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A new traffic model with the consideration of coupling effect with two lanes*

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This paper introduces a new two-lane high-order continuum model by embedding the two delay time scales continuum traffic model presented by Xue (2003 *Phys. Rev.* E **68** 066123) into the multi-lane model proposed by Daganzo (1997 *Transpn. Res.* B **31** 83) with the consideration of the coupling effect between the vehicles of two lanes in instantaneous traffic situation and lane-change effect. In the novel model, the coupling effect of two lanes and phenomena of lane change, which were not discussed in Daganzo's model and Xue's model, are taken into account. Numerical simulation shows that it is in accordance with real traffic flow. This obviously indicates that the new phenomenon and behaviour are analogous results as single lane presented by Xue, and the proposed model is more reasonable on two lanes. Furthermore, the generation rate between two lanes is also investigated. The results show that the formation and diffusion of traffic shock wave can be better simulated on two lanes.

Keywords: traffic model, two lanes, numerical simulation
PACC: 0520, 5110, 0547

1. Introduction

The Lighthill–Whitham–Richards (LWR) theory, proposed by Lighthill and Whitham[1] and Richards[2] independently, is the earliest and most fundamental first-order continuum traffic model in transportation science. Within the framework of this theory, traffic flows on a long homogeneous freeway are governed by the following conservation equation:

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = s(x,t), \tag{1}$$

where k is traffic density, q is traffic flow rate, $s(x,t)$ is the flow generation rate, while x and t are space and time, respectively. If the freeway has no on-off ramp, then $s(x,t)$=0, otherwise, $s(x,t) \neq 0$. The flow–density relationship between flow rate q and density k is:

$$q = ku. \tag{2}$$

Furthermore, an equilibrium speed–density relationship is assumed in LWR theory as follows:

$$u = u_{\rm e}(k). \tag{3}$$

Combining the above three equations, we can obtain the analytical solutions for various simple traffic flow problems. Shock waves and traffic jams were revealed from these solutions.

However, it is impossible for the LWR model to describe the traffic flows in nonequilibrium states correctly since the speed is assumed to be always determined by the equilibrium speed-density Eq.(3). Therefore, high-order continuum traffic flow models, which incorporate a dynamic equation or momentum equation to represent general car-following behaviour, have been developed by many researchers. The acceleration and inertia of driving are taken into account in the momentum equation. Hence, the high-order models make up for some deficiencies in the simple continuum models and improve the ability in reproducing complex traffic behaviour.[3−11]

Nevertheless, the above existing high-order models have a fundamental flaw that one of the characteristic speeds resulted from the momentum and conservation equations is larger than the macroscopic flow velocity, which implies that vehicles would affect the vehicles in front of them.[12] This evidently violates the anisotropy of traffic flow and does not accord with real traffic.

On the basis of the full velocity difference (FVD)

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car-following model, which is one of car-following models presented with the consideration of velocity difference,[12−18] Jiang *et al*[12] proposed a new dynamic equation in which the density gradient term is replaced by a speed gradient term. Recently, Jiang and Wu[19] further analysed the structural properties of the solutions to the new model, which showed that the model's characteristic speeds were not larger than the macroscopic flow velocity. But the propagation velocity of disturbance c_0 in this model is constant independent of the density. In fact, the propagation velocity of disturbance in a Payne–Whitham-like model should depend on the density.[20,21] Subsequently, Xue and Dai[22] improved the model by treating c_0 as a function of the density, which indicates that the undesirable 'wrong-way travel' phenomenon and gas-like behaviour have been eliminated and the formation and diffusion of traffic shock can be better simulated in their model. In addition, other different kinds of models were developed to describe the traffic.[23−35]

In the above models, the following car is not allowed to overtake the leading car. Hence, these models are applicable only to the traffic flow on freeways with single lane. Daganzo[36] developed a continuum theory of traffic dynamics for freeways with special lanes. The LWR model is extended to two lanes allocated for two sorts of vehicles (lane 1 for faster vehicles and lane 2 for slower vehicles). His conservation equation is:

