Vol 18 No 2, February 2009
1674-1056/2009/18(02)/0522-09

Chinese Physics B

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Evidence of self-affine multiplicity fluctuation of particle production in ^{84}Kr–emulsion interactions at 1.7 A GeV*

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(Received 28 March 2008; revised manuscript received 12 June 2008)

Self-affine multiplicity fluctuation is investigated by using the two-dimensional factorial moment methodology and the concept of the Hurst exponent (H). Investigation on the experimental data of compound particles and target fragments emitted in ^{84}Kr–AgBr interactions at 1.7 A GeV reveals that the best power law behaviours are exhibited at H=0.7 and 0.6 respectively, and the data for shower particles produced in ^{84}Kr–emulsion interactions at 1.7 A GeV indicate that the best power law behaviour occurs at H=0.6, all of which show the self-affine multiplicity fluctuation patterns. The multifractality and the non-thermal phase transition occurring during producing the compound particles, the target fragments, and the shower particles in the ^{84}Kr–AgBr interaction and the ^{84}Kr–emulsion interaction are also discussed. The multifractality is observed during producing compound particles, target fragments, and shower particles. In the target fragment production, an evidence of non-thermal phase transition is observed, but in the shower particle production and the compound particle production, no evidence of non-thermal phase transition is observed.

Keywords: relativistic heavy-ion collision, non-statistical fluctuation, nuclear emulsion
PACC: 2570N, 2460, 2940R

1. Introduction

The study of non-statistical fluctuations in relativistic nuclear collisions has recently attracted a great deal of attention due to the possibility of disentangling important information about the mechanism of multiparticle production in such collisions. To understand the origin of non-statistical fluctuations, the method of scaled factorial moments (SFMs), proposed by Bialas and Peschanski,[1] is used for extracting the dynamical contribution to the fluctuations in multiplicity distributions in high energy collisions. A power-law growth of SFM with a decreasing phase space interval would suggest the existence of intermittent fluctuations in particle spectra. It has been conjectured that intermittency that is a general mechanism for producing large local fluctuations, may be a manifestation of phase transition.[2,3] But no unambiguous experimental evidence, which can show a phase transition and similar mechanism occurring in the particle production process, is available.

Most investigations of non-statistical fluctuations in high-energy nuclear collisions which have been carried out concentrate on the produced pions with a common belief that these particles are most informative, for they are produced immediately after collisions. Very few attempts have concentrated on knocked-out target protons (or recoiled protons) at medium energies (30–400 MeV), termed grey tracks in emulsion terminology, which are supposed to carry some information about the interaction dynamics because the time scale of the emission of these particles is the same as that of producing pions, and is of the order of 10^{-22} s. It is universally received that these recoiled protons are in the low energy part of the intranuclear cascade formed in high energy interactions. It is natural to combine grey track particles and shower track particles (mostly produced by pions) in a collision as a new kind of particle called compound particles to probe the reaction dynamics in high-energy nuclear interactions. So far, only a few investigations of non-statistical fluctuations in relativistic heavy-ion collisions have been made with this parameter.[4−10]

Up to now most of the intermittent studies have been carried out with the emphasis on the pioniza-

*Project supported by the National Natural Science Foundation of China (Grant Nos 10475054 and 10675077), the Research Foundation of Ministry of Education of China (Grant No 205026), the Natural Foundation of Shanxi Province, China (Grant No 2008011005) and Shanxi Provincial Foundation for Returned Scholars, China (Grant No 20031046).
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tion, while study of target fragmentation has been little. But the study of the target fragments may also provide important information for a good understanding of force and dynamics involved during and immediately after nuclear impact. One of the well-known models explaining the origin of target fragments is the cascade-evaporation model.[11] In the cascade-evaporation model, the particles corresponding to shower and grey tracks are emitted from the nucleus soon after the impact, leaving the hot residual nucleus in an excited state. Slow or black (in emulsion terminology) particles ($\beta < 0.3$) are evaporated from the hot excited target nucleus, and the emissions of these particles are believed to be isotropic in the rest system of the target. However, the isotropy may be a little disturbed due to the motion of the target. The cascade-evaporation model is based on the assumption that statistical equilibrium is established in the decaying system and that lifetime is much longer than the time taken to redistribute its excitation energy among different nucleons in the nucleus. However, different experimental data indicate the existence of a non-equilibrium process involved in the emission of black particles.[12−17]

In the early studies, the intermittent types of fluctuation were investigated in a one-dimensional space, though the real process takes place in a three-dimensional space. To remove the error due to dimensional reduction the analysis should be performed in a higher dimension. Even in a two-dimensional intermittency analysis, the usual procedure is to divide the phase space equally into subspaces for each dimension, and the studies in the two dimensions all show that the particle production process is self-similar. However, the experimental observations regarding momentum values in different directions show that the phase space in high-energy collision multiparticle production is anisotropic, which was indicated early by Van Hove.[18] According to Mandelbrot,[19] when a given pattern is differently scaled in different directions, it is called self-affine fractal.

