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Properties of asymmetrically evolved community networks*

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This paper studies a simple asymmetrically evolved community network with a combination of preferential attachment and random properties. An important issue about community networks is to discover the different utility increments of two nodes, where the utility is introduced to investigate the asymmetrical effect of connecting two nodes. On the other hand, the connection of two nodes in community networks can be classified as two nodes belonging to the same or to different communities. The simulation results show that the model can reproduce a power-law utility distribution $P(u) \sim u^{-\sigma}$, $\sigma = 2 + \frac{1}{p}$, which can be obtained by using mean-field approximation methods. Furthermore, the model exhibits exponential behaviour with respect to small values of a parameter denoting the random effect in our model at the low-utility region and a power-law feature with respect to big values of this parameter at the high-utility region, which is in good agreement with theoretical analysis. This kind of community network can reproduce a unique utility distribution by theoretical and numerical analysis.

Keywords: community networks, asymmetrical, utility
PACC: 0590, 0200

1. Introduction

Complex networks are composed of a large set of various kinds of connected nodes which are ubiquitous in nature and society,[1] including the internet, the world wide web, communication networks, food webs, biological neural networks, electrical power grids, social and economic relations networks, traffic and transportation networks,[2−6] co-authorship and citation networks of scientists, cellular and metabolic networks, etc. In recent years, many properties of complex networks have been reported in the literature.[7,8] A good deal of motivations has been accounted for the ubiquity of scale-free property of degree distribution in many real systems, which can be firstly reproduced in the model proposed by Barabási and Albert.[7] That is, the degree distribution $P(k)$ gives a power-law behaviour, i.e., $P(k) \sim k^{-\gamma}$, where k is the degree of the node and γ is a power-law exponent. There are two main mechanisms that can be regarded as being responsible for the emergence of a scale-free pattern. One principal concept is that such distribution promotes the appearance of hubs, namely there exist some nodes with particularly high degrees. By concentrating connections, hubs play a critical role in defining the connectivity of community networks as well as other topological properties such as minimal paths. Another concept is growth, which can be informally understood as that many scale-free networks are evolved in a dynamical way.

Recent studies have shown that disparate systems can be described as community networks which assemble nodes and edges with nontrivial topological properties.[9−24] Communities appear in networks where nodes join together in tight inner-communities that have few connections between inter-communities.[9−22] Evolved community networks have been presented or studied by many researchers. In Refs.[9–10], based on the different dynamic models, Gao *et al* interpreted cascading failures based on the scale-free community networks. In Ref.[13], Nakazato K and Arita T built the growth mechanism which is a combination of preferential and non-preferential attachment in a higher level. In Refs.[12, 14], Li and Maini proposed a weighted community network model which exhibits community structure with pref-

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erential attachment property. However, many previous models often exhibit a symmetrical picture for the weight evolution, where the weight on the edge is symmetrically defined. In other words, it is considered that an edge's weight symmetrically evolved on its two endpoints (i.e., the weight symmetrically influences on the strengths increments of its two endpoints). This scenario, however, misses the fact that weights are asymmetrically evolved in complex networks with community structure. To fill this gap, we will present a model to study community network evolution under the asymmetrically evolved mechanism in this paper. In contrast with previous models where weights are symmetrically evolved, a parameter utility u is introduced to describe the asymmetrical property after building a connection between two nodes. In Ref.[25] Zheng and Gao have studied properties of asymmetrically evolved scale-free networks. To our knowledge, the asymmetrically evolved community networks have not been presented in the literature up to now. Inspired by Ref.[25], we utilize the attraction utility to interpret the properties of asymmetrically evolved community networks. There exist the asymmetrically evolved and preferential attachment phenomena in some real-world networks with community structure. For example, in scientific collaboration networks, a famous multidisciplinary researcher (with more inter-community edges) is more likely to collaborate with the researchers in other academic fields (communities) which are not familiar to him, as compared with the single-topic researchers. Conversely, a new researcher in one academic field (community), who wants to enter another academic field, is more likely to collaborate with these interdisciplinary researchers.[12,14] However, it needs to note that such connection may have different contributions or benefits during the cooperation process. Generally, in scientific collaboration networks, the interdisciplinary researcher has more contributions than a new researcher. Similarly, this phenomenon can also be discovered in traffic systems. When a new bus station is built, it is usually preferential to connect the traffic hub which can provide multi-modal exchange traffic station (e.g., subways, bus stations, etc.) rather than a single-modal exchange traffic station. It is only because that the multi-modal exchange traffic station can provide several alternative exchange choices for travellers during the traffic process. Especially, it needs to point out that different exchange choices generally have different attractions for different travellers. These phenomena are interpreted not only by the effect of community structure with preferential attachment property but also by the effect of asymmetrically preferential utility. Due to their different importance, their attractions of two endpoints may vary asymmetrically after connection. It can be mathematically expressed as $\Delta u_{is}^1 \neq \Delta u_{js}^1$, while connecting node i to node j in the same community s. Meanwhile connecting node i (in community s) to node j in different communities l ($l \neq s$), it also can be mathematically expressed as $\Delta u_{is}^2 \neq \Delta u_{jl}^2$. In this paper, we propose an asymmetrically evolved community network model which is based on the preferential attachment property and asymmetrically preferential utility.

