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Hawking radiation from the charged and magnetized BTZ black hole via covariant anomaly*

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This paper discusses Hawking radiation from the charged and magnetized Bañados–Teitelboim–Zanelli (BTZ) black hole from the viewpoint of anomaly, initiated by Robinson and Wilczek recently. It reconstructs the electromagnetic field tensor and the Lagrangian of the field corresponding to the source with electric and magnetic charges to redefine an equivalent charge and gauge potential. It employs the covariant anomaly cancellation method to determine the compensating fluxes of charge flow and energy-momentum tensor, which are shown to match with those of the 2-dimensional blackbody radiation at the Hawking temperature exactly.

Keywords: Hawking radiation, covariant anomaly, event horizon, BTZ black hole
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1. Introduction

It is believed that Hawking radiation from black hole is one of the quantum properties inherent in horizons and a profound understanding of it may hint at how to construct a quantum theory of gravity. Since the original calculation of Stephen Hawking[1] that was based on Bogoliubov transformations, there have emerged many derivations on Hawking radiation.[2−5] Recently, Robinson and Wilczek brought forward a novel method[6] from the anomaly point of view. The key trick of this approach is that any higher dimensional space time can be reduced to a two-dimensional theory effectively by the universal dimensional reduction technique. In the two-dimensional reduction, when one omits the ingoing modes, which classically have nothing to do on the observable physics, to formulate the effective field theory outside the event horizon (EH), it becomes chiral and exhibits anomaly at the quantum level. The underlying theory, however, should be covariant under the infinitesimal coordinate translations, and one thus should introduce a compensating flux to cancel the anomaly quantum mechanically. The result shows that the introduced flux of energy–momentum tensor is nothing but that of Hawking radiation. Due to the universality of this method, it was soon generalized to charged black holes[7] by using the gauge anomaly in addition to gravitational anomaly and further applied to rotating black holes[8,9] by analogizing the azimuthal quantum number m to the electric charge e.[10−16] However for the charged and magnetized black hole, because there not only exists gauge flux to electric charge but also exists gauge flux to magnetic charge, its quantum anomaly has not been treated until now. To overcome this difficulty in this paper, we reconstruct the electromagnetic field tensor and the Lagrangian of the background spacetime by regarding that the electric and magnetic charges concentrate on a conducting sphere with a constant ratio. As a result, the equivalent charge and gauge potential of the charged and magnetized BTZ black hole are redefined.[17]

On the other hand, the Bañados–Teitelboim–Zanelli (BTZ) black holes have attracted great attention recently, because the BTZ theory refers to special solutions of (2+1)-dimensional anti-de Sitter gravity and the charged BTZ black holes are simply the analogous solutions in (2+1)-dimensional anti-de Sitter–Maxwell gravity. Hence it can provide a simplified model for exploring some conceptual issues, such as the quantum corrected thermodynamics,[18,19] AdS/CFT correspondence,[20] string theory[21] and so

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on. To further understand the basic properties of BTZ black hole, in this paper, we concentrate on studying Hawking radiation from the charged and magnetized BTZ black hole from the anomaly point of view. It should be pointed out that the flux of Hawking radiation here is determined only by the covariant anomaly rather than the consistent one. This covariant anomaly cancellation method was put forward very recently and was believed conceptually to be clean and economical.

The remainder of this paper is arranged as follows. In Section 2, we reconstruct the electromagnetic field tensor to redefine the equivalent charge and gauge potential of the charged and magnetized BTZ black holes. Then in Section 3, we determine its Hawking radiation by the covariant gauge anomaly and gravitational anomaly. Finally, we give our concluding remarks in Section 4.

2. The equivalent charge and gauge potential of the charged and magnetized BTZ black hole

In a (2+1)-dimensional anti-de Sitter–Maxwell gravity with a negative cosmological constant $\Lambda = -1/l^2$, the line element of the charged and magnetized non-rotating BTZ black hole takes the form as[22]

$$\mathrm{d}s^2 = -\tilde{f}(r)\mathrm{d}t_B^2 + \tilde{f}(r)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\phi^2, \tag{1}$$

where the lapse function is

$$\tilde{f}(r) = \frac{r^2}{l^2} - M - (Q_\mathrm{e}^2 + Q_\mathrm{g}^2)\ln\frac{r^2}{l^2}, \tag{2}$$

where M denotes the ADM mass, Q_e and Q_g respectively represent the electric charge and magnetic charge of the BTZ black hole (the BTZ unit $G_3 = 1/8$ is adopted throughout this paper). Since this BTZ black hole is charged and magnetized, it is inconvenient for us to discuss the gauge anomalies with respect to electric charge and magnetic charge directly. But while the black hole is taken as a conducting sphere with constant ratio of electric charge to magnetic charge to redefine the electromagnetic tensor and Lagrangian, we will find an equivalent charge of the electric and magnetic charges and meanwhile the corresponding gauge potential also can be redefined. In this case the anomaly cancellation method is applicable.