This paper reports on an investigation about the nature of dynamical fluctuations of target fragments and compound particles emitted in ^{84}Kr–AgBr interactions, and shower particles emitted in ^{84}Kr–emulsion collisions at 1.7 A GeV in the framework of a two-dimensional SFM methodology, with the anisotropy of phase spaces taken into consideration. The dependence of the anomalous fractal dimension d_q on the order of the moment q is investigated. In order to examine the occurrence of non-thermal phase transition in the interactions, the variation of λ_q with the order of the moment q is also investigated.

2. Experimental details

Stacks of Ilford G-5 nuclear emulsion plates are horizontally exposed to a ^{84}Kr beam at 1.7 A GeV at Bevalac Berkeley. XSJ-1 and XSJ-2 microscopes with a 100× oil immersion objective and 16× ocular lenses are used to scan the plates. The tracks are picked up at a distance of 5 mm from the edge of the plates and carefully followed until they either interact with emulsion nuclei or escape from the plates. Interactions which take place within 30 μm from the top or bottom surface of the emulsion plates are not considered for final analysis. The primary tracks are all followed back to ensure that the events chosen do not include interactions from the secondary tracks of other interactions. When they are so observed the corresponding events are removed from the sample. Details of scanning and classification of events can be found in our previous paper.[20−23]

According to the emulsion terminology,[24] the particles emitted from interactions are classified as follows.

(a) Black particles. They are target fragments with ionization greater than or equal to $9I_0$, where I_0 is the minimum ionization of a single charged particle. Their ranges are less than 3 mm, speeds less than 0.3 c, where c is the light speed in vacuum, and energies less than 26 MeV.

(b) Grey particles. Most of them are recoiled protons in a kinetic energy range $26 \leq E \leq 375$ MeV and a few of them are kaons of kinetic energies $20 \leq E \leq 198$ MeV and pions with kinetic energies $12 \leq E \leq 56$ MeV. They have the ionization $1.4I_0 \leq I < 9I_0$. Their ranges in emulsion are greater than 3 mm and speeds within $0.3\,\mathrm{c} \leq v \leq 0.7\,\mathrm{c}$.

The grey and black particles together are called heavy ionizing particles.

(c) Shower particles. They are produced as single-charged relativistic particles having speeds greater than or equal to 0.7 c. Most of them are pions contaminated by a small proportion of fast protons and K mesons.

(d) The projectile fragments. They have varieties of tracks each with a constant ionization, long range, and small emission angle.

To ensure that the targets in the emulsion are silver or bromine nuclei, we have chosen only the events with at least eight heavy ionizing tracks ($N_h \geq 8$) for target fragments and compound particles analysis. To reduce the influence of statistical fluctuations, we have chosen only the events with at least ten shower particles ($N_s \geq 10$) for shower particles analysis.

3. Methodology

The intermittent behaviour of produced particles is analysed by using the method of SFMs. The non-uniformity of particle spectra influences the scaling behaviour of SFMs. Bialas and Gazdzicki[25] introduced 'cumulative' variable which drastically reduced the distortion of intermittency due to non-uniformity of single-particle density distribution. Following them, the cumulative variable $X(x)$ is related to the single-particle density distribution $\rho(x)$ through

$$X(x) = \int_{x_1}^{x} \rho(x')\mathrm{d}x' \Big/ \int_{x_1}^{x_2} \rho(x')\mathrm{d}x', \tag{1}$$

where x_1 and x_2 are two extreme points of the distribution $\rho(x)$. The variable $X(x)$ varies between 0.0 and 1.0, with $\rho(X(x))$ kept almost constant. The values of x_1 and x_2 are –1 and 1 in cosθ-space, 0 and 2π in ϕ-space, and $\eta_{\min}$ and $\eta_{\max}$ in η-space, respectively.

The self-affine multiplicity fluctuation analysis is performed in a two-dimensional phase space. Denote the two phase space variables as x_1 and x_2, then horizontal SFM of order q will be defined as[1]

$$F_q(\delta x_1 \delta x_2) = \frac{1}{M'} \sum_{m=1}^{M'} \frac{n_m(n_m-1)...(n_m-q+1)}{\langle n_m \rangle^q}, \tag{2}$$

where $\delta x_1 \delta x_2$ is the size of a two-dimensional cell, n_m is the multiplicity in the mth cell, $\langle n_m \rangle$ is the average multiplicity of all events in the mth cell, and M' is the number of two-dimensional cells into which the considered phase space has been divided.

To fix δx_1, δx_2 and M', we consider a two-dimensional region $\Delta x_1 \Delta x_2$ and divide it into subcells with widths

$$\delta x_1 = \Delta x_1 / M_1, \tag{3}$$

and

$$\delta x_2 = \Delta x_2 / M_2 \tag{4}$$

in the x_1 and x_2 directions, where $M_1 \neq M_2$ and $M' = M_1 \times M_2$.