The rest of the paper is organized as follows: an asymmetrically evolved community network model is presented in Section 2. Simulation results and theoretical analysis are given in Section 3. Finally, some conclusions are shown in Section 4.

2. The model generation

The asymmetrically evolved community networks with preferential attachment property and asymmetrically preferential utility is generated by the following algorithms:

(i) Initialization: Start from h communities denoted by H_1, H_2, $H_3, ..., H_h$ (each community with ϕ_0 initial nodes), and $\frac{h(h-1)}{2}$ inter-communities edges connecting each community to other $h-1$ communities. It is assured that there exists one edge between two communities. Moreover, we assume that the edges of inner-community are completely connected.

(ii) Growth: At each time step, a new node is added to a community and connects to ϕ ($1 < \phi < \phi_0$) nodes through ϕ inner-edges in the same community (the inner-edges are defined as edges that only connect nodes in same community), and with probability α connects to η ($1 < \eta < \phi$) nodes in other $h-1$ communities through η inter-edges (the inter-edges are defined as edges that connect nodes among different communities), simultaneously.

(iii) Preferential utility:

(a) Inner-community: When choosing two nodes in the same community (denoted as the s-th community), we assume that probability $\Pi\left(u_{is}^1\right)$ depends on inner-community utility u_{is}^1 (new node j connected to

node i in the same community s), in such a way that

$$\Pi\left(u_{is}^{1}\right)=\frac{u_{is}^{1}}{\sum u_{js}^{1}}. \tag{1}$$

(b) Inter-community: When choosing two nodes of two different communities, we assume that the probability $\Pi\left(u_{is}^{2}\right)$ depends on inter-community utility u_{is}^{2} (new node j in community l will connect to node i in different communities $s\,(s\neq l)$), in a similar way that

$$\Pi\left(u_{is}^{2}\right)=\frac{u_{is}^{2}}{\sum_{jl(l\neq s)}u_{jl}^{2}}. \tag{2}$$

(iv) Random:

(a) Inner-community: Connecting to node i in community s, the utility increments update as: $\Delta u_{js}^{1}=1$ for new node j in community s; and $\Delta u_{is}^{1}=1$ with probability p_1 for node i in community s or $\Delta u_{is}^{1}=0$ with probability $1-p_1$ for node i in community s, simultaneously.

(b) Inter-community: Connecting to node i in community s, the utility increments update as: $\Delta u_{jl}^{2}=1$ for new node j in community l $(l\neq s)$; and $\Delta u_{is}^{2}=1$ with probability p_2 for node i in community s or $\Delta u_{is}^{2}=0$ with probability $1-p_2$ for node i in community s, simultaneously.

