_{\mathrm{o}}^{I}(\alpha,\alpha^*)=\frac{4\exp(-2\,|\alpha|^2)}{\pi[1-(1-2\eta)^M]}\sum_{\substack{m,n=0\\ m\neq n}}^{\left[\frac{M-1}{2}\right]}(-1)^{2m+1}\sqrt{\binom{M}{2n+1}\binom{M}{2m+1}}\eta^{m+n+1}(1-\eta)^{M-n-m-1}$$
$$\times\left[\sqrt{\frac{(2n+1)!}{(2m+1)!}}(2\alpha)^{2(m-n)}\mathrm{L}_{2n+1}^{2(m-n)}(4\,|\alpha|^2)+\sqrt{\frac{(2m+1)!}{(2n+1)!}}(-2\alpha^*)^{2(n-m)}\mathrm{L}_{2m+1}^{2(n-m)}(4\,|\alpha|^2)\right]. \tag{14b}$$

By virtue of numerical computation, the Wigner functions of the EOBSs are plotted for different values of the parameters $\eta$($\eta$=0.1,0.5 and 0.9) and $M$ ($M$=6 and 7) against Re($\alpha$) and Im($\alpha$) in Figs.1 and 2, respectively. We discuss the variation of the Wigner function with $\eta$ in phase space.

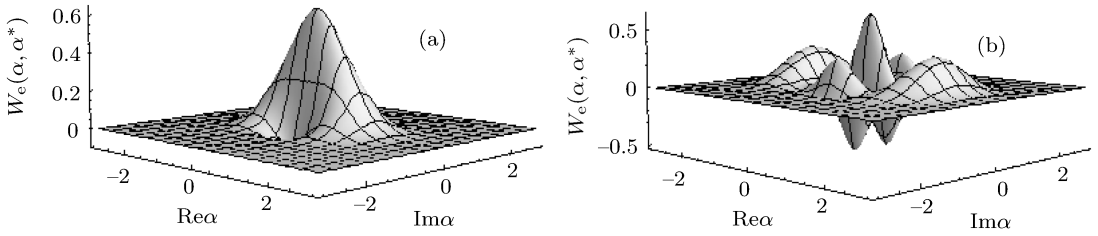



**Fig.1.** Wigner functions of the even binomial state with Re$\alpha$ and Im$\alpha$ for different values of $\eta$: (a) $\eta$=0.1 (coherent state limit), (b) $\eta$=0.5, with $M$=6 for all cases.

In Fig.1(a) we have the Wigner function $W_{\mathrm{e}}(\alpha,\alpha^*)$ corresponding to the EBS for the coherent state limit ($\eta$=0.1). The distribution has an almost Gaussian-like shape due to the existence of a little negativity, but for the OBS it shows larger negativity in Fig.2(a). These characters indicate that the non-classicality of the OBS is more pronounced, namely the sub-Poissonian statistics of the OBS is stronger than that of the EBS.

In Fig.1(b) the Wigner function of the EBS displays one prominent upward peak and two downward peaks at the origin of phase space, however there exist two upward peaks and only one prominent downward peak in Fig.2(b) which shows the plots for the OBS $W_{\mathrm{o}}(\alpha,\alpha^*)$ with the same values of $\eta$ and different values of $M$ in the phase space. This means that the EBS and the OBS are orthogonal. The conclusion is also valid for other values of $\eta$ and $M$ in this case.

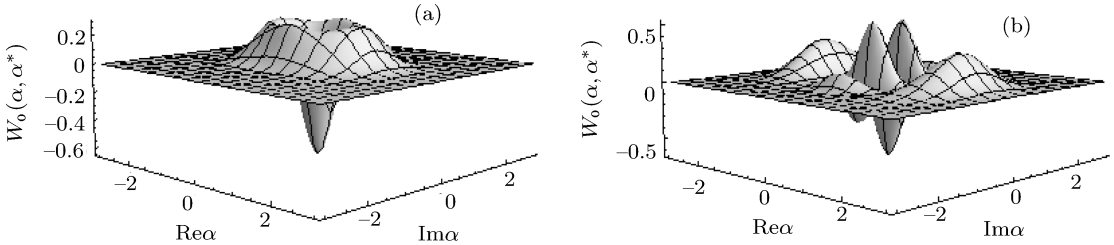



**Fig.2.** Wigner functions of the odd binomial state with Re$\alpha$ and Im$\alpha$ for different values of $\eta$: (a) $\eta$=0.1 (coherent state limit), (b) $\eta$=0.5, with $M$=7 for all cases.