For a source with electric charge and magnetic charge, one can define the electromagnetic tensor as[17]

$$F_{\mu\nu} = \nabla_\nu A_\mu - \nabla_\mu A_\nu + G^+_{\mu\nu}, \tag{3}$$

where $G^+_{\mu\nu}$ is the Dirac string term. And the Maxwell equations are

$$\nabla_\nu F^{\mu\nu} = 4\pi\rho_\mathrm{e}u^\mu, \tag{4}$$

$$\nabla_\nu F^{+\mu\nu} = 4\pi\rho_\mathrm{g}u^\mu, \tag{5}$$

where $F^{+\mu\nu}$ is the dual tensor of $F^{\mu\nu}$, ρ_e and ρ_g stand for the densities of electric and magnetic charges respectively, while u^μ represents the 4-velocity. If we define a new real anti-symmetric tensor

$$\tilde{F}^{\mu\nu} = F^{\mu\nu}\cos\beta + F^{+\mu\nu}\sin\beta, \tag{6}$$

where β denotes a real constant angle, then equations (4) and (5) will change to

$$\nabla_\nu\tilde{F}^{\mu\nu} = 4\pi(\rho_\mathrm{e}\cos\beta + \rho_\mathrm{g}\sin\beta)u^\mu, \tag{7}$$

$$\nabla_\nu\tilde{F}^{+\mu\nu} = 4\pi(-\rho_\mathrm{e}\sin\beta + \rho_\mathrm{g}\cos\beta)u^\mu. \tag{8}$$

Let

$$\rho_\mathrm{e}\cos\beta + \rho_\mathrm{g}\sin\beta = \rho_h, \tag{9}$$

$$-\rho_\mathrm{e}\sin\beta + \rho_\mathrm{g}\cos\beta = 0, \tag{10}$$

besides yielding $\rho_\mathrm{e}/\rho_\mathrm{g} = \cot\beta$, the Maxwell equations can be rewritten as

$$\nabla_\nu\tilde{F}^{\mu\nu} = 4\pi\rho_h u^\mu, \tag{11}$$

$$\nabla_\nu\tilde{F}^{+\mu\nu} = 0. \tag{12}$$

That is

$$\frac{\partial}{\partial x^\nu}(\sqrt{-g}\tilde{F}^{\mu\nu}) = 4\pi\sqrt{-g}J^\mu, \tag{13}$$

where $J^\mu = \rho_h u^\mu$. Obviously, equation (13) is similar to the Maxwell equation corresponding to the source only with electric charge. Therefore, if we consider the black hole as a conducting sphere while the electric and magnetic charges are concentrated on the black hole with the density rate as $\rho_\mathrm{e}/\rho_\mathrm{g} = \cot\beta$, we have

$$Q_\mathrm{h}^2 = Q_\mathrm{e}^2 + Q_\mathrm{g}^2, \tag{14}$$

where Q_e and Q_g are the electric charge and magnetic charge of the hole and Q_h stands for the equivalent charge corresponding to the density ρ_h.

Correspondingly, the Lagrangian of the electromagnetic field changes to

$$L_h = -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{15}$$

where

$$\tilde{F}_{\mu\nu} = \nabla_\nu \tilde{A}_\mu - \nabla_\mu \tilde{A}_\nu, \tag{16}$$

in which the nonvanishing component of the gauge potential is

$$\tilde{A}_{\rm t} = -Q_{\rm h} \ln \frac{r}{l}. \tag{17}$$

And the lapse function can be rewritten as

$$f(r) = \frac{r^2}{l^2} - M - Q_{\rm h}^2 \ln \frac{r^2}{l^2}, \tag{18}$$

according to the null hyper surface equation, which can yield the outer horizon r_+ and inner horizon r_-. In addition, the surface gravity at the EH is given by

$$\kappa = \frac{1}{2}\partial_{\rm r} f\,\big|_{r=r_+} = \frac{2r_+^3 - l^4 Q_{\rm h}^2}{2l^2 r_+^2}. \tag{19}$$

We find that equations (17), (18) and (19) are similar to those of the general BTZ black hole only with electric charge, thus we next base on them to study Hawking radiation from the charged and magnetized BTZ black hole from the viewpoint of anomaly.