In our model, the new node is considered to be positive and its utility is increased by 1, while connecting to an old node in community networks. While the chosen node is regarded to be passive, its utility is increased by 1 with probabilities p_1 and p_2. Although utility increments Δu_{is}^{1}, Δu_{is}^{2} are regarded to be taken only two different values (i.e. Δu_{is}^{1}, $\Delta u_{is}^{2}=0,1$), some new lights are shed to investigate asymmetrically evolved community networks. The initial utility of node is equal to its degree in our model. After

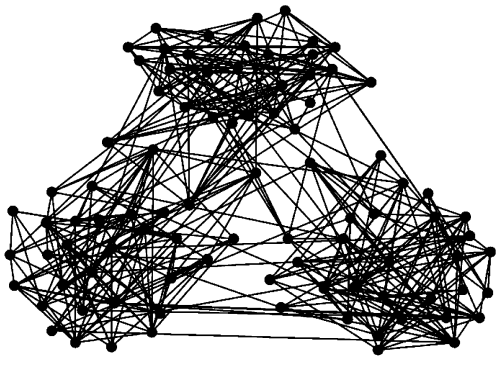


Fig.1. The simple asymmetrically evolved community networks with $\phi_0=6$, $\phi=4$, $\eta=1$, $\alpha=0.1$, $h=3$, $p_1=0.8$, $p_2=0.6$ and $hN=90$ nodes. (Here the nodes in each community are randomly placed).

t time steps, total community networks provide with $hN=h\phi_0+t$ nodes and $\frac{h\phi_0\left(\phi_0-1\right)}{2}+\frac{h\left(h-1\right)}{2}+\phi t+$ integer $(\alpha\eta t)$ edges. Here parameters α and η control the growth of inter-community edges. If α and η are small, the inner-community edges will be dense but inter-community edges will be sparse. Therefore, this model will generate the community networks.[12,14] For clarity, we perform the numerical simulation to generate a sample of asymmetrically evolved community network with 90 nodes as shown in Fig.1.

3. Theoretical analysis and simulation results

Using the mean-field approximation, we analyse the utility scaling property of asymmetrically evolved community networks. Here we can treat u_{is}^{1}, u_{is}^{2} and t as continuous variables. The utilities u_{is}^{1}, u_{is}^{2} of node i in community s can be computed analytically as follows:

$$\frac{\partial u_{is}^{1}}{\partial t}=\frac{\phi p_1}{h}\frac{u_{is}^{1}}{\sum_j u_{js}^{1}}, \tag{3}$$

taking into account that

$$\sum\nolimits_j u_{js}^{1}=\frac{\phi}{h}t+\frac{\phi}{h}p_1 t+\phi_0\left(\phi_0-1\right)\approx\frac{\phi}{h}\left(1+p_1\right)t. \tag{4}$$

For large t in the sense of mathematical expectation, we have

$$\frac{\partial u_{is}^{1}}{\partial t}\approx\frac{p_1 u_{is}^{1}}{\left(1+p_1\right)t}. \tag{5}$$

Under initial condition u_{is}^{1}, u_{is}^{2}, $u_{is}^{1}\left(t_\omega\right)=\phi$ (the node i in community s is added to the system at time t_ω), the solution of this equation is

$$u_{is}^{1}\left(t_\omega\right)=\phi\left(\frac{t}{t_\omega}\right)^{\frac{p_1}{1+p_1}}. \tag{6}$$

Similarly

$$\frac{\partial u_{is}^{2}}{\partial t}=\frac{h-1}{h}\alpha\eta p_2\frac{u_{is}^{2}}{\sum_{jl(l\neq s)}u_{jl}^{2}}, \tag{7}$$

taking into account that

$$\sum_{jl(l\neq s)}u_{jl}^{2}=\frac{h\left(h-1\right)}{2}\alpha\eta t+\frac{h\left(h-1\right)}{2}\alpha\eta p_2 t+\left(h-1\right)^{2}. \tag{8}$$

