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Generation of entanglement molecules via weak coherent field in cavity QED*

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This paper presents a method for generating entanglement molecules, which is introduced by Dur (2001 *Phys. Rev.* A). In this scheme, N ladder-type three-level atoms are sent through a resonant weak coherent cavity field, then the system states are measured. And the system field may collapse onto some possible types of entanglement molecules. Meanwhile it discusses about the interaction time from the experimental point of view, and compare the result with the previous scheme proposed by Huang (2004 *J. Phys.* B: *At. Mol. Opt. Phys.*).

Keywords: weak coherent state, resonant interaction, entanglement molecules
PACC: 4250

It is well known that entanglement is at the heart of quantum information and quantum communication. Cavity quantum electrodynamics (QED), with Rydberg atoms crossing superconducting microwave cavities, provides one of the candidates for generating entangled states and quantum information processing.[1] In Ref.[1], a complementary experiment involving a coherent superposition of vacuum and one photon states has been performed. Zheng[2] proposed a scheme for the teleportation of atomic states with a weak coherent cavity field, the scheme is based on the resonant interaction of atom with a weak coherent state, which is a coherent superposition of 0 and 1 photon states. Experimental realizations of two-atom Einstein–Podolsky–Rosen (EPR) states through the resonant atom–cavity interaction[3] and two-atom collision[4] have been reported. In recent years, the basic problem of characterizing and quantifying quantum entanglement for bipartite system has been solved, so much interests are focused on the multipartite system.[5−8] Liao *et al*[5] showed their interest in three-body entanglement. Dur[6] introduced a family of multiqubit state named 'entanglement molecules', for which one can choose for any pair of qubits independently whether a maximally entangled state can be distilled without help of the remaining parties or not, this property neither belongs to GHZ state[7] nor W state. Such as three-atom entanglement molecules are defined by Eq.(1), when any atom is traced out, the remaining two atoms are always entangled.

$$\begin{aligned}|\Psi_3\rangle =& c_{e_1}c_{e_2}c_{g_3}|e_1\rangle|e_2\rangle|g_3\rangle \\ &+ c_{g_1}c_{g_2}c_{g_3}|g_1\rangle|g_2\rangle|g_3\rangle \\ &+ c_{e_1}c_{g_2}c_{e_3}|e_1\rangle|g_2\rangle|e_3\rangle \\ &+ c_{g_1}c_{e_2}c_{e_3}|g_1\rangle|e_2\rangle|e_3\rangle\,. \end{aligned} \tag{1}$$

Huang and Zhan[8] presented a scheme to prepare a kind of entanglement molecule via cavity QED and investigated its entanglement properties. They considered the nonresonant interaction of ladder-type three-level atoms with a single-mode cavity field. In contrast to the previous scheme, we propose a method to prepare N-atom entangled state via the resonant interaction of ladder-type three-level atoms with a weak coherent cavity field. After the atom field entangled states have been created, we detect the system and the atom field collapses onto the entanglement molecules. The result shows that the time required to complete the process is greatly shortened, which is important in view of decoherence.

We consider one ladder-type three-level atom resonant interaction with a weak single-mode cavity field. The atomic states are denoted by $|e\rangle$, $|i\rangle$ and $|g\rangle$. $|e\rangle$ is the excited state, $|i\rangle$ is the middle state and $|g\rangle$ is the ground state, as is shown in Fig.1. Suppose the atom-

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field coupling strength $g_1 = g_2 = g$ and the atomic transition frequency between the states $|e\rangle$ and $|i\rangle$ is resonant with the cavity field. On the other hand, the transition frequency between the middle $|i\rangle$ and the lowest state $|g\rangle$ is largely detuned from the cavity frequency so that the lowest state $|g\rangle$ is not affected during atom–cavity interaction. Then in the interaction picture, the atom–cavity interaction is described by the Jaynes–Cummings Hamiltonian[9,10]

$$H_I = \hbar g(a^\dagger|i\rangle\langle e| + a|e\rangle\langle i|), \tag{2}$$

where $a^\dagger$ and a are the creation and annihilation operators of cavity field respectively.