$$\frac{\partial P}{\partial t}+\frac{\partial Q}{\partial x}=S, \tag{4}$$

where $P=(k_1,k_2)^{\mathrm{T}}$, $Q=[q_1(k_1,\ k_2),\ q_2(k_1,\ k_2)]^{\mathrm{T}}$, $S=(s_1,\ s_2)^{\mathrm{T}}$, k_m, q_m and s_m represent the traffic density, the flow rate and the generation rate on lane $m(=1,2)$, respectively. To carry out analyses graphically, simple continuum model was only adopted by Daganzo, hence the non-equilibrium traffic flow dynamics cannot be described in his model. Another multi-lane traffic model, considering the specific traffic conditions in Chinese cities (e.g., untidy flows and lower speeds on roads), was also proposed by Wu.[37] But Wu's model tends to equalize all lanes' densities through averaging them, which conflicts with the real traffic. Lately, Tang and Huang[38,39] developed a continuum model for freeways with two lanes which applies the momentum equation proposed by Jiang *et al*[12] into the Daganzo's modelling framework for multi-lane traffic flow.[36] However, this model allows only the faster vehicles to change lanes. Recently, Huang, Tang and Gao[40] further extended the work of Tang and Huang[38,39] by allowing vehicles on both lanes to change their lanes according to the traffic condition on the spot and at that time, which contains the speed gradient-based momentum equations derived from a car-following theory suited to two-lane traffic flow. Tang, Jiang and Wu[41] also extended speed gradient model for traffic flow on two-lane freeways. And other different two-lane models were presented to reproduce traffic phenomenon.[42−47] But the coupling effect of the interactions between vehicles on two lanes has not been taken into account in these models.

In this paper, we extend the two delay time scales model proposed by Xue *et al*[22] to describe asymmetric two-lane traffic with the consideration of coupling effect, which allows these vehicles on both lanes to change their lanes according to the traffic condition on the spot and at that time. We also investigate lane change rate resulting in the shock wave effect in two-lane freeways.

The paper is organized as follows. In Section 2, we establish a continuum model with anisotropy by introducing the drivers' reaction time t_{r} and the vehicle relaxation time T to the forward anticipation velocity with the consideration of coupling effect on both lanes. A numerical simulation is described in Section 3, which validates the correctness of the theoretical analysis, and some nonequilibrium phenomena are investigated, such as stop-and-go waves, shock waves and phase transition. Section 4 concludes the paper.

2. Modelling and analysis

Two delay time scales model proposed by Xue's assumption of forward anticipation[22] was considered that the forward anticipation of velocity u was accomplished in the relaxation time T, while this process was completed by an adjustment with drivers' reaction time t_{r} to the anticipated one u_{e}. The stream velocity u in the relaxation time T just reached the anticipated value u_{e} corresponding to the density in time $t+t_{\mathrm{r}}$ at the location $x+ut_{\mathrm{r}}$, where t_{r} is drivers' reaction time. The velocity-density relation is deduced as

$$u(x+uT,t+T)=u_{\mathrm{e}}(k(x+ut_{\mathrm{r}},t+t_{\mathrm{r}})). \tag{5}$$

Using the Taylor series expansion for Eq.(5) and applying Eq.(1), we get[22]

$$\frac{\partial u}{\partial t}+(u-c(k))\frac{\partial u}{\partial x}=\frac{u_{\mathrm{e}}(k)-u}{T}, \tag{6}$$

$$c(k) = -k\frac{t_{\rm r}}{T}\frac{{\rm d}u_{\rm e}}{{\rm d}k} \geq 0.$$

On two-lane freeways, the behaviour of lane change will occur. To model this effect, the interactions between vehicles on two lanes must be taken into account to describe the flow rate changing from the other lane and are set as the coupling effect term in this paper. Accordingly, the coupling effect term is added in the Eq.(5) to describe the lateral lane's effect and flow rate changing from the other lane.

$$\begin{aligned} &u_m(x + u_m T_m, t + T_m) \\ &= u_{em}[k_m(x + u_m t_{rm}, t + t_{rm}), \beta_{nm} k_n], \\ &\qquad m, n = 1, 2, \quad \text{and} \quad m \neq n. \end{aligned} \tag{7}$$

