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Robust generation of high-fidelity entangled states for multiple atoms*

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A scheme is presented for generating entangled states of multiple atoms in a cavity. In the scheme the atoms simultaneously interact with a cavity mode, with the first atom driven by two classical fields and the other atoms driven by a classical field. Our scheme is valid even if the cavity decay rate is larger than the effective coupling strength, which is important for experiment. The generation of entangled states is conditional on the detection of a photon decaying from the cavity and thus the fidelity of the entangled state is insensitive to the detection inefficiency. Furthermore, the scheme can be applied to the case with any number of atoms in principle.

Keywords: cavity QED, entangled state, optical cavity
PACC: 4250

1. Introduction

Entanglement not only provides the possibility to test quantum nonlocality against local hidden variable theories,[1] but also has practical applications in quantum information.[2,3] The characterization of entanglement is a central topic in quantum information. For the three-qubit case, there are two inequivalent classes of entangled states, i.e. the Greenberger–Horne–Zeilinger (GHZ) class[4] and the W class.[5] Unlike the GHZ states, W states are robust against qubit loss in the sense that even some of qubits are lost the remaining qubits are still in entangled states. Furthermore, the entanglement of W states is robust against qubit flip noise and immune to global dephasing.[6]Owing to the robustness W states may lead to stronger nonclassicality than GHZ states[7] and have practical applications in quantum communication.[8−10]

Recently, W states of three photons[11] and eight ions[6] have been experimentally observed. More recently, the entanglement persistency of three-photon W states has been experimentally demonstrated.[12] The cavity QED (cavity quantum electrodynamic) system is another qualified candidate for entanglement engineering and quantum information processing since atoms are ideal for the storage of information and photons are suitable for transferring the information. Three-particle GHZ states have been experimentally realized in cavity QED systems.[13] Schemes have been proposed to generate W states for three or more atoms via atom-cavity interaction.[14−16] However, these schemes work under the ideal conditions and none of them has been realized experimentally.

In this paper, we suggest an efficient scheme for generating multi-atom W states in an optical cavity. Our scheme has several advantages. First, it can work in the regime with a cavity decay rate larger than the effective atom-cavity coupling strength, greatly relaxing the requirement for the cavity quality. Second, the generation of the entangled states is conditional on the detection of a photon decaying from the cavity and thus the entanglement is insensitive to the detector inefficiency. Third, in principle the scheme can be applied to the case with any number of atoms. Last, the interaction time does not need accurately adjusting.

The remainder of the present paper is organized as follows. In Section 2, we propose a scheme for generating W states of multiple atoms trapped in a cavity via Raman coupling. In the scheme the cavity decay is continuously monitored. In Section 3, we show that a pure entangled state can be created even if the photodetector is imperfect. In Section 4, we discuss the physical realization of the scheme and analyse decoherence effects.

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2. Generation of W states for multiple atoms

We consider n atoms trapped in an optical cavity. The atoms have one excited state $|r\rangle$ and two ground states $|e\rangle$ and $|g\rangle$. The transition $|g\rangle \rightarrow |r\rangle$ is coupled with the quantized cavity mode of coupling strength g. The transition $|e\rangle \rightarrow |r\rangle$ of the first atom is driven by a classical laser field with the Rabi frequency Ω_1 as shown in Fig.1. Meanwhile, the transition $|e\rangle \rightarrow |r\rangle$ of the other atoms is driven by another classical laser field with the Rabi frequency Ω_2 as shown in Fig.2.

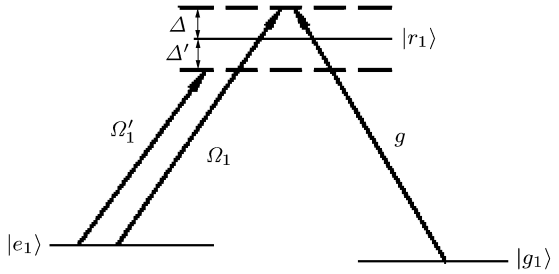


Fig.1. The atomic level configuration and transitions for the first atom. The transition $|g_1\rangle \rightarrow |r_1\rangle$ is coupled with the cavity mode of coupling constant g and detuning Δ. The transition $|e_1\rangle \rightarrow |r_1\rangle$ is driven by two classical laser fields with the detunings Δ and $-\Delta'$.