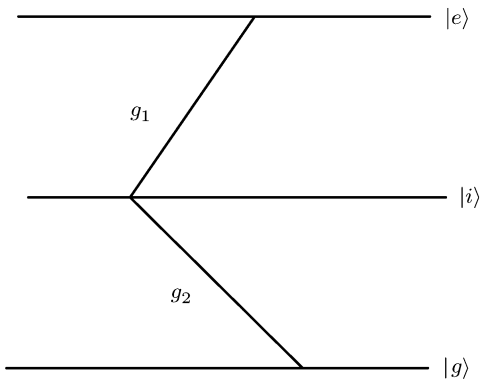


Fig.1. Schematic diagram of the type three level atom resonantly coupling to a weak coherent field, $|e\rangle$, $|i\rangle$ and $|g\rangle$ denote the states of the atom, $g_1 = g_2 = g$ is the coupling coefficient.

We assume that the cavity field is initially prepared in a coherent field $|\alpha\rangle$,[11]

$$|\alpha\rangle = \mathrm{e}^{-|\alpha^2|/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle\,. \tag{3}$$

When the average photon number $|\alpha|^2$ is low (about 0.2), the terms with $n > 1$ are negligible. Thus to a good approximation, the coherent state $|\alpha\rangle_w$ referred to as 'weak coherent field state' in this paper is given by the superposition of the vacuum state and one-photon state,[1,2] The fidelity of the coherent state $|\alpha\rangle_w$ is $F(\alpha) = |\langle\alpha|\alpha\rangle_w|^2$. Figure 2 shows that the fidelity is 0.982 corresponding to the average photon number $|\alpha|^2$=0.2.

$$|\alpha\rangle_w \simeq \mathrm{e}^{-|\alpha^2|/2}(|0\rangle + \alpha\,|1\rangle). \tag{4}$$

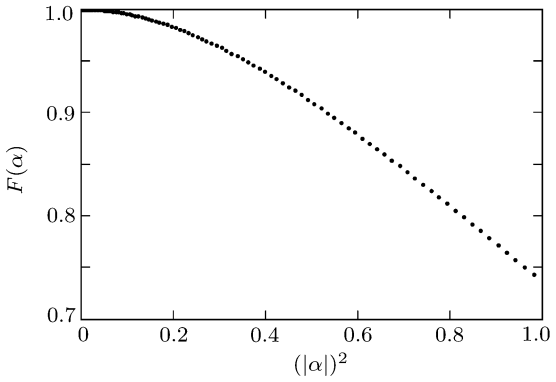


Fig.2. The fidelity of coherent state $|\alpha\rangle_w$ curve corresponding to the average photon number.

Figure 3 displays the setup for preparing and detecting the multipartite entangled states, The Rydberg atoms are produced in zone B or B′ by the excitation of velocity-selected atomic beam effusing from oven O or O′. The atoms cross the cavity C fed by the classical microwave source S and the cavity C is sandwiched between the Ramsey cavities F_i or F'_i $(i = 1, 2)$. D or D′ stands for the state selective field ionization detector.

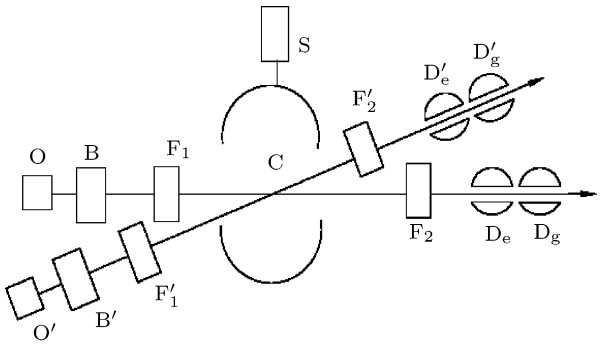


Fig.3. The setup for preparing and detecting the entanglement molecules, The Rydberg atoms are produced in zone B (B′) by the excitation of velocity-selected atomic beam effusing from oven O (O′). The atoms cross the cavity C sandwiched between the Ramsey cavities F_i or F'_i (i=1, 2) fed by the classical microwave source S. D_e (D_g) stands for the state selective field ionization detector.