Equation (7) means that the stream velocity u_m in the relaxation time T_m just reaches the anticipated value u_{em} (u_{em} is functions of k_1 and k_2) corresponding to the density in time $t + t_{rm}$ at the location $x+ut_{rm}$ on lane m, where t_{rm} is drivers' reaction time on lane m. And here β_{nm} is coupling coefficient from lane n to lane m. On the basis of principle of coupling interaction, we consider $\beta_{nm} = \beta_{mn} = \beta$. Using the Taylor series expansion for Eq.(7) and neglecting higher-order terms we obtain

$$\begin{aligned} &u_m(x, T_m) + \frac{\partial u_m}{\partial t} T_m + u_m T_m \frac{\partial u_m}{\partial x} + O(T_m^2) \\ =&u_{em}(k_m, \beta k_n) + \frac{\partial u_{em}}{\partial k_m} \\ &\times \left(t_{rm} \frac{\partial k_m}{\partial t} + u_m t_{rm} \frac{\partial k_m}{\partial x} \right) + O(t_{rm}^2). \end{aligned} \tag{8}$$

Equation (8) can be rearranged as

$$\begin{aligned} \frac{\partial u_m}{\partial t} + u_m \frac{\partial u_m}{\partial x} =& \frac{u_{em}(k_m, \beta k_n) - u_m}{T_m} + \frac{t_{rm}}{T_m} \frac{\partial u_{em}}{\partial k_m} \\ &\times \left(\frac{\partial k_m}{\partial t} + u_m \frac{\partial k_m}{\partial x} \right). \end{aligned} \tag{9}$$

Applying Eq.(4), we have

$$\begin{aligned} &\frac{\partial u_m}{\partial t} + (u_m - c_{m0}) \frac{\partial u_m}{\partial x} = \frac{u_{em}(k_m, \beta k_n) - u_m}{T_m} + \gamma_m s_m, \\ &c_{m0} = -k_m \frac{t_{rm}}{T_m} \frac{\partial u_{em}}{\partial k_m} \geq 0, \quad \text{and} \quad \gamma_m = \frac{t_{rm}}{T_m} \frac{\partial u_{em}}{\partial k_m}. \end{aligned} \tag{10}$$

The dynamic model for two-lane traffic consists of the following equations:

$$\frac{\partial k_m}{\partial t} + \frac{\partial q_m}{\partial x} = s_m(x, t), \tag{11}$$

$$\frac{\partial u_m}{\partial t} + (u_m - c_{m0}) \frac{\partial u_m}{\partial x} = \frac{u_{em}(k_m, \beta k_n) - u_m}{T_m} + \gamma_m s_m,$$

$$m, n = 1, 2, \quad \text{and} \quad m \neq n, \tag{12}$$

$$s_1(x, t) + s_2(x, t) = 0, \tag{13}$$

where $c_{m0} = -k_m \frac{t_{rm}}{T_m} \frac{\partial u_{em}}{\partial k_m} \geq 0$ represents the propagation speed of the disturbance on lane m. Moreover, s_1 and s_2 may have different effect on the velocity dynamic equation in Eq.(12), which can be modelled in our model by setting $\gamma_m = \frac{t_{rm}}{T_m} \frac{\partial u_{em}}{\partial k_m}$ (γ_m is a function of equilibrium flow's differential coefficient to the density k_m). Also, equation (12) shows that one lane's (lane m) dynamic equation is affected by its lateral lane's coupling effect of traffic density (βk_n) and its generation rate term (s_m) with lane change. Equation (12) states that there is no on-off ramp in the freeway. But it does not mean that $s_1(x, t) = 0$ and $s_2(x, t) = 0$ must hold for all times and spaces, because lane changes are allowed in this study. Note that the model given by Eqs.(11) and (12) for a specific lane, if the coupling effect and its generation rate are not taken into account, is formally the same as the continuum model on single lane proposed by Xue.[22] However, attention should be paid to such a difference that all velocities in Eqs.(11) and (12) are functions of k_1 and k_2. According to Liu's idea[48] and Tang's method,[41] we suppose that the terms of the generation rate have the form as follows:

$$\begin{aligned} &s_1 = (Q_{\rm e1} - Q_{\rm e2}) k_1 (1 - (k_2/k_{\rm 2jam})^\theta), \\ &\qquad s_2 = -s_1, \quad \text{if} \quad k_1 \geq k_2, \end{aligned} \tag{14}$$

$$\begin{aligned} &s_2 = (Q_{\rm e2} - Q_{\rm e1}) k_2 (1 - (k_1/k_{\rm 1jam})^\phi), \\ &\qquad s_1 = -s_2, \quad \text{if} \quad k_1 < k_2, \end{aligned} \tag{15}$$

where Q_{ei} is the equilibrium flow ($Q_{ei} = k u_{ei}(k)$) and θ, ϕ are a parameter that reflects the intensity of the lane-change effect from one lane to another, $0 \leq \theta$, $\phi \leq 1$, and depends on the condition of road. The special case $Q_{\rm e1} = Q_{\rm e2}$ corresponds to symmetric lane change rules, while $Q_{\rm e1} \neq Q_{\rm e2}$ corresponds to asymmetric lane change rules, k_{ijam} is the jam density of lane i.