The comparison between Fig.1 and Fig.2 emphasizes that the Wigner function distributions of the EOBSs are real which can take negative values in limited regions of phase space. Therefore the nonclassical properties of the EOBSs can be established in the whole ranges of $\eta$ and $M$. Moreover, we note that the negative part of the distribution is larger when $\eta$ increases, which means that the nonclassicality is more pronounced. In other words, the stronger sub-Poissonian statistics occurs. On the other hand, with $\eta$ increasing the oscillatory behaviours between the positive part and the negative part of $W_{\mathrm{e,o}}(\alpha,\alpha^*)$ of the EOBSs are more pronounced and irregular distributions than those of the usual binomial state, namely the EOBSs exhibit stronger quantum interference effects. In conclusion, the behaviour of the Wigner function $W_{\mathrm{e,o}}(\alpha,\alpha^*)$ accords with the quantum features of these states.

Now, we come to investigate the marginal distributions of Wigner functions for the EOBSs. As is well known, the one-mode Wigner operator $\Delta(p,q)$ can be composed by virtue of IWOP technique, since its marginal integration is the position projector

$$\int_{-\infty}^{\infty}\mathrm{d}p\Delta(p,q)=|q\rangle\langle q|\,, \tag{15}$$

and the momentum projector

$$\int_{-\infty}^{\infty}\mathrm{d}q\Delta(p,q)=|p\rangle\langle p|\,, \tag{16}$$

where $|q\rangle$ and $|p\rangle$ are the standard eigenvectors of the coordinate operator $Q$ and the momentum operator $P$, respectively.

If setting $\alpha=(q+\mathrm{i}p)/\sqrt{2}$ in expressions (13a) and (13b), then using Eq.(15) we can directly obtain the marginal distribution of the Wigner function for the state $|\eta,M\rangle_{\mathrm{e}}$ in the $q$ direction

$$P_{\mathrm{e}}(q)=\frac{2\exp(-q^2)}{[1+(1-2\eta)^M]\sqrt{\pi}}\left|\sum_{n=0}^{\left[\frac{M}{2}\right]}\sqrt{\binom{M}{2n}\eta^{2n}(1-\eta)^{M-2n}}\frac{1}{\sqrt{(2n)!2^n}}\mathrm{H}_{2n}(q)\right|^2. \tag{17a}$$

Using expressions (13a) and (13b) and Eq.(16), we can obtain another marginal distribution in the $p$ direction

$$P_{\mathrm{e}}(p)=\frac{2\exp(-q^2)}{[1+(1-2\eta)^M]\sqrt{\pi}}\left|\sum_{n=0}^{\left[\frac{M}{2}\right]}\sqrt{\binom{M}{2n}\eta^{2n}(1-\eta)^{M-2n}}\frac{1}{\sqrt{(2n)!2^n}}\mathrm{H}_{2n}(p)\right|^2, \tag{17b}$$

where we have used the formula for Hermite polynomials[10]

$$\mathrm{H}_n(q)=\exp\left(\frac{q^2}{2}\right)\left(q-\frac{\mathrm{d}}{dq}\right)^n\exp\left(-\frac{q^2}{2}\right). \tag{18}$$

From expressions (17a) and (17b), we find that $P_{\mathrm{e}}(q)$ (or $P_{\mathrm{e}}(p)$) is the probability of finding a particle with a given value of position $q$ (or a value of momentum $p$) in the state $|\eta,M\rangle_{\mathrm{e}}$ described by expression (2). Similarly, we have

$$P_{\mathrm{o}}(q)=\frac{2\exp(-q^2)}{[1-(1-2\eta)^M]\sqrt{\pi}}\left|\sum_{n=0}^{\left[\frac{M-1}{2}\right]}\sqrt{\binom{M}{2n+1}\eta^{2n+1}(1-\eta)^{M-(2n+1)}}\frac{1}{\sqrt{(2n+1)!2^{2n+1}}}\mathrm{H}_{2n+1}(q)\right|^2, \tag{19a}$$

and

$$P_{\mathrm{o}}(p)=\frac{2\exp(-q^2)}{[1-(1-2\eta)^M]\sqrt{\pi}}\left|\sum_{n=0}^{\left[\frac{M-1}{2}\right]}\sqrt{\binom{M}{2n+1}\eta^{2n+1}(1-\eta)^{M-(2n+1)}}\frac{1}{\sqrt{(2n+1)!2^{2n+1}}}\mathrm{H}_{2n+1}(p)\right|^2, \tag{19b}$$

which are the marginal distributions of the Wigner function for the state $|\eta, M\rangle_{\rm o}$. $P_{\rm o}(q)$ (or$P_{\rm o}(p)$) is the probability of finding a particle in the whole space with a given value of position $q$ (or the value of momentum $p$) in the state $|\eta, M\rangle_{\rm o}$, it is also the physical meaning of the Wigner functions of the EOBSs.