All the atoms to be sent must be initially prepared by laser excitation and microwave manipulation[11] and the first atom is in the following type of superposition state:

$$|\Phi_1\rangle = c_{e_1}|e\rangle + c_{i_1}|i\rangle\,, \tag{5}$$

where $|c_{e1}|^2 + |c_{i_1}|^2 = 1$. As the state $|g\rangle$ is not affected during the atom–cavity interaction and the atom–cavity interaction is described by Hamiltonian of Eq.(2). After an interaction time τ, the atom–

cavity system evolves into

$$\begin{aligned}|\Psi_1\rangle =& c_{g_1}|g_1\rangle|\alpha\rangle_w + \mathrm{e}^{-|\alpha|^2/2}\{|i_1\rangle|0\rangle \\ &+ \alpha[\cos(g\tau)|i_1\rangle|1\rangle \\ &- \mathrm{i}\sin(g\tau)|e_1\rangle|0\rangle]\}. \end{aligned} \tag{6}$$

We control all the atoms such that they pass through the cavity with the same velocity, thus the effective interaction time for each atom with the cavity mode is assumed to be equal. Let $g\tau = \pi$, then the state $|i_1\rangle|\alpha\rangle_w$ undergoes the transition

$$\begin{aligned}|i_1\rangle|\alpha\rangle_w \rightarrow& \mathrm{e}^{-|\alpha|^2/2}(|i_1\rangle|0\rangle - \alpha|i_1\rangle|1\rangle \\ \simeq& |i_1\rangle|-\alpha\rangle_w . \end{aligned} \tag{7}$$

Then equation (6) can be simplified to

$$|\Psi_1\rangle = c_{g_1}|g_1\rangle|\alpha\rangle_w + c_{i_1}|i_1\rangle|-\alpha\rangle_w . \tag{8}$$

After the passage of the first atom, we send the second atom to inject into the cavity. Now the state of the whole system combined by the second atom and the cavity can be expanded as

$$\begin{aligned}|\Psi_2\rangle =& (c_{g_1}c_{g_2}|g_1\rangle|g_2\rangle + c_{i_1}c_{i_2}|i_1\rangle|i_2\rangle)|\alpha\rangle_w \\ &+ (c_{g_1}c_{i_2}|g_1\rangle|i_2\rangle + c_{i_1}c_{g_2}|i_1\rangle|g_2\rangle)|-\alpha\rangle_w . \end{aligned} \tag{9}$$

After the second atom leaves the cavity, the third atom prepared as the others is injected into the cavity. Then the atom–cavity state becomes

$$|\Psi_3\rangle = |\widetilde{\Psi}_3\rangle_\alpha|\alpha\rangle_w + |\widetilde{\Psi}_3\rangle_{-\alpha}|-\alpha\rangle_w,$$

where

$$\begin{aligned}|\widetilde{\Psi}_3\rangle_\alpha =& (c_{g_1}c_{g_2}|g_1\rangle|g_2\rangle + c_{i_1}c_{i_2}|i_1\rangle|i_2\rangle)c_{g_3}|g_3\rangle \\ &+ (c_{g_1}c_{i_2}|g_1\rangle|i_2\rangle + c_{i_1}c_{g_2}|i_1\rangle|g_2\rangle)c_{i_3}|i_3\rangle , \end{aligned} \tag{10}$$

$$\begin{aligned}|\widetilde{\Psi}_3\rangle_{-\alpha} =& (c_{g_1}c_{g_2}|g_1\rangle|g_2\rangle + c_{i_1}c_{i_2}|i_1\rangle|i_2\rangle)c_{i_3}|i_3\rangle \\ &+ (c_{g_1}c_{i_2}|g_1\rangle|i_2\rangle + c_{i_1}c_{g_2}|i_1\rangle|g_2\rangle)c_{g_3}|g_3\rangle . \end{aligned} \tag{11}$$

We send N atoms through the cavity one by one, the kth atom is in the following type of superposition state:

$$|\Phi_k\rangle = c_{e_k}|e_k\rangle + c_{i_k}|i_k\rangle , \tag{12}$$

where $|c_{e_k}|^2 + |c_{i_k}|^2 = 1$, and k denotes the k-th atom.

After atom N exits from the cavity, the state of the system is

$$|\Psi_N\rangle = |\widetilde{\Psi}_N\rangle_\alpha|\alpha\rangle_w + |\widetilde{\Psi}_N\rangle_{-\alpha}|-\alpha\rangle_w, \tag{13}$$

where

$$\begin{aligned}|\widetilde{\Psi}_N\rangle_\alpha =& |\widetilde{\Psi}_{N-1}\rangle_\alpha c_{g_N}|g_N\rangle \\ &+ |\widetilde{\Psi}_{N-1}\rangle_{-\alpha}c_{i_N}|i_N\rangle, \end{aligned} \tag{14}$$

$$\begin{aligned}|\widetilde{\Psi}_N\rangle_{-\alpha} =& |\widetilde{\Psi}_{N-1}\rangle_\alpha c_{i_N}|i_N\rangle \\ &+ |\widetilde{\Psi}_{N-1}\rangle_{-\alpha}c_{g_N}|g_N\rangle. \end{aligned} \tag{15}$$