In real traffic, the lane change rate, say from lane 1 to 2, should be zero when either $k_2 = k_{\rm 2jam}$ (lane 2 is in complete jam) or k_1=0 (there is no vehicle on lane 1). Furthermore, it obviously depends on the equilibrium speed density relationship on the present lane and on the coupling density of another lane. These properties are fulfilled in Eqs.(14) and (15).

The system composed of Eqs.(11) and (12) can be expressed in a vector manner as follows:

$$\frac{\partial V}{\partial t}+[A]\frac{\partial V}{\partial x}=E, \tag{16}$$

where

$$V=\begin{bmatrix}k_1\\u_1\\k_2\\u_2\end{bmatrix},\quad E=\begin{bmatrix}s_1\\(u_{1e}-u_1)/T+\gamma_1 s_1\\s_2\\(u_{2e}-u_2)/T_2+\gamma_2 s_2\end{bmatrix},$$

$$[A]=\begin{bmatrix}u_1 & k_1 & 0 & 0\\0 & u_1-c_{10} & 0 & 0\\0 & 0 & u_2 & k_2\\0 & 0 & 0 & u_2-c_{20}\end{bmatrix}.$$

Comparing our new model with the simple continuum model presented by Daganzo, we can see that the new model differs from the others in adding two new equations of motion, in which the speed gradient term replaces the density gradient term as the anticipation term with $c_{i0}=c(k_i)$. This replacement in the new model solves the characteristic speed problem that exists in the previous high-order models and therefore enables the new model to satisfy the anisotropic property of traffic flow. Next we apply matrices to analyse character speeds of the system (16). The eigenvalues, λ, of the $[A]$ matrix are found by setting

$$\det([\boldsymbol{A}]-\lambda[\boldsymbol{I}])=0, \tag{17}$$

where $[\boldsymbol{I}]$ is identity matrix. From Eq.(17), we have

$$\lambda_{11}=u_1,\quad \lambda_{12}=u_1-c_{10},$$

$$\lambda_{21}=u_2,\quad \lambda_{22}=u_2-c_{20}.$$

From the above-mentioned $c_{i0}\geq 0$ (i=1, 2), it is found that the characteristic speeds of each subsystem are not larger than their macroscopic flow speeds, which shows that the rear disturbances on each lane cannot propagate forward, but it does not guarantee that the rear disturbance in a system consisting of two lanes will not affect the driving behaviour of the leading vehicle because lane changing is allowed in these two-lane traffic system.

3. Numerical simulation

In this section, using the finite difference method, we numerically solve the model to demonstrate the model's ability of capturing complex traffic phenomena. Let index j and index i represent the time interval and the road section, respectively. Then the difference equations correspond to Eqs. (11) and (12) are as follows:

$$\begin{aligned}k_{mi}^{j+1}=&k_{mi}^{j}+\frac{\Delta t}{\Delta x}k_{mi}^{j}\left(u_{mi}^{j}-u_{m(i+1)}^{j}\right)\\&+\frac{\Delta t}{\Delta x}u_{mi}^{j}\left(k_{m(i-1)}^{j}-k_{mi}^{j}\right)\\&+\Delta t s_{mi}^{j},\quad m=1,2.\end{aligned} \tag{18}$$

(I) When traffic is heavy, $u_{mi}^{j}<c_{m0}$,

$$\begin{aligned}u_{mi}^{j+1}=&u_{mi}^{j}+\frac{\Delta t}{\Delta x}\left[c_{m0}-u_{mi}^{j}\right]\left(u_{m(i+1)}^{j}-u_{mi}^{j}\right)\\&+\frac{\Delta t}{T}\left(u_{em}-u_{mi}^{j}\right)\\&+\gamma_m\Delta t s_{mi}^{j},\quad m=1,2.\end{aligned} \tag{19}$$