3. Quantum anomaly and Hawking radiation of the charged and magnetized BTZ black hole

As mentioned above, before the effective field theory is constructed, one should reduce the arbitrary dimensional space time to an effective two-dimensional theory via considering the action of scalar field in the black hole background. In the case of the charged and magnetized (2+1)-dimensional BTZ black hole, after performing the scalar field decomposition and tortoise coordinate transformation, we find that physics near the EH can be described by an infinite collection of massless two-dimensional scalar field effectively on the metric

$$\mathrm{d}s^2 = -f\mathrm{d}t_{\rm B}^2 + f^{-1}\mathrm{d}r^2, \tag{20}$$

as the near horizon limit is taken.

In the effective two-dimensional reduction, each partial wave mode has both gauge and general coordinate symmetries. Once the classically insignificant ingoing modes are integrated out to formulate the effective field outside the EH, it becomes chiral and suffers from gauge and gravitational anomaly with respect to the gauge and general coordinate symmetries respectively. At the quantum level however, there is no anomaly, and one thus should introduce a compensating flux. To accomplish this, Refs.[8, 9] adopted the consistent gauge and gravitational anomalies together with the regularity requirement, which require the covariant fluxes to vanish at the EH. This implies that one must input two anomalies to restore the gauge or general covariance. More recently, Ref.[23] suggested that the same work can be done by only employing the covariant anomaly. One unique reason for this arises from the functional form of covariant anomaly, this is governed solely by the gauge or diffeomorphism transformation properties, but not so for the consistent one. Hence we thereafter adopt this simplified approach of cancelling the covariant anomaly to determine the flux of Hawking radiation. We first discuss the gauge anomaly to charge flow. For convenience, we split the region outside the EH into $[r_+,\, r_+ + \varepsilon]$ and $[r_+ + \varepsilon,\, +\infty]$. When the classically irrelevant ingoing modes are omitted, there is not anomaly in $[r_+ + \varepsilon,\, +\infty]$ and the covariant charge flow flows

$$\nabla_\mu \tilde{J}^\mu_{(0)} = 0, \tag{21}$$

while in the near-horizon region $[r_+,\, r_+ + \varepsilon]$, due to the breakdown of the gauge symmetries, the covariant charge flow is not conserved but suffers from gauge anomaly, which takes the form as[24]

$$\nabla_\mu \tilde{J}^\mu_{({\rm H})} = \pm\frac{Q_{\rm h}^2}{4\pi\sqrt{-g}}\varepsilon_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{22}$$

where $+(-)$ corresponds to the left (right) handed field, $\varepsilon^{\mu\nu}$ is the anti-symmetric tensor ($\varepsilon^{10} = \varepsilon^{01} = -1$). Note that here the gauge charge is equivalent charge, we hence substitute e with $Q_{\rm h}$. Integrating Eqs.(21) and (22) for the $\mu = r$ component, one gets

$$\tilde{J}^{\rm r}_{(0)} = a_0, \tag{23}$$

$$\tilde{J}^{\rm r}_{({\rm H})} = a_{\rm h} + Q_{\rm h}^2[\tilde{A}_{\rm t}(r) - \tilde{A}_{\rm t}(r_+)]/2\pi. \tag{24}$$

where a_0 and $a_{\rm h}$ are integration constants, which respectively stand for the values of the covariant fluxes of charge flow at infinity and the EH. Since the region outside the EH has been divided into two parts, thus

the corresponding covariant fluxes can be written as $\tilde{J}^{\mu}_{(2)} = \tilde{J}^{u}_{(0)}\Theta_{+}(r) + \tilde{J}^{\mu}_{(\mathrm{H})}H(r)$, where $\Theta_{+} = \Theta\left(r - r_{\mathrm{h}} - \varepsilon\right)$ is the scalar step function and $H = 1 - \Theta_{+}$ is the scalar top hat function. Basing on them, the Ward identity changes to

$$\begin{aligned}\partial_{\mathrm{r}}\tilde{J}^{\mathrm{r}} =&\partial_{\mathrm{r}}\left(\frac{Q_{\mathrm{h}}^{2}}{2\pi}\tilde{A}_{\mathrm{t}}H\right) + \left(\tilde{J}^{\mathrm{r}}_{(0)} - \tilde{J}^{\mathrm{r}}_{(\mathrm{H})} + \frac{Q_{\mathrm{h}}^{2}\tilde{A}_{\mathrm{t}}(r)}{2\pi}\right)\\ &\times\delta\left(r - r_{+} - \varepsilon\right)\end{aligned} \tag{25}$$