Here M_1 and M_2 are the scale factors that satisfy the expression

$$M_1 = M_2^H, \tag{5}$$

where the parameter H ($0 < H \leq 1$) is called the Hurst exponent.[26] It is the parameter characterizing the self-affinity property of dynamical fluctuations. It is clear from expression (5) that the scale factors M_1 and M_2 cannot be integers simultaneously, so that the size of the elementary phase space cell should be taken as a continuously varying value.

The following method[26] has been adopted for performing the analysis with non-integral values of the scale factor (M). For simplicity, consider a one-dimensional space (y) and let

$$M = N + a, \tag{6}$$

where N is an integer and $0 \leq a < 1$. When we use the elementary bin of width $\delta y = \Delta y / M$ as a scale to measure the region Δy, we get N of elementary bins and a smaller bin of width $a\Delta y/M$ left. Placing the smaller bin at the last (or first) position of the region and taking the average only over the first (or last) N bins yield

$$F_{qi}(\delta y) = \frac{1}{M} \sum_{m=1}^{N} \frac{n_{mi}(n_{mi}-1)\ldots(n_{mi}-q+1)}{\langle n_m \rangle^q}, \tag{7}$$

where n_{mi} is the multiplicity in m-th cell of i-th event, M determined from expression (6) can be arbitrary positive real number and vary continuously. The averaged horizontal SFM then becomes

$$\langle F_q(\delta y) \rangle = \frac{1}{N_{ev}} \sum_{i=1}^{N_{ev}} F_{qi}(\delta y). \tag{8}$$

The horizontally averaged moment is sensitive to the shape of the single-particle density distribution and depends on the correlation between the cells. There is another vertically averaged factorial moment,[27] which is normalized locally and therefore carries information only about fluctuation within each cell. The vertical SFM is defined as

$$\langle F_q(\delta y) \rangle = \frac{1}{M} \sum_{m=1}^{N} \frac{1}{N_{ev}} \sum_{i=1}^{N_{ev}} \times \frac{n_{mi}(n_{mi}-1)\ldots(n_{mi}-q+1)}{\langle n_m \rangle^q}, \tag{9}$$

where

$$\langle n_m \rangle = \frac{1}{N_{ev}} \sum_{i=1}^{N_{ev}} n_{mi}, \tag{10}$$

the average content of bin m over the ensemble of events.

Self-affine multiplicity fluctuation would manifest itself as a power-law scaling of $\langle F_q \rangle$ with the cell size of the form

$$\langle F_q(\delta y) \rangle \propto (\delta y)^{-a_q} \quad \text{as} \quad \delta y \to 0 \tag{11}$$

or a linear relation

$$\ln\langle F_q \rangle = -a_q \ln \delta y + b_q. \tag{12}$$

The invariant quantity of the scaling $a_q > 0$ is called the intermittency exponent, and it is a measure of the fluctuation strength.

To probe the anisotropic structure of phase space we have calculated SFMs for the q-th order (q=2, 3, 4, 5), with the value of Hurst exponent being variable. The partition numbers along $\cos\theta$ (for target fragments and compound particles) or pseudorapidity $\eta = -\ln\tan(\theta/2)$ (for shower particles) and ϕ directions are chosen as M_ϕ=3, 4, ..., 30, and $M_{\cos\theta}$ (or M_η) given by

$$M_{\cos\theta}(\text{ or } M_\eta) = M_\varphi^H. \tag{13}$$

We have not considered the first two data points corresponding to M_ϕ=1, 2 to reduce the effect of momentum conservation[28] which tends to spread the particles in opposite directions, thereby reducing the values of the factorial moments. This effect becomes weaker as M increases.

4. Experimental results

We have analysed the dependences of the natural logarithm of average value of vertical and horizontal SFMs ($\ln\langle F_q \rangle$) along the Y axis on the natural logarithm of ($\delta X_{\cos\theta}\delta X_\phi$) along the X axis for compound particles emitted from ^{84}Kr–AgBr interaction at 1.7 A GeV, with Hurst exponent being at different values: 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1.0. For each case the linear behaviours are observed in two or three regions, separately. In order to find the partitioning condition under which the scaling behaviour is displayed best, we have performed linear fitting in the first region, and have estimated the value of χ^2 per degree of freedom (DOF) for each linear fitting. Interestingly, the best linear behaviour is shown at H=0.7 but not at H=1 in both vertical and horizontal analyses for each order of moment for the data set. The plots of $\ln\langle F_2 \rangle$ obtained from vertical analysis against $\ln(\delta X_{\cos\theta}\delta X_\phi)$ at H=0.7 and 1.0 are shown in Fig.1. Table 1 presents the values of χ^2 per DOF and the intermittency exponents obtained from vertical analyses for ^{84}Kr–AgBr interactions at different values of H and orders of moment. Figure 2 shows the dependences of $\ln\langle F_q \rangle$ obtained from horizontal SFM analysis on $\ln(\delta X_{\cos\theta}\delta X_\phi)$ for different orders of moment at H=0.7 and 1.0. The values of χ^2 per DOF and the intermittency exponents from horizontal analyses for ^{84}Kr–AgBr interactions at different values of H and orders of moment are listed in Table 2. From Tables 1 and 2 it follows that the values of χ^2 per DOF from both vertical and horizontal analyses are smaller at H=0.7 for different orders of moment. So the dynamical fluctuation pattern in ^{84}Kr–AgBr interactions is not self-similar but self-affine. From Tables 1 and 2 it also follows that the value of vertical intermittency exponent is smaller than that of horizontal exponent