(II) When traffic is light, $u_{mi}^{j}\geq c_{m0}$,

$$\begin{aligned}u_{mi}^{j+1}=&u_{mi}^{j}+\frac{\Delta t}{\Delta x}\left[c_{m0}-u_{mi}^{j}\right]\left(u_{mi}^{j}-u_{m(i-1)}^{j}\right)\\&+\frac{\Delta t}{T}\left(u_{em}-u_{mi}^{j}\right)\\&+\gamma_m\Delta t s_{mi}^{j},\quad m=1,2.\end{aligned} \tag{20}$$

To investigate congestion and dissipation of the traffic flow, we adopt two Riemann initial conditions as follows:[39]

$$\text{(i)}\quad \begin{cases}k_{1\mathrm{u}}^{1}=0.03,\\k_{2\mathrm{u}}^{1}=0.04,\end{cases}\quad \begin{cases}k_{1\mathrm{d}}^{1}=0.12,\\k_{2\mathrm{d}}^{1}=0.18,\end{cases} \tag{21a}$$

$$\text{(ii)}\quad \begin{cases}k_{1\mathrm{u}}^{2}=0.12,\\k_{2\mathrm{u}}^{2}=0.18,\end{cases}\quad \begin{cases}k_{1\mathrm{d}}^{2}=0.03,\\k_{2\mathrm{d}}^{2}=0.04,\end{cases} \tag{21b}$$

where $k_{1\mathrm{u}}^{1}$, $k_{2\mathrm{u}}^{1}$, $k_{1\mathrm{d}}^{1}$, $k_{2\mathrm{d}}^{1}$ and $k_{1\mathrm{u}}^{2}$, $k_{2\mathrm{u}}^{2}$, $k_{1\mathrm{d}}^{2}$, $k_{2\mathrm{d}}^{2}$ are the upstream and downstream densities on two lanes respectively. Equation (21a) corresponds to the appearance of shock waves when free-flow traffic meets stopped vehicles, while equation (21b) corresponds to the rarefaction wave as a queue dissolves. Two initial conditions are[40]

$$u_{1\mathrm{u}}^{1,2}=u_{1e}(k_{1\mathrm{u}}^{1,2},k_{2\mathrm{u}}^{1,2}),\quad u_{1\mathrm{d}}^{1,2}=u_{1e}(k_{1\mathrm{d}}^{1,2},k_{2\mathrm{d}}^{1,2}), \tag{22a}$$

$$u_{2\mathrm{u}}^{1,2}=u_{2e}(k_{1\mathrm{u}}^{1,2},k_{2\mathrm{u}}^{1,2}),\quad u_{2\mathrm{d}}^{1,2}=u_{2e}(k_{1\mathrm{d}}^{1,2},k_{2\mathrm{d}}^{1,2}). \tag{22b}$$

Substituting the $k_m+\beta k_n$ into the equilibrium speed-density relationship developed by Del Castillo and Benitez,[40,49] the equilibrium speed-density relationships are as follows:

$$\begin{aligned}u_{\mathrm{e}1}(k)=u_{1\mathrm{f}}\Bigg[1-\exp\Bigg(1-\exp\Bigg(&\frac{c_{1\mathrm{jam}}}{u_{1\mathrm{f}}}\\&\times\left(\frac{k_{1\mathrm{jam}}+\beta k_{2\mathrm{jam}}}{k_1+\beta k_2}-1\right)\Bigg)\Bigg)\Bigg],\end{aligned} \tag{23a}$$

$$u_{e2}(k) = u_{2f}\left[1-\exp\left(1-\exp\left(\frac{c_{2jam}}{u_{2f}} \times\left(\frac{\beta k_{1jam}+k_{2jam}}{\beta k_1+k_2}-1\right)\right)\right)\right], \quad (23b)$$

where k_{1jam},k_{2jam} and u_{1f}, u_{2f} are the jam densities and free-flow velocity of lane 1 and lane 2, respectively. c_{1jam} and c_{2jam} are the propagation velocity of a disturbance under the jam density k_{1jam} and k_{2jam}, respectively.