If $\alpha=0$, no inter-community edges are added in the evolution of community networks, and total number of inter-community edges is always $\frac{h\left(h-1\right)}{2}$. If $\alpha\neq0$, under initial condition $u_{is}^{2}\left(t_\omega\right)=\alpha\eta$ (the node i in community s is added to community networks at time

t_ω with inter-community connectivity) in the sense of mathematical expectation, the solution of this equation is

$$u_{is}^2(t) = \alpha\eta\left(\frac{t+\xi^1}{t_\omega+\xi^1}\right)^{\frac{p_2}{1+p_2}}, \tag{9}$$

where $\xi^1 = \dfrac{(h-1)\,h}{2\alpha\eta}$. If α is not so small, such that $2\dfrac{1}{h}\alpha\eta t >> h-1$ for large t, then

$$\frac{\partial u_{is}^2}{\partial t} \approx \frac{p_2 u_{is}^2}{(1+p_2)\,t}. \tag{10}$$

Under the initial condition $u_{is}^2(t_\omega) = \alpha\eta$ (the node i in community s is added to community networks at time t_ω with inter-community connectivity) in the sense of mathematical expectation the solution of this equation is

$$u_{is}^2(t_\omega) \approx \alpha\eta\left(\frac{t}{t_\omega}\right)^{\frac{p_2}{1+p_2}}. \tag{11}$$

For simplicity, we assume that $p_1 = p_2 = p$, then the total utility distributions of node i in community s is

$$\begin{aligned} u_i(t) =& u_{is}^1(t) + u_{is}^2(t) \\ \approx& \phi\left(\frac{t}{t_\omega}\right)^{\frac{p_1}{1+p_1}} + \alpha\eta\left(\frac{t}{t_\omega}\right)^{\frac{p_2}{1+p_2}} \\ \approx& (\phi+\alpha\eta)\left(\frac{t}{t_\omega}\right)^{\frac{p}{1+p}}. \end{aligned} \tag{12}$$

Using Eq.(12), we find that the probability $P(u_i(t) < u)$ of a node whose utility $u_i(t)$ is smaller than u can be written as

$$P(u_i(t) < u) = P\left(t_\omega > \left(\frac{u}{\phi+\alpha\eta}\right)^{-\left(1+\frac{1}{p}\right)} t\right). \tag{13}$$

Assuming that we add the nodes (including the initial nodes) at equal time intervals to community networks, the probability density of t_ω is

$$P_i(t_\omega) = \frac{1}{h\phi_0 + t}. \tag{14}$$

Substituting Eq.(14) into Eq.(13), we obtain

$$\begin{aligned} & P\left(t_\omega > \left(\frac{u}{\phi+\alpha\eta}\right)^{-\left(1+\frac{1}{p}\right)} t\right) \\ =& 1 - \frac{(\phi+\alpha\eta)^{\left(1+\frac{1}{p}\right)} t}{u^{\left(1+\frac{1}{p}\right)}(h\phi_0+t)}. \end{aligned} \tag{15}$$

The probability density for $P(u)$ can be obtained by using

$$\begin{aligned} P(u) =& \frac{\partial P(u_i(t) < u)}{\partial u} \\ =& \frac{(\phi+\alpha\eta)^{\left(1+\frac{1}{p}\right)}(1+p)\,t}{(h\phi_0+t)\,p} u^{-\left(2+\frac{1}{p}\right)}. \end{aligned} \tag{16}$$

In the following, we perform simulations by using our model. In Fig.2, similar to strength distribution, total utility distribution $P(u)$ is an important statistical character as shown in Fig.2. The results are approximately consistent with the distributions which start from the exponential curve with the small probabilities p_1, p_2 in low-utility region and then crosses over to the power law with the large probabilities p_1, p_2 in high-utility region. The graph corresponds to the average 15 realizations of community networks. Moreover, Figures 3(a) and 3(b) depict the total utility distribution $P(u)$ as a function of total utility u for different inner-community edge ϕ (see Fig.3(a)), different inter-community edge η and probability α (see Fig.3(b)), respectively. The graph shows that the utility distributions exhibit a power-law behaviour under the condition of $p_1 = 0.8$ and $p_2 = 0.6$. The graph corresponds to the average 15 realizations of community networks. As shown in Fig.4, we obtain a power-law utility distribution $P(u) \sim u^{-\sigma}$, $\sigma = 2 + \dfrac{1}{p}$ independent of parameters ϕ, α and η, which is in good agreement with theoretical analysis. It displays that the utility distributions exhibit a power law property with $P(u) = u^{-\sigma}$, $\sigma = 2 + \dfrac{1}{p}$ $(0 < p < 1)$ independent of parameters α, ϕ and η. The graph corresponds to the average 15 realizations of community networks.