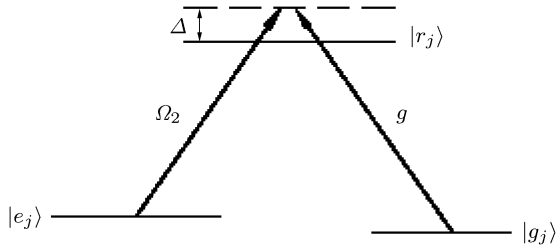


Fig.2. The atomic level configuration and transitions for the j-th ($j > 1$) atom. The transition $|g_j\rangle \rightarrow |r_j\rangle$ is coupled with the cavity mode of coupling constant g_j and detuning Δ. The transition $|e_j\rangle \rightarrow |r_j\rangle$ is driven by a classical laser field with the detuning Δ.

Here we assume that the first atom is located at one antinode of the cavity mode and the other atoms are located at another antinode of the cavity mode. In this case we can address selectively the first atom and the other atoms by two focused external laser beams, respectively. Assume that the two classical fields and the cavity mode are detuned from the respective transitions by the same amount Δ. Consider the case of $\Delta \gg \Omega_1,\ \Omega_2,\ g,\ \gamma$, where γ is the atomic spontaneous emission rate. If the atoms are initially populated in the ground states, the probability for the atoms to be populated in the upper level $|r\rangle$ is negligible and the atomic spontaneous emission is suppressed during the evolution. The classical fields and the cavity mode induce the Raman coupling between the two ground stats $|e\rangle$ and $|g\rangle$. In addition, the transition $|e\rangle \rightarrow |r\rangle$ of the first atom is driven by a classical field with the Rabi frequency Ω_1' and detuning $-\Delta'$. On condition that $\Delta' \gg \Omega_1',\ g$ this classical field induces only a Stark shift in the state $|e\rangle$. The effective Hamiltonian is

$$\begin{aligned} H_{\mathrm{e}} = & \left(\frac{\Omega_1^2}{\Delta} - \frac{{\Omega'}_1^2}{\Delta'}\right)|e_1\rangle\langle e_1| + \sum_{j=2}^{n}\frac{\Omega_2^2}{\Delta}|e_j\rangle\langle e_j| \\ & + \sum_{j=1}^{n}\frac{g^2}{\Delta}a^\dagger a\,|g_j\rangle\langle g_j| \\ & + \frac{\Omega_1 g}{\Delta}\left(aS_1^+ + a^\dagger S_1^-\right) \\ & + \sum_{j=2}^{n}\frac{\Omega_2 g}{\Delta}\left(aS_j^+ + a^\dagger S_j^-\right), \end{aligned} \tag{1}$$

where a and $a^\dagger$ are the annihilation and creation operators of the cavity mode, and $S_j^+ = |e_j\rangle\langle g_j|$ and $S_j^- = |g_j\rangle\langle e_j|$ are the rising and lowering operators for the j-th atom. The first term describes the Stark shifts for the first atom induced by the two classical fields driving it, the second term describes the Stark shifts for other $n-1$ atoms induced by the corresponding classical field, the third term describes the photon-number dependent Stark shifts induced by the cavity mode, and the other terms describes the Raman coupling between the two ground states. We define the 'excitation' number operator as

$$N_e = \sum_{j=1}^{n}|e_j\rangle\langle e_j| + a^\dagger a. \tag{2}$$

Since N_{e} commutes with H_{e}, the 'excitation' number conserves during the interaction.