## 3. Tomograms of the EOBSs

In recent years, the tomograms in quantum mechanics and quantum optics have been used to investigate the possibility of describing a quantum state with a positive probability distribution. A direct description of quantum states by means of quantum tomograms for the system observables is interesting from both the theoretical and experimental points of view. So, we derive the tomograms of the states $|\eta, M\rangle_{\rm e,o}$ which are defined as

$$T_{\rm e,o}(q,f,g)=\iint_{\infty}{\rm d}p'{\rm d}q'\delta(q-fq'-gp')W_{\rm e,o}(q',p'), \tag{20}$$

where $W_{\rm e,o}(q',p')$ are the Wigner functions given by expressions (13) and (14) for the states $|\eta, M\rangle_{\rm e,o}$. However, it is very difficult to evaluate the integration if we directly substitute expressions (13a) and (13b) into expression (20). Luckily, we can use the following Radon transform between the Wigner operator and the projection operator $|q\rangle_{f,g\ f,g}\langle q|$ of intermediate coordinate-momentum state:[12]

$$|q\rangle_{f,g\ f,g}\langle q|=\iint_{\infty}{\rm d}p'{\rm d}q'\delta(q-fq'-gp')\Delta(q',p'), \tag{21}$$

where the state $|q\rangle_{f,g}$ is expressed as

$$|q\rangle_{f,g}=\frac{1}{[\pi(f^2+g^2)]^{1/4}}\exp\left[-\frac{q^2}{2(f^2+g^2)}+\frac{\sqrt{2}q}{f-{\rm i}g}a^{\dagger}-\frac{f+{\rm i}g}{2(f-{\rm i}g)}\right]a^{\dagger 2}|0\rangle, \tag{22}$$

which is the eigenvector of the operator $(fQ+gP)$ corresponding to the eigenvalue $q$; $f$and $g$ are two real parameters. Thus, the tomograms of the states $|\eta, M\rangle_{\rm e,o}$ given by expressions (3) and (4) are

$$\begin{aligned}T_{\rm e,o}(q,f,g)&=\iint_{\infty}{\rm d}p'{\rm d}q'\delta(q-fq'-gp')_{\rm e,o}\langle\eta,M|\,\Delta(q',p')\,|\eta,M\rangle_{\rm e,o}\\&=\left|{}_{f,g}\langle q\,|\eta,M\rangle_{\rm e,o}\right|^2,\end{aligned} \tag{23}$$

which shows that for the tomogram approach there exists an intermediate coordinate-momentum state, $|q\rangle_{f,g}$, and the Radon transform of the Wigner operator is just a pure-state matrix $|q\rangle_{f,g\ f,g}\langle q|$. Using the completeness relation of coherent states, we can obtain the tomogram amplitude of the state $|\eta, M\rangle_{\rm e}$ as

$$\begin{aligned}{}_{f,g}\langle q\,|\eta,M\rangle_{\rm e}=&{}_{f,g}\langle q|\int\frac{{\rm d}^2\alpha}{\pi}|\alpha\rangle\langle\alpha|\,\eta,M\rangle_{\rm e}\\=&[\pi(f^2+g^2)]^{-1/4}\sqrt{\frac{2}{[1+(1-2\eta)^M]}}\sum_{n=0}^{\left[\frac{M}{2}\right]}\sqrt{\binom{M}{2n}\eta^{2n}(1-\eta)^{M-2n}}\\&\times\sqrt{\frac{1}{(2n)!}}\left[\sqrt{\frac{f-{\rm i}g}{2(f+{\rm i}g)}}\right]^{2n}{\rm H}_{2n}\left(\frac{q}{\sqrt{f^2+q^2}}\right)\exp\left[-\frac{q^2}{2(f^2+q^2)}\right],\end{aligned} \tag{24}$$

where we also have used the following integral formula:[12]

$$\begin{aligned}\int\frac{{\rm d}^2z}{\pi}\exp\left[\varsigma|z|^2+\xi z+\eta z^*+fz^2+gz^{*2}\right]=&\frac{1}{\sqrt{\varsigma^2-4fg}}\exp\left[\frac{-\varsigma\xi\eta+\xi^2g+\eta^2f}{\varsigma^2-4fg}\right],\\{\rm Re}(\varsigma\pm f\pm g)<0,\quad&\text{and}\quad{\rm Re}\left(\frac{\varsigma^2-4fg}{\varsigma\pm f\pm g}\right)<0,\end{aligned} \tag{25}$$

and the generating function of $\mathrm{H}_n(q)$ expressed as

$$\mathrm{H}_n(q) = \frac{\mathrm{d}^n}{\mathrm{d}q^n}\exp(2qt - q^2)\bigg|_{t=0}. \tag{26}$$