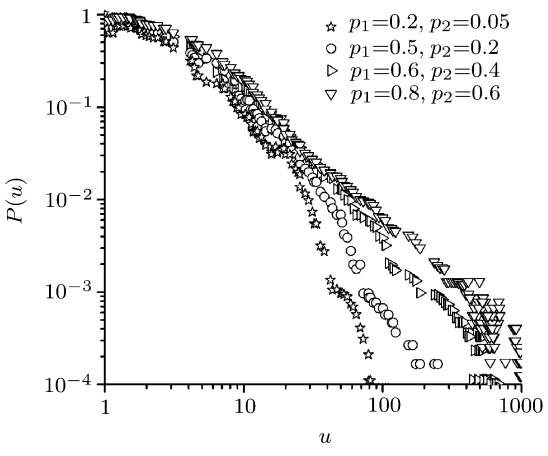


Fig.2. Double logarithmic plot of probability distributions of community utility $P(u)$ in the asymmetrically evolved community networks of total nodes $hN = 6000$ with the different probabilities p_1 and p_2 (for $p_1 = 0.2$, $p_2 = 0.05$, $p_1 = 0.5$, $p_2 = 0.2$, $p_1 = 0.6$, $p_2 = 0.4$ and $p_1 = 0.8$, $p_2 = 0.6$, respectively).