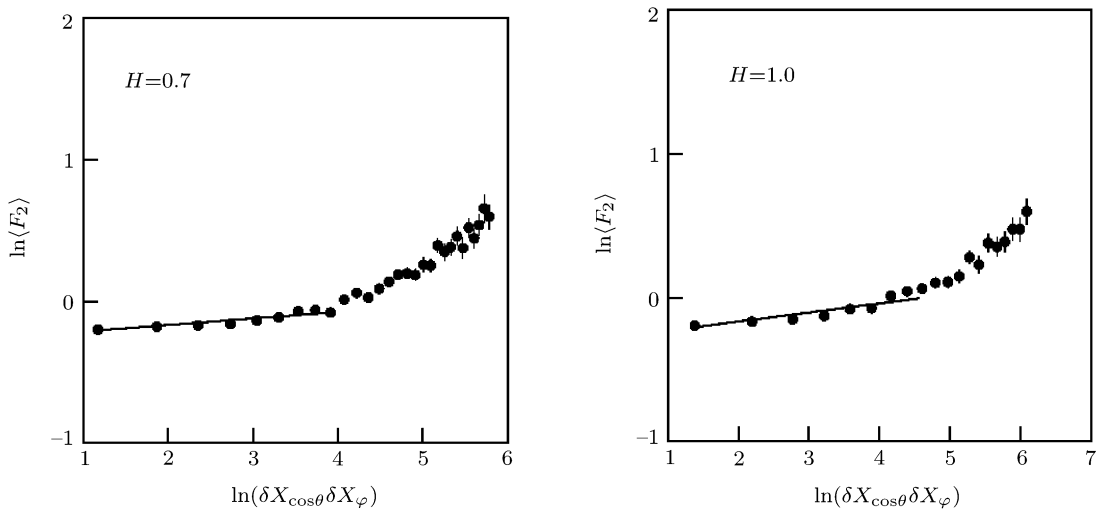


Fig.1. Plots of $\ln\langle F_2 \rangle$ obtained from vertical analysis against $\ln(\delta X_{\cos\theta}\delta X_\phi)$ at H=0.7 and 1.0 in the case of compound particles emitted from ^{84}Kr–AgBr interactions at 1.7 A GeV.

for the same H and order of moment q, which is natural because vertical SFM reveals the fluctuation within each cell but horizontal SFM shows the fluctuation within each event.

Table 1. The values of χ^2 per DOF and vertical intermittency exponent for some specific values of Hurst exponent H in ^{84}Kr–AgBr interactions at 1.7 A GeV.

	q=2		q=3		q=4		q=5	
Value Of H	χ^2 per DOF	Intermittency exponents (a_q)	χ^2 per DOF	Intermittency exponents (a_q)	χ^2 per DOF	Intermittency exponents (a_q)	χ^2 per DOF	Intermittency exponents (a_q)
0.3	1.890	0.079±0.007	2.680	0.211±0.018	3.129	0.388±0.034	1.562	0.524±0.064
0.4	3.953	0.069±0.006	6.936	0.156±0.013	5.731	0.338±0.025	1.893	0.432±0.060
0.5	1.754	0.064±0.005	3.029	0.171±0.012	4.313	0.312±0.028	0.922	0.326±0.062
0.6	2.639	0.057±0.005	4.606	0.136±0.012	3.270	0.219±0.025	0.100	0.254±0.055
0.7	1.390	0.047±0.005	2.072	0.114±0.010	1.570	0.189±0.024	0.494	0.262±0.052
0.8	2.888	0.052±0.004	3.164	0.142±0.013	2.818	0.252±0.033	1.407	0.293±0.069
0.9	3.033	0.059±0.004	2.818	0.144±0.014	1.544	0.242±0.033	1.692	0.378±0.062
1.0	4.036	0.063±0.005	5.302	0.167±0.014	3.832	0.283±0.033	2.268	0.403±0.061