Thus we can obtain the tomogram of the state $|\eta, M\rangle_{\mathrm{e}}$ as

$$T_{\mathrm{e}}(q,f,g) = [\pi(f^2+g^2)]^{-1/2}\frac{2}{1+(1-2\eta)^M}\left|\sum_{n=0}^{\left[\frac{M}{2}\right]}\sqrt{\binom{M}{2n}\eta^{2n}(1-\eta)^{M-2n}}\sqrt{\frac{1}{(2n)!}}\right.$$
$$\left.\times\left[\sqrt{\frac{f-\mathrm{i}g}{2(f+\mathrm{i}g)}}\right]^{2n}\mathrm{H}_{2n}\left(\frac{q}{\sqrt{f^2+q^2}}\right)\exp\left[-\frac{q^2}{2(f^2+q^2)}\right]\right|^2. \tag{27}$$

In the same way, we can derive the tomogram function of the state $|\eta, M\rangle_{\mathrm{o}}$ as

$$T_{\mathrm{o}}(q,f,g) = [\pi(f^2+g^2)]^{-1/2}\frac{2}{1-(1-2\eta)^M}\left|\sum_{n=0}^{\left[\frac{M-1}{2}\right]}\sqrt{\binom{M}{2n+1}\eta^{2n+1}(1-\eta)^{M-2n-1}}\sqrt{\frac{1}{(2n+1)!}}\right.$$
$$\left.\times\left[\sqrt{\frac{f-\mathrm{i}g}{2(f+ig)}}\right]^{2n+1}\mathrm{H}_{2n+1}\left(\frac{q}{\sqrt{f^2+q^2}}\right)\exp\left[-\frac{q^2}{2(f^2+q^2)}\right]\right|^2. \tag{28}$$

The tomogram, as a real and positive distribution function of $\alpha$, is similar to the $Q$ function of the corresponding state in quantum optics, so by virtue of the numerical computations we plot the tomograms $T_{\mathrm{e,o}}(q, f=\cos\theta, g=\sin\theta)$ of the EOBSs versus $(q,\theta)$ for the indicated values of parameter $\eta$ in Fig.3.

From Fig.3, with the coordinate $q$ changing, the tomogram of the EBS displays $\pi$-periodic variations and one prominent peak in each period. Furthermore, we can see that at the extreme of the even number state ($\eta = 0.9$), the tomographic distribution becomes narrower and higher than that for the even coherent state ($\eta = 0.1$) in the phase space. In other words, with $\eta$ increasing the interference effects begin to become stronger. On the other hand, we also find that the OBS has a similar variation in tomographic feature, such as $\pi$-periodic variation, with $\eta$. Therefore, we conclude that the EBS and the OBS show similar variations in interference effect and nonclassical property.

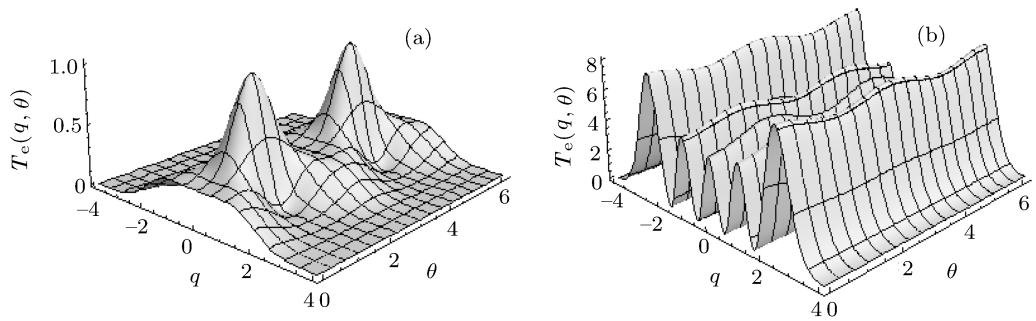



**Fig.3.** Tomographic maps of the even binomial state with $q$ and $\theta$ for different values of $\eta$: (a) $\eta$=0.1 (coherent state limit), (b) $\eta$=0.9 (number state limit), with $M$=4 for all cases.

## 4. Conclusions

We use the coherent state representation of Wigner operator and the intermediate coordinate-momentum representation to derive the Wigner functions and the tomograms of the EOBSs. It is found that the variation of the Wigner function distribution with parameters $\eta$ and $M$ (the tomogram with the

parameters $q$ and $\theta$) reveals interference properties of the EOBSs well. The feature indicates that the EBS and the OBS exhibit different nonclassical properties. The results of the tomograms for the EOBSs which are obtained by means of the projection operator of the intermediate coordinate-momentum state $|q\rangle_{f,g\,f,g}\langle q|$ may be useful for experiments as a reference. Therefore, one can measure the module-square of the wave functions of the EOBSs in coherent state representation experimentally, then the tomograms for the EOBSs are obtained and also the states $|\eta, M\rangle_{\mathrm{e,o}}$ are measured.

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