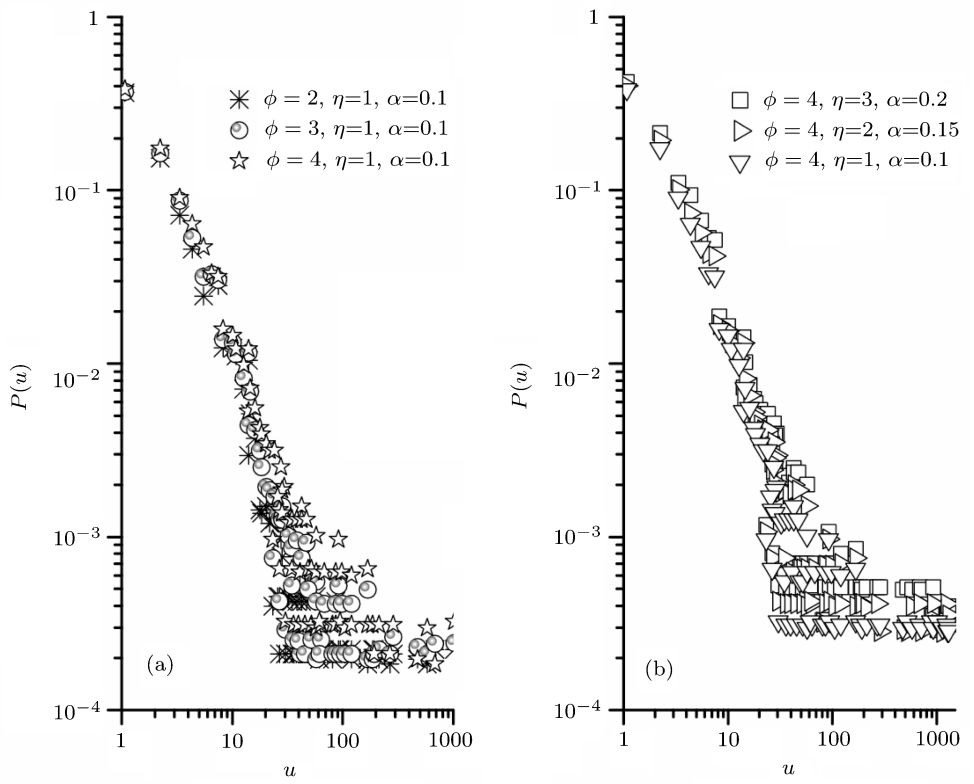


Fig.3. Log-log plot the utility distributions $P(u)$ in asymmetrically evolved community networks of total nodes $hN = 6000$. Figure 3(a) is for different inner-community edges ϕ (for $\phi = 2$, $\eta = 1$, α=0.1, ϕ=3, η= 1, α=0.1 and $\phi = 4$, $\eta = 1$, $\alpha = 0.1$ respectively). Figure 3(b) is for different inter-community edges η and probability α (for $\phi = 4$, $\eta = 3$, $\alpha = 0.2$; $\phi = 4$, $\eta = 2$, $\alpha = 0.15$ and $\phi = 4$, $\eta = 1$, $\alpha = 0.1$ respectively).

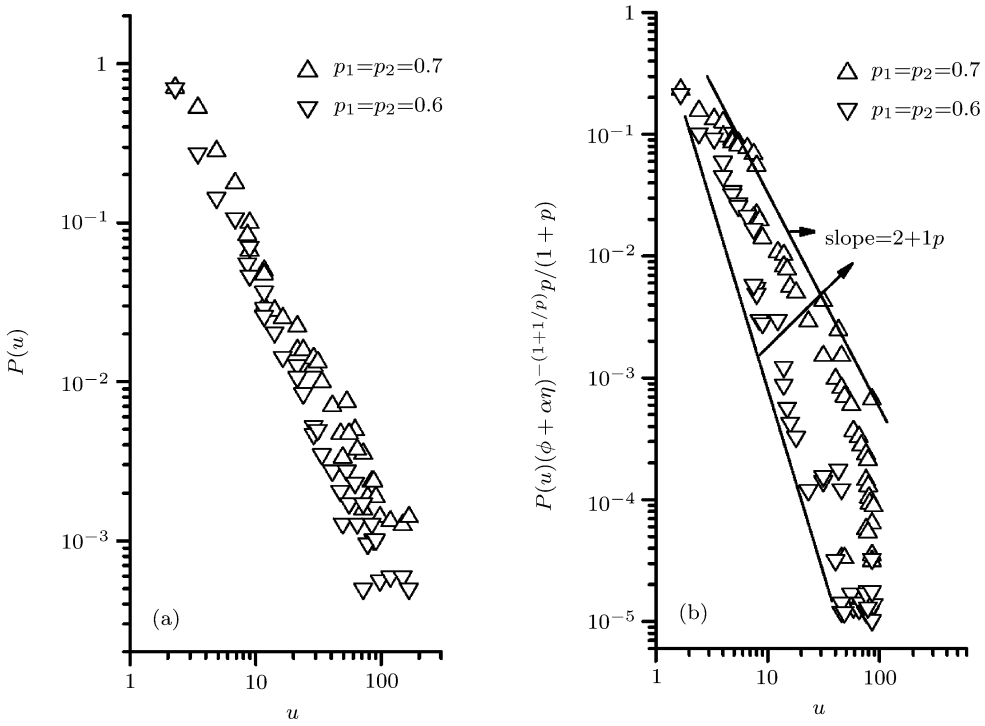


Fig.4. Figure 4(a): the utility distributions in the asymmetrically evolved community networks of total nodes $hN = 2000\,(p_1 = p_2 = 0.6,\ p_1 = p_2 = 0.7)$. Figure 4(b): the distributions $P(u)\,(\phi+\alpha\eta)^{-\frac{1+p}{p}}\,\frac{p}{1+p}$ of total nodes $hN = 2000\,(p_1 = p_2 = 0.6,\ p_1 = p_2 = 0.7)$.

4. Conclusion

In this paper, we have proposed and studied a simple asymmetrically evolved community network model. Numerical results have indicated that utility distribution of asymmetrically evolved community networks is approximately consistent with a distribution which combines an exponential curve with a power law behaviour for different parameters in our model. Therefore, the asymmetrically evolved community networks can provide good examples for studying evolving and growing phenomena in complex networks.

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