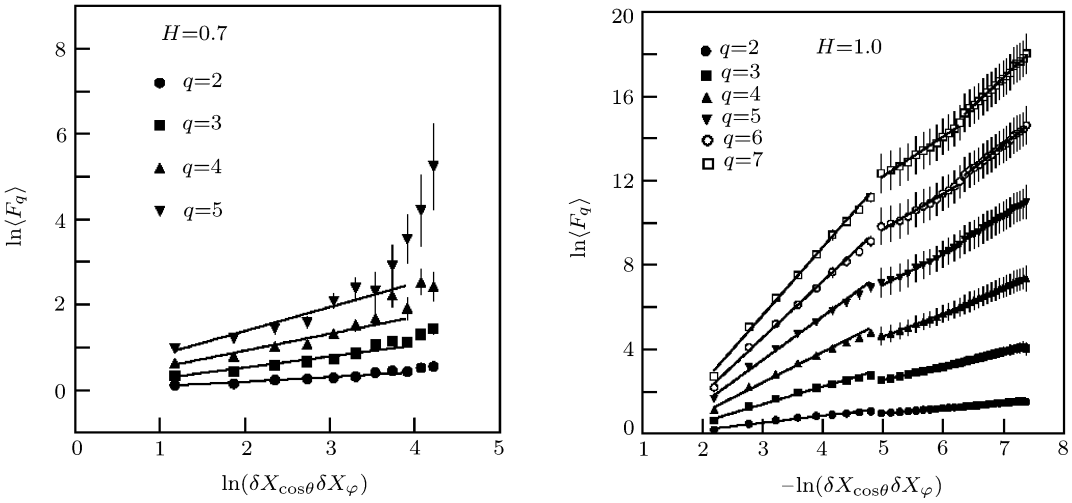


Fig.2. Plots of $\ln\langle F_q\rangle$ obtained from horizontal analysis against $\ln(\delta X_{\cos\theta}\delta X_{\phi})$ at H=0.7 and 1.0 in the case of compound particles emitted from ^{84}Kr–AgBr interactions at 1.7 A GeV.

Table 2. The values of χ^2 per DOF and horizontal intermittency exponent for some specific values of Hurst exponent H in ^{84}Kr–AgBr interactions at 1.7 A GeV.

	q=2		q=3		q=4		q=5	
Value Of H	χ^2 per DOF	Intermittency exponents (a_q)	χ^2 per DOF	Intermittency exponents (a_q)	χ^2 per DOF	Intermittency exponents (a_q)	χ^2 per DOF	Intermittency exponents (a_q)
0.3	2.958	0.132±0.007	3.181	0.363±0.017	3.694	0.619±0.027	4.787	0.925±0.042
0.4	3.180	0.125±0.008	3.322	0.318±0.017	3.270	0.526±0.027	4.018	0.783±0.041
0.5	3.197	0.118±0.008	2.767	0.282±0.017	3.161	0.458±0.029	4.393	0.663±0.045
0.6	1.887	0.115±0.008	2.393	0.275±0.018	3.507	0.432±0.032	3.844	0.620±0.051
0.7	1.599	0.109±0.008	1.914	0.262±0.019	1.793	0.396±0.034	1.655	0.560±0.055
0.8	2.635	0.115±0.008	1.311	0.268±0.020	2.163	0.430±0.037	3.272	0.596±0.061
0.9	1.307	0.120±0.009	0.937	0.263±0.022	0.951	0.384±0.044	1.796	0.562±0.074
1.0	6.981	0.336±0.007	5.421	0.845±0.017	3.796	1.446±0.030	2.577	2.078±0.043

For shower particles produced in ^{84}Kr–emulsion interactions and target fragments emitted from ^{84}Kr–AgBr collisions, we have also studied the variation of the natural logarithm of average value of horizontal SFMs ($\ln\langle F_q\rangle$) with the natural logarithm of $\ln(\delta X_{\cos\theta}\delta X_\phi)$ (for target fragments) or $\ln(\delta X_\eta\delta X_\phi)$ (for shower particles) for different values of Hurst exponent. For each case the linear behaviour is observed in two or three regions. Following the same method as that in the discussion of compound particles production, we have performed linear fitting in the first region. For both shower particle production and target fragment production the best linear behaviour is achieved at H=0.6 but not at H=1.0 for each order of moment. Figure 3 presents the variations of $\ln\langle F_q\rangle$ with $\ln(\delta X_{cos\theta}\delta X_\phi)$ for target fragments at H=0.6 and 1.0 for different orders of moment. Figure 4 shows the dependences of $\ln\langle F_q\rangle$ on $\ln(\delta X_\eta\delta X_\phi)$ for shower particles at H=0.6 and 1.0 for different orders of moment. Table 3 represents the values of χ^2 per DOF and the intermittency exponent for target fragments produced in ^{84}Kr–AgBr interactions and shower particles emitted from ^{84}Kr–emulsion collisions at different values of H and orders of moment q. From the table it is clearly seen that the values of χ^2 per DOF are smaller at H=0.6 for different orders of moment for both target fragment production and shower particle production. So the dynamical fluctuation patterns for target fragment production and shower particle production both are not self-similar but self-affine.