We now consider the cavity decay. Suppose that the photon decays are continuously monitored. On condition that no detection of photon has been registered the system is governed by the non-Hermitian Hamiltonian

$$H_{\mathrm{n}} = H_{\mathrm{e}} - \mathrm{i}\kappa a^\dagger a, \tag{3}$$

where κ is the cavity decay rate. Assume that the system is initially in the state

$$|\psi(0)\rangle = |0e_1g_2\ldots g_n\rangle, \tag{4}$$

where $|0\rangle$ represents the vacuum state of the cavity mode. In this case only the first atom has one "excitation" and thus the whole system evolves in the

subspace with the "excitation" number being 1. Set

$$\Omega_2 = \sqrt{n}g, \quad \text{and} \quad \frac{\Omega_1^2}{\Delta} - \frac{\Omega'^2_1}{\Delta'} = \frac{\Omega_2^2}{\Delta}. \tag{5}$$

Suppose that $2\sqrt{\lambda_1^2+\lambda_2^2} < \kappa$, where $\lambda_1 = \Omega_1 g/\Delta$ and $\lambda_2 = \sqrt{n-1}\Omega_2 g/\Delta = \sqrt{(n-1)n}g^2/\Delta$. After an interaction time t the evolution of the system is given by

$$\begin{aligned}|\psi(t)\rangle =& \frac{1}{\lambda_1^2+\lambda_2^2}\left\{\lambda_2^2 + \lambda_1^2 \mathrm{e}^{-\kappa t/2}\left[\frac{\mathrm{e}^{\alpha t/2}+\mathrm{e}^{-\alpha t/2}}{2} - \frac{\kappa}{\alpha}\frac{\mathrm{e}^{-\alpha t/2}-\mathrm{e}^{\alpha t/2}}{2}\right]\right\}|0e_1g_2\cdots g_n\rangle \\ & - \frac{\lambda_1\lambda_2}{\lambda_1^2+\lambda_2^2}\left\{1 - \mathrm{e}^{-\kappa t/2}\left[\frac{\mathrm{e}^{\alpha t/2}+\mathrm{e}^{-\alpha t/2}}{2} - \frac{\kappa}{\alpha}\frac{\mathrm{e}^{-\alpha t/2}-\mathrm{e}^{\alpha t/2}}{2}\right]\right\}|\phi\rangle \\ & + 2\mathrm{i}\frac{\lambda_1}{\alpha}\mathrm{e}^{-\kappa t/2}\frac{\mathrm{e}^{-\alpha t/2}-\mathrm{e}^{\alpha t/2}}{2}|1g_1g_2\cdots g_n\rangle\,, \end{aligned} \tag{6}$$

where

$$\begin{aligned}|\phi\rangle =& \frac{1}{\sqrt{n-1}}(|0g_1e_2g_3\ldots g_n\rangle \\ & + |0g_1g_2e_3g_4\ldots g_n\rangle + \ldots \\ & + |0g_1\ldots g_{n-1}e_n\rangle), \end{aligned} \tag{7}$$

$$\alpha = \sqrt{\kappa^2 - 4(\lambda_1^2+\lambda_2^2)}. \tag{8}$$

Here we have discarded the common phase factor $\mathrm{e}^{-\mathrm{i}\Omega_2^2 t/\Delta}$. Under the condition of $\mathrm{e}^{-(\kappa-\alpha)t/2} \ll 1$ and $\mathrm{e}^{-(\kappa+\alpha)t/2} \ll 1$, the coefficients of the terms multiplied by the damping factors $\mathrm{e}^{-(\kappa-\alpha)t/2}$ and $\mathrm{e}^{-(\kappa+\alpha)t/2}$ are much smaller than 1 and thus can be discarded. In this case the quantized cavity mode is in the vacuum state. The first atom exchanges energy with the other atoms via interaction with the quantized cavity mode and classical fields. In other words, the quantized cavity mode and classical fields play a role in transferring the 'excitation'. With the help of the cavity mode and classical fields, the first atom shares the 'excitation' with the other atoms. This leads to the entangled state

$$|\psi_a\rangle = \frac{\lambda_2}{\lambda_1^2+\lambda_2^2}\left[\lambda_2|e_1g_2\ldots g_n\rangle - \frac{\lambda_1}{\sqrt{n-1}}\left(|g_1e_2g_3\ldots g_n\rangle + |g_1g_2e_3g_4\ldots g_n\rangle + \ldots + |g_1\ldots g_{n-1}e_n\rangle\right)\right]. \tag{9}$$