In this way, the atom-field entangled states have been created. The remaining problem is how we can detect the cavity field. We assume that the measurement has been done in a time interval short enough so that the cavity dissipation effects can be ignored. If the cavity field is measured and found to be in the state $|\alpha\rangle_w$, the field collapses onto the entanglement molecule $|\widetilde{\Psi}_N\rangle_\alpha$. In order to do that we inject a field with the amplitude α into the cavity.[12] This undergoes the transitions $|\alpha\rangle_w \rightarrow |2\alpha\rangle_w$ and $|-\alpha\rangle_w \rightarrow |0\rangle$. Then we send an auxiliary atom a' effusing from oven O$'$, which is initially in the state $|i\rangle_{a'}$, across the cavity. The state $|i\rangle_{a'}|0\rangle$ undergoes no transitions. On the other hand, the state $|i\rangle_{a'}|2\alpha\rangle_w$ undergoes the transition

$$\begin{aligned}|i\rangle_{a'}|2\alpha\rangle_w \rightarrow& \mathrm{e}^{-2|\alpha|^2}\Bigg\{|i\rangle_{a'}|0\rangle + \sum_{n=1}^{\infty}\frac{(2\alpha)^n}{\sqrt{n!}} \\ &\times [\cos(g\sqrt{n}\tau)|i\rangle_{a'}|n\rangle \\ &- \mathrm{i}\sin(g\sqrt{n}\tau)|e\rangle_{a'}|n-1\rangle]\Bigg\}, \end{aligned} \tag{16}$$

where τ is the interaction time. Thus, the output $|\widetilde{\Psi}_N\rangle_\alpha|e\rangle_{a'}$ which can be detected in D$'$ corresponds to the case that the atom is found in the state $|\alpha\rangle_w$. The success probability is

$$P = \mathrm{e}^{-4|\alpha|^2}\sum_{n=1}^{\infty}\frac{(2\alpha)^{2n}}{n!}\sin^2(g\sqrt{n}\tau). \tag{17}$$

It is necessary to make a comparison of our scheme with the previous schemes. In contrast to the scheme in Ref.[8], our scheme uses a resonant interaction. The nonresonant interaction time[8] for each atom with the cavity is $\tau_{\text{previous}} = \dfrac{\pi\Delta}{g^2}$, while in our scheme the resonant interaction time is $\tau_{\text{present}} = \dfrac{\pi}{g}$. In general, Δ is chosen to be $10g$,[13] thus $\tau_{\text{present}} = 0.1\tau_{\text{previous}}$. This means that the generation process in the present scheme can be completed in a faster way, which is important for cavity QED experiments.

Finally we make a discussion in view of experiment. The most sensitive points are the dissipative processes due to atomic spontaneous emission and cavity decay. But if the total time for the generation process is much smaller than the atomic lifetime

and cavity decay time, the dissipative processes are supposed to be dominated by the atom–cavity interaction. Kuhr *et al*[14] built a microwave Fabry–Perot resonator made of diamond-machined copper mirrors coated with superconducting niobium, whose damping time ($T_c = 130$ ms at 51 GHZ and 0.8 K) corresponds to a finesse of 4.6×10^9. Assume that the radiative time for Rydberg atoms is about T_{ar}=300 ms.[1] In Ref.[14], corresponding to $Q = 4.2 \times 10^{10}$, the cavity with a photon storage time is $T_{cr} \approx$130 ms. The decoherence time of the cavity field is $T_l = \dfrac{T_{cr}}{|\alpha|^2} = 650$ ms with average photons $|\alpha|^2 = 0.2$. For a typical atom–cavity coupling strength $g = 2\pi \times 25.5$ kHz. The interaction time of each atom with the cavity field is about $\tau \approx 19.6 \times 10^{-6}$ s. If the travelling times of the atoms are considered as $T_{tr} = 10\tau$, then the travelling time $T_{tr} = 19.6 \times 10^{-5}$ s. Thus the cavity field decoherence time $T_l = 3.3 \times 10^3 T_{tr}$. Therefore, based on cavity QED techniques, the entanglement molecules involving many atoms might be realizable.

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