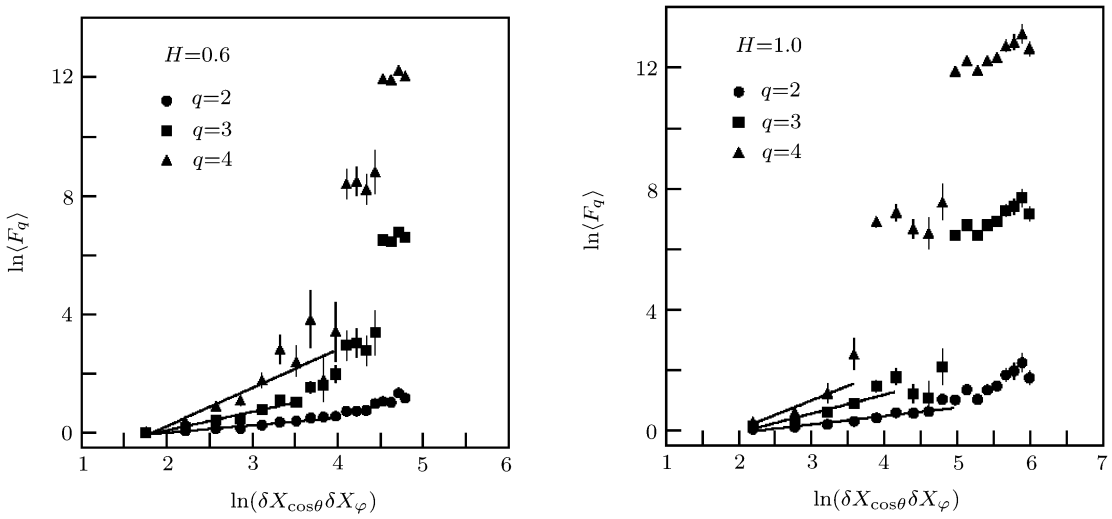


Fig.3. Plots of $\ln\langle F_q\rangle$ obtained from horizontal analyses against $\ln(\delta X_{\cos\theta}\delta X_\phi)$ at H=0.6 and 1.0 in the case of target fragments emitted from ^{84}Kr–AgBr interactions at 1.7 A GeV.

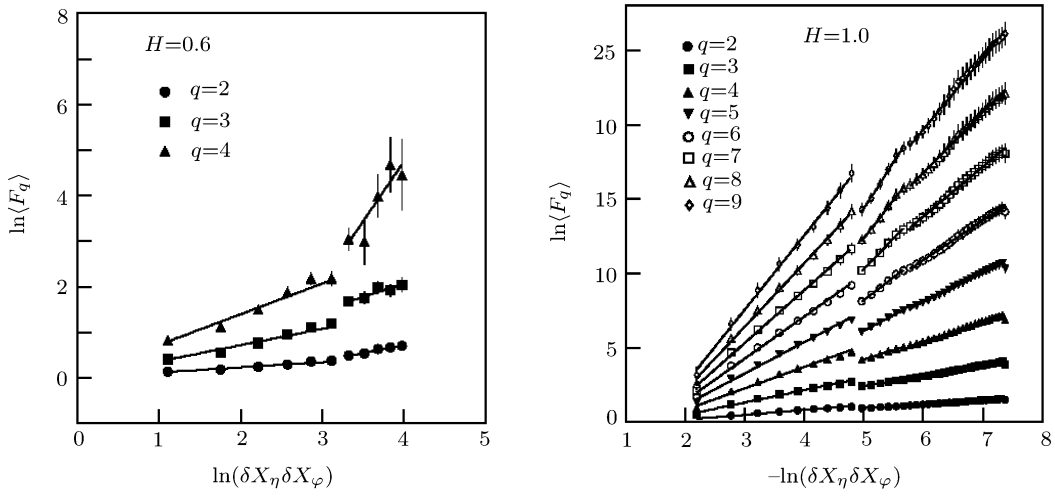


Fig.4. Plots of $\ln\langle F_q\rangle$ obtained from horizontal analyses against $\ln(\delta X_\eta\delta X_\phi)$ at H=0.6 and 1.0 in the case of shower particles emitted from ^{84}Kr–emulsion interactions at 1.7 A GeV.