Choosing $\Delta' = 2\Delta$, $\Omega_1 = (n-1)\Omega_2 = (n-1)\sqrt{n}g$ and $\Omega_1' = \sqrt{2(n-2)}ng$ we have $\lambda_1 = \sqrt{n-1}\lambda_2$. Then the state $|\psi_a\rangle$ can be rewritten as

$$|\psi_a\rangle = \frac{1}{n}\left[|e_1g_2\ldots g_n\rangle - |g_1e_2g_3\ldots g_n\rangle - |g_1g_2e_3g_4\ldots g_n\rangle - \ldots - |g_1\ldots g_{n-1}e_n\rangle\right], \tag{10}$$

which is a W state. The probability for obtaining such a state is $1/n$.

3. Purification of entanglement

Up to now, we have not considered the nonunit detection efficiency η. Due to the imperfection of the photodetector, a photon may escape from the cavity but no click is observed, which reduces the fidelity. Therefore, the atomic system is in the mixed state

$$\rho = (1-\eta)\left(1-\frac{1}{n}\right)|g_1g_2\ldots g_n\rangle\langle g_1g_2\ldots g_n| + |\phi\rangle\langle\phi|\,. \tag{11}$$

In order to purify the entanglement we use a third ground state $|f\rangle$. After the transformation $|g_j\rangle \to |f_j\rangle$

we have

$$\rho' = (1-\eta)\left(1-\frac{1}{n}\right)|f_1 f_2 \ldots f_n\rangle\langle f_1 f_2 \ldots f_n| + |\phi'\rangle\langle\phi'|, \tag{12}$$

where

$$|\phi'\rangle = \frac{1}{n}(|e_1 f_2 \ldots f_n\rangle - |f_1 e_2 f_3 \ldots f_n\rangle - \ldots - |f_1 \ldots f_{n-1} e_n\rangle). \tag{13}$$

Then we drive the transition $|e\rangle \to |r\rangle$ for the first atom and the other atoms by using two classical fields with the same Rabi frequency Ω and detuning Δ, respectively. On condition that no detection of photon is registered the evolution of the system is governed by the non-Hermitian Hamiltonian

$$H'_{\mathrm{n}} = H'_{\mathrm{e}} - \mathrm{i}\kappa a^{\dagger}a, \tag{14}$$

where

$$H'_{\mathrm{e}} = \sum_{j=1}^{n}\frac{\Omega^2}{\delta}|e_j\rangle\langle e_j| + \sum_{j=1}^{n}\frac{g^2}{\delta}a^{\dagger}a|g_j\rangle\langle g_j| + \sum_{j=1}^{n}\frac{\Omega g}{\delta}\left(aS_j^{+} + a^{\dagger}S_j^{-}\right). \tag{15}$$

Set $\Omega = g$ and consider the imperfection of the photodetector. After an interaction time τ the state of the whole system evolves into

$$(1-\eta)\left(1-\frac{1}{n}\right)|0 f_1 f_2 \ldots f_n\rangle\langle 0 f_1 f_2 \ldots f_n| + \frac{1}{n^2}a(|0 g_1 f_2 \ldots f_n\rangle\langle 0 g_1 f_2 \ldots f_n| + |0 f_1 g_2 f_3 \ldots f_{n-1} f_n\rangle\langle 0 f_1 g_2 f_3 \ldots f_{n-1} f_n| + \ldots + |0 f_1 f_2 \ldots f_{n-1} g_n\rangle\langle 0 f_1 f_2 \ldots f_{n-1} g_n|) + |\phi''\rangle\langle\phi''|, \tag{16}$$