Free boundary conditions are applied here,[21,22] $\partial k_m/\partial x = 0$, $\partial u_m/\partial x = 0$. Consider a section of freeway 20 km long, divide it into 100 cells, and take the time interval as 1 s. According to real observation and the parameter identification process, the values of the parameters are chosen as follows:[22,39] $u_{1f} = 40$ m/s, t_{r1}=1 s, T_1=8 s, k_{1jam}=0.15 veh/m, $c_{1jam} = 7$ m/s; $u_{2f} = 30$ m/s, t_{r2}=0.75 s, T_2=7 s, k_{2jam}=0.2 veh/m, $c_{2jam} = 6$ m/s; $\beta = 0.1$, $\theta = \phi = 1$.

Note that the jam density of lane 2 is higher than that of lane 1. The computational results under the two Riemann initial conditions (21a) and (21b) are shown in Figs.1–6, respectively.

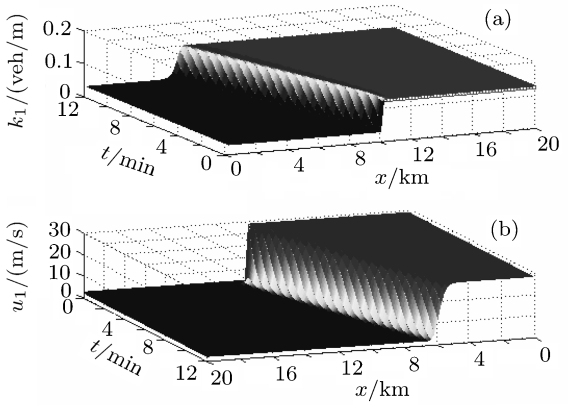


Fig.1. Shock waves under the Riemann initial conditions of Eq.(21a): (a) temporal evolution of density $k_1(x,t)$ and (b) velocity $u_1(x,t)$ on lane 1.

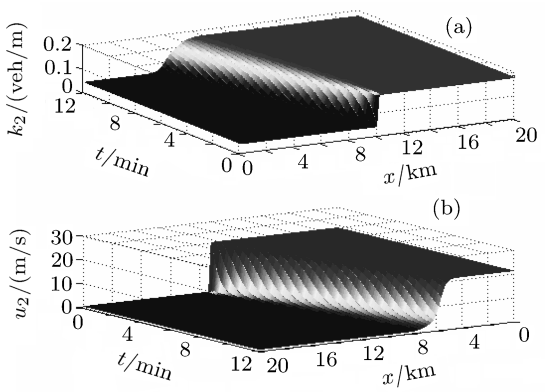


Fig.2. Shock waves under the Riemann initial conditions of Eq.(21a): (a) temporal evolution of density $k_2(x,t)$ and (b) velocity $u_2(x,t)$ on lane 2.

Figures 1, 2, 4 and 5 show that different Riemann initial conditions lead to different fronts between the congested and free-flow traffic, which results in proper figures of traffic flow density changing on both lanes with lane change (see Figs.3 and 6). The wave shapes of two lanes are in accordance with that of the single lane by Xue[22] (See Figs.1 and 2 in Ref.[22]), which indicates that our new two-lane model is reasonable. Figures 1 and 2 show how the backward-moving shock wave front evolves under the condition (21a) on lane 1 and lane 2, respectively. This means that traffic becomes more congested, which we often see in rush hours. Figures 4 and 5 show how the rarefaction wave front evolves under the condition (21b) on lane 1 and lane 2, respectively. It is a queue in the process of dissolution, which is consistent with our daily experiences in real traffic. Figures 3 and 6 show that the lane change behaviour occurs at large densities more frequently than at small densities on both lanes. The

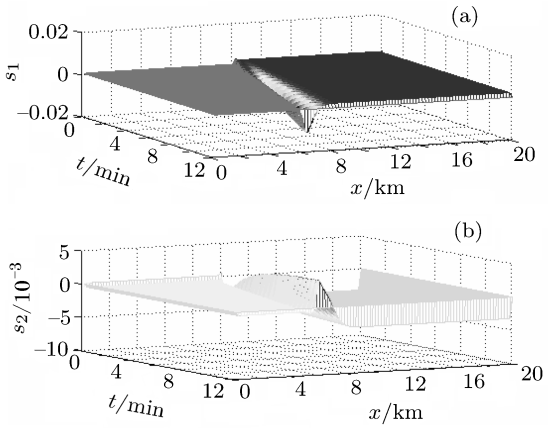


Fig.3. Lane change rate under the Riemann initial conditions of Eq.(21a): (a) s_1 is the generation rate on lane 1 and (b) s_2 is the generation rate on lane 2.