Table 3. The values of χ^2 per DOF and horizontal intermittency exponent at some specific values of H for target fragment production and shower particle production in ^{84}Kr–AgBr and ^{84}Kr–emulsion interactions at 1.7 A GeV.

		q=2		q=3		q=4	
Type of particles	Value Of H	χ^2 per DOF	Intermittency exponents (a_q)	χ^2 per DOF	Intermittency exponents (a_q)	χ^2 per DOF	Intermittency exponents (a_q)
Target fragment	0.3	5.878	0.286±0.011	1.723	0.587±0.053	0.991	0.941±0.142
	0.4	5.236	0.271±0.012	4.743	0.663±0.053	0.365	0.896±0.155
	0.5	4.016	0.255±0.010	3.824	0.704±0.047	2.407	1.290±0.392
	0.6	3.820	0.250±0.012	3.308	0.626±0.049	0.783	1.137±0.164
	0.7	5.455	0.222±0.009	2.957	0.519±0.041	1.012	1.128±0.135
	0.8	5.471	0.239±0.011	3.799	0.617±0.054	0.739	1.117±0.170
	0.9	4.168	0.263±0.013	2.300	0.592±0.060	3.216	1.046±0.190
	1.0	5.499	0.277±0.014	3.194	0.626±0.063	2.768	0.964±0.217
Shower particle	0.3	3.401	0.138±0.006	4.589	0.442±0.018	4.046	0.870±0.036
	0.4	2.487	0.123±0.006	3.321	0.394±0.019	3.445	0.746±0.038
	0.5	2.083	0.111±0.007	2.340	0.353±0.020	2.872	0.685±0.041
	0.6	1.911	0.112±0.007	2.290	0.358±0.022	1.787	0.673±0.044
	0.7	3.815	0.112±0.008	4.547	0.344±0.023	4.573	0.639±0.048
	0.8	4.391	0.110±0.008	3.732	0.331±0.025	3.125	0.525±0.052
	0.9	3.562	0.117±0.008	2.141	0.341±0.027	0.800	0.569±0.061
	1.0	2.161	0.330±0.004	1.544	0.835±0.011	0.886	1.444±0.022

The power-law behaviour of the SFMs implies the existence of some kind of fractal pattern[29] in the dynamics of the particle production in their final state. Therefore, it is natural to study the fractal nature of compound particles and target fragments emitted from ^{84}Kr–AgBr interactions and shower particles produced in ^{84}Kr–emulsion collisions under the self-affine scaling scenario. The anomalous fractal dimension d_q of fractals and multifractals is related to the intermittency exponent a_q as

$$d_q = \frac{a_q}{q-1}. \tag{14}$$

The variation of d_q with order q is shown in Fig.5(a) for compound particle production, obtained from both vertical and horizontal analyses, and Fig.5(b) for target fragment production and shower particle production, obtained from horizontal analysis. From the plots it is seen that d_q is linearly dependent on order q for compound particles and target fragments emitted from ^{84}Kr–AgBr interactions and shower particles produced in ^{84}Kr–emulsion collisions. This suggests that some cascade-type mechanism may be responsible for the emission of compound particles and target fragments in ^{84}Kr–AgBr interactions and shower particles in ^{84}Kr–emulsion collisions.

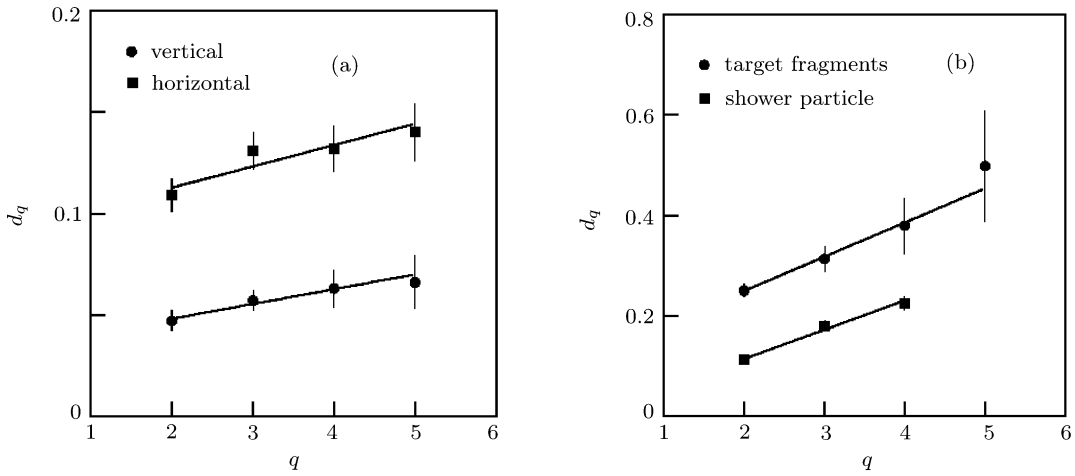


Fig.5. Plots of d_q against q for compound particles produced in ^{84}Kr–AgBr interactions at H=0.7 (a) and for target fragments and shower particles emitted from ^{84}Kr–AgBr interactions and ^{84}Kr–emulsion collisions at H=0.6 respectively (b).