where

$$|\phi''\rangle = \frac{1}{n}\mathrm{e}^{-\kappa\tau/2}\left\{\left[\left(\frac{\mathrm{e}^{\beta\tau/2}+\mathrm{e}^{-\beta\tau/2}}{2} - \frac{\Gamma}{\beta}\frac{\mathrm{e}^{-\beta\tau/2}-\mathrm{e}^{\beta\tau/2}}{2}\right)|0e_1\rangle - \frac{2g}{\beta}\frac{\mathrm{e}^{-\beta\tau/2}-\mathrm{e}^{\beta\tau/2}}{2\mathrm{i}}|1g_1\rangle\right]|f_2 \ldots f_n\rangle - \left[\left(\frac{\mathrm{e}^{\beta\tau/2}+\mathrm{e}^{-\beta\tau/2}}{2} - \frac{\Gamma}{\beta}\frac{\mathrm{e}^{-\beta\tau/2}-\mathrm{e}^{\beta\tau/2}}{2}\right)|0e_2\rangle - \frac{2g}{\beta}\frac{\mathrm{e}^{-\beta\tau/2}-\mathrm{e}^{\beta\tau/2}}{2\mathrm{i}}|1g_2\rangle\right]|f_1 f_3 \ldots f_n\rangle - \ldots - \left[\left(\frac{\mathrm{e}^{\beta\tau/2}+\mathrm{e}^{-\beta\tau/2}}{2} - \frac{\Gamma}{\beta}\frac{\mathrm{e}^{-\beta\tau/2}-\mathrm{e}^{\beta\tau/2}}{2}\right)|0e_n\rangle - \frac{2g}{\beta}\frac{\mathrm{e}^{-\beta\tau/2}-\mathrm{e}^{\beta\tau/2}}{2\mathrm{i}}|1g_n\rangle\right]|f_1 f_2 \ldots f_{n-1}\rangle\right\}, \tag{17}$$

$$\beta = \sqrt{\kappa^2 - 4g^4/\delta^2}, \tag{18}$$

and

$$a = (1-\eta)\left\{1 - \frac{1}{4}\mathrm{e}^{-\kappa\tau}\left[\mathrm{e}^{\beta\tau/2}\left(1+\frac{\Gamma}{\beta}\right) + \mathrm{e}^{-\beta\tau/2}\left(1-\frac{\Gamma}{\beta}\right)\right]^2 + \mathrm{e}^{-\kappa\tau}\frac{g_1^2}{\beta^2}\left(\mathrm{e}^{-\beta\tau/2} - \mathrm{e}^{\beta\tau/2}\right)^2\right\}. \tag{19}$$

The registering of a click at the photodetector projects the atoms to the W state

$$|\phi_a\rangle = \frac{1}{\sqrt{n}}(|g_1 f_2 f_3 \ldots, f_n\rangle - |f_1 g_2 f_3 \ldots f_n\rangle - \ldots - |f_1 f_2 \ldots f_{n-1} g_n\rangle). \tag{20}$$

Here we have used the normalized state. During this stage, if we wish to wait for a time τ_{d} to detect the photon the probability of success is

$$P = \frac{\eta}{n}\left\{1 - \frac{1}{4}\mathrm{e}^{-\kappa\tau_{\mathrm{d}}}\left[\mathrm{e}^{\beta\tau_{\mathrm{d}}/2}\left(1+\frac{\Gamma}{\beta}\right) + \mathrm{e}^{-\beta\tau_{\mathrm{d}}/2}\left(1-\frac{\Gamma}{\beta}\right)\right]^2 + \mathrm{e}^{-\kappa\tau_{\mathrm{d}}}\frac{g_1^2}{\beta^2}\left(\mathrm{e}^{-\beta\tau_{\mathrm{d}}/2} - \mathrm{e}^{\beta\tau_{\mathrm{d}}/2}\right)^2\right\}. \tag{21}$$

If the time $\tau_{\rm d}$ is long enough so that $\mathrm{e}^{-(\kappa-\beta)\tau_{\rm d}} \ll 1$ and the probability of reaching such a state is η/n. As the success of the scheme is conditional on the detection of a photon, the imperfection of the photodetector reduces only the successful probability, but does not affect the fidelity of the entangled state.