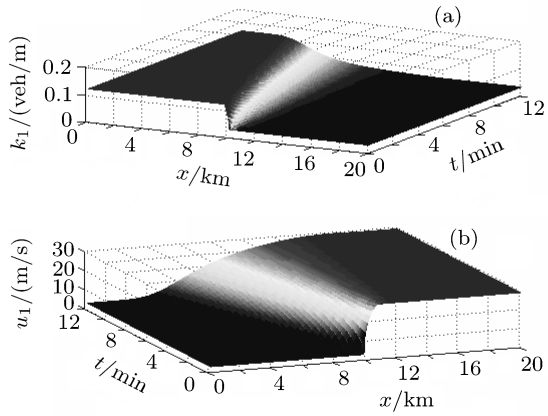


Fig.4. Rarefaction waves under the Riemann initial conditions of Eq.(21b): (a) temporal evolution of density $k_1(x,t)$ and (b) velocity $u_1(x,t)$ on lane 1.

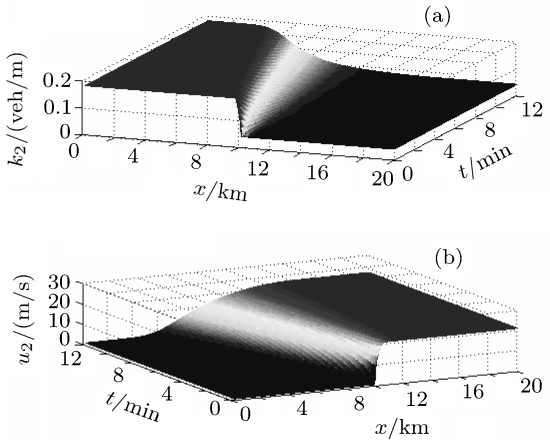


Fig.5. Rarefaction waves under the Riemann initial conditions of Eq.(21b): (a) temporal evolution of density $k_2(x,t)$ and (b) velocity $u_2(x,t)$ on lane 2.

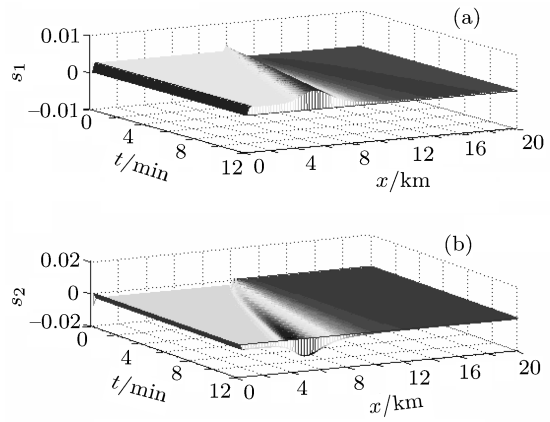


Fig.6. Lane change rate under the Riemann initial conditions of Eq.(21b): (a) s_1 is the generation rate on lane 1 and (b) s_2 is the generation rate on lane 2.

results are consistent with the properties of traffic flow in Refs.[50, 51]. These effects are also consistent with the diverse nonlinear dynamic phenomena observed in realistic traffic flow.

4. Conclusion

Many previously existing continuum high-order models mainly describe single lane traffic while leaving multi-lane problems discussed insufficiently. Daganzo has already presented the two-lane traffic model, but the effect of acceleration and inertia is not taken into account in his model. Therefore, his model cannot represent the nonequilibrium feature. In this paper, we establish a new two-lane model that can avoid the defect of Daganzo's model, which derives the equation of motion with a nonconstant propagation velocity of disturbance in the traffic flow by considering coupling effect for two lanes. The model has anisotropy, which is confirmed by the formation and diffusion of a traffic shock wave in numerical simulations and validated by using the model. Moreover, the model has an analogy with Zhang's, Aw and Rascle's, Jiang and Wu's, Xue's and Tang's models. Also, numerical simulation indicates that the formation and diffusion of traffic shock can be better simulated on two lanes. And complex nonequilibrium phenomena such as stop-and-go, ghost jams, local clusters and phase transition of the new two-lane model will be studied in our future study.

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