It has been suggested[30] that intermittent behaviour may be a projection of a non-thermal phase transition believed to occur during the evolution of the collision, which in turn would be responsible for anomalous events. A non-thermal phase transition in heavy-ion collisions is a kind of transition in which the new phase need not be characterized by thermodynamical behaviour. If a phase transition of this nature occurs then the function

$$\lambda_q = (a_q + 1)/q \tag{15}$$

is predicted[30,31] to have a minimum at some value $q = q_c$. The region $q < q_c$, namely the normal phase, is dominated by many relatively small fluctuations, whereas the region $q > q_c$, namely the spin glass phase, contains rarely occurring very large fluctuations. The condition for the coexistence of the two phases of the cascade is the presence of a minimum of intermittency parameter λ_q at a certain value q_c for the order q. We have discussed the features of coexistence of the two phases of the cascade in the emissions of compound particles and target fragments in ^{84}Kr–AgBr interactions and shower particles in ^{84}Kr–emulsion collisions.

Figure 6 presents the dependences of λ_q on order q for emissions of compound particles, target fragments, and shower particles. From Fig.6(a) it is seen that for compound particle production the values of λ_q obtained from both vertical and horizontal analyses decrease with the increase of order q, and non minimum value of λ_q is observed in the data set, which indicates non-coexistence of two different phases in the emission of compound particles in ^{84}Kr–AgBr interactions. From Fig.6(b) it is seen that for shower particle production the values of λ_q from horizontal analysis decrease with the increase of order q, which also indicates non-coexistence of two different phases in the emission of shower particles in ^{84}Kr–emulsion interactions, but for target fragment production a clear minimum value of λ_q occurs at q=4, which implies the existence of non-thermal phase transitions in the process of target fragments production in ^{84}Kr–AgBr interactions.

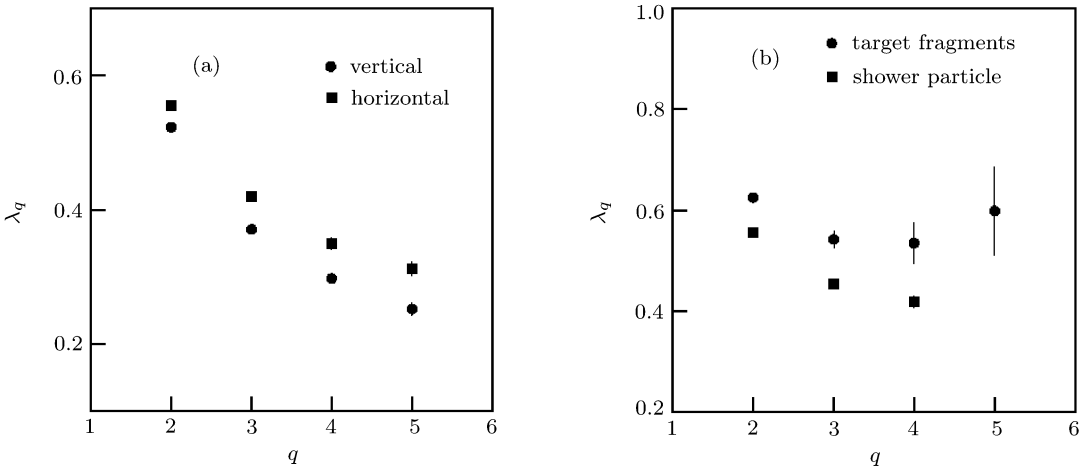


Fig.6. Plots of λ_q against q with compound particles at H=0.7 in ^{84}Kr–AgBr interactions (a), and target fragments and shower particles at H=0.6 in ^{84}Kr–AgBr and emulsion collisions (b), respectively.

5. Conclusions

From the present study it may be concluded that the effects of the intermittency are observed in the target fragment production and the compound particle production from ^{84}Kr–AgBr interactions and in the shower particle production from ^{84}Kr–emulsion collisions at 1.7 A GeV. The best power law behaviour is exhibited at H=0.7 for the target fragment production using both vertical and horizontal SFM analyses, but for the shower particle production and the target fragment production the best power law behaviour is exhibited at H=0.6 from horizontal SFM analysis. All the above analyses suggest that the dynamical fluctuation pattern is not self-similar but self-affine. The anomalous fractal dimension of the intermittency is found to increase with the increase of order of moment, which suggests the presence of multifractality in the emission of compound particles and target fragments from ^{84}Kr–AgBr interactions and in the emission of shower particles from ^{84}Kr–emulsion collisions. For

the shower particle production and the compound particle production, non-minimum value of λ_q is observed in the data set, which suggests that there is no coexistence of the normal phase and the spin glass phase, but for the target fragment production a clear minimum value of λ_q is observed in the data set, which suggests that there is a coexistence of the normal phase and the spin-glass phase.

Acknowledgment

We are thankful to Professor I. Otterlund of Lund University in Sweden for supplying the emulsion plates.

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