4. Physical realization

We now briefly address the experimental feasibility of the proposed scheme. The required atomic level configuration can be achieved in Cs atoms. The hyperfine levels $|F=4,m=1\rangle$, $|F=4,m=0\rangle$, and $|F=4,m=-1\rangle$ of $6\mathrm{S}_{1/2}$ can act as the states $|e\rangle$, $|g\rangle$, and $|f\rangle$, respectively, while the hyperfine level $|F'=5,m'=1\rangle$ of $6\mathrm{P}_{3/2}$ can function as $|r\rangle$. The cavity mode is right-circularly polarized, and the classical fields driving the transition $|e\rangle \rightarrow |r\rangle$ are π-polarized. In order to perform the transformation $|g\rangle \rightarrow |f\rangle$ we excite the transitions $|g\rangle \rightarrow |h\rangle$ and $|f\rangle \rightarrow |h\rangle$ by using a pair of classical fields, where $|h\rangle$ is an auxiliary excited state. The hyperfine level $|F'=5,m'=-1\rangle$ of $6\mathrm{P}_{3/2}$ can act as $|h\rangle$. The two classical fields are detuned from the respective transitions by the same amount. In the case where the detuning is much larger than the respective coupling strength, the upper level $|h\rangle$ can be adiabatically eliminated and the two classical fields just induce the Raman transition between the states $|g\rangle$ and $|f\rangle$. We can obtain the required transformations by adjusting the phases of the classical fields and interaction time appropriately. As long as the classical pulse is strong enough the atom-cavity coupling is negligible during the application of the pulse.

In a recent cavity QED experiment on Cs atoms trapped in an optical cavity, the corresponding atom-cavity coupling strength is $g = 2\pi \times 34\,\mathrm{MHz}$.[17] The decay rates for the excited atomic states and the cavity mode are $\gamma = 2\pi \times 2.6\,\mathrm{MHz}$ and $\kappa = 2\pi \times 4.1\,\mathrm{MHz}$, respectively. Set $n=3$, $\Omega_1 = 2\sqrt{3}g$, $\Omega_2 = \sqrt{3}g$, $\Delta = 30\Omega_1$, and $\Delta = 30g$, then the Raman coupling strengths λ_1 and λ_2 will be $\lambda_1 = \Omega_1 g/\Delta \approx 2\pi \times 1.13\,\mathrm{MHz}$ and $\lambda_2 = \sqrt{2}\Omega_2 g/\Delta \approx 2\pi \times 0.8\,\mathrm{MHz}$. The parameters α and β are about $2\pi \times 3.02\,\mathrm{MHz}$ and $2\pi \times 3.42\,\mathrm{MHz}$. The atoms are virtually excited. The effective decoherence rate due to the atomic spontaneous emission is $\gamma' = \gamma\Omega_1^2/\Delta^2 = 2\pi \times 650\,\mathrm{Hz}$. The interaction times t and τ are on the order of $t \approx 5/(\kappa-\alpha) \approx 4.63\,\mu\mathrm{s}/2\pi$ and $\tau \approx 5/(\kappa-\beta) \approx 7.35\,\mu\mathrm{s}/2\pi$. The infidelity induced by the decoherence is on the order of $\gamma'(t_1+\tau) \approx 7.8 \times 10^{-3}$. The present scheme is valid for the Lamb–Dicke regime. In a recent experiment,[18] the localization of the Lamb–Dicke limit of the axial motion was demonstrated for a single atom trapped in an optical cavity.

5. Conclusions

We have proposed a scheme for generating multi-atom W states in an optical cavity. Our scheme can work in the regime where the cavity decay rate is much larger than the effective atom-cavity coupling strength, which is important in view of experiment. The interaction time does not need accurately adjusting. The generation of entangled atoms is conditional upon the detection of a photon and thus the fidelity of the entanglement is insensitive to the detection inefficiency.

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