in which the Wess–Zumino term imposes the covariant flux as an anomaly-free one $\tilde{J}^{\mathrm{r}'} = \tilde{J}^{\mathrm{r}} - \tilde{A}_{\mathrm{t}}He^{2}/2\pi$, that is, the first term in the right-hand side will be cancelled by the quantum effect of the classically insignificant ingoing modes that we have ignored. Hence to restore the gauge covariance at the quantum level, the properties of delta function will force

$$a_{0} = a_{\mathrm{h}} - Q_{\mathrm{h}}^{2}\tilde{A}_{\mathrm{t}}(r_{+})/2\pi, \tag{26}$$

where we have adopted Eqs.(23) and (24). After employing the covariant boundary conditions that require the covariant flux to vanish at the EH, the total flux of charge current, which is required to restore the underlying gauge symmetry at the EH by cancelling the gauge anomaly at the quantum level, can be expressed explicitly as

$$a_{0} = \frac{Q_{\mathrm{h}}^{2}}{2\pi}Q_{\mathrm{h}}\ln\frac{r_{+}}{l}. \tag{27}$$

Actually as we will show next, this is nothing but the charge flow associating with the Hawking thermal radiation.

Similarly, we can find the flux of energy-momentum tensor radiated from the charged and magnetized BTZ black hole. When the modes that propagate along light-like direction are excluded to formulate the effective field outside the EH, due to the ill-definition of the time coordinate, the energy-momentum tensor is divergent, hence the effective field suffers from gravitational anomaly that takes the form of the breakdown of the conservation of energy-momentum tensor. The covariant anomalous equation takes the form as[24]

$$\begin{aligned}\nabla_{\mu}\tilde{T}^{\mu}_{\nu} =&\frac{1}{96\pi\sqrt{-g}}g_{\nu\alpha}\varepsilon^{\beta\alpha}\partial_{\beta}R = \tilde{B}_{\nu}\\ =&\frac{1}{\sqrt{-g}}\partial_{\mu}\tilde{N}^{\mu}_{\nu}.\end{aligned} \tag{28}$$

For the 2-dimensional space-time, we find

$$\tilde{N}^{\mathrm{r}}_{\mathrm{t}} = [2ff'' - (f')^{2}]/192\pi. \tag{29}$$

As before, we can split the region outside the horizon into $[r_{+},\, r_{+}+\varepsilon]$ and $[r_{+}+\varepsilon,\infty]$ and write the covariant energy-momentum tensor as $\tilde{T}^{\mu}_{\nu} = \tilde{T}^{u}_{\nu(0)}\Theta_{+}(r) + \tilde{T}^{\mu}_{\nu(\mathrm{H})}H(r)$ by the scalar step function and scalar top hat function. Since the existence of an effective background gauge potential, the covariant energy-momentum tensor in the region $[r_{+}+\varepsilon,\,\infty]$ satisfies the modified conservation equation

$$\nabla_{\mu}\tilde{T}^{\mu}_{\nu(0)} = \tilde{F}_{\mu\nu}\tilde{J}^{\mu}_{(0)}. \tag{30}$$

Adopting Eq.(23), we can solve it as

$$\tilde{T}^{\mathrm{r}}_{\mathrm{t}(0)} = e_{0} + a_{0}\tilde{A}_{\mathrm{t}}, \tag{31}$$

where e_{0} is an integration constant that represents the value of energy-momentum tensor flux at infinity. On the other hand, in the near-horizon region $[r_{+},\, r_{+}+\varepsilon]$, there not only exists gauge anomaly but also exhibits gravitational anomaly, the Ward identity thus should be

$$\nabla_{\mu}\tilde{T}^{\mu}_{\nu(\mathrm{H})} = \tilde{F}_{\mu\nu}\tilde{J}^{\mu}_{(\mathrm{H})} + \tilde{B}_{\nu}. \tag{32}$$

Solving it by inserting Eq.(24), we obtain

$$\tilde{T}^{\mathrm{r}}_{\mathrm{t}(\mathrm{H})} = e_{\mathrm{H}} + \int_{r_{+}}^{\mathrm{r}}\mathrm{d}r\partial_{\mathrm{r}}\left[a_{0}\tilde{A}_{\mathrm{t}} + \frac{Q_{\mathrm{h}}^{2}}{4\pi}\tilde{A}_{\mathrm{t}}^{2} + \tilde{N}^{\mathrm{r}}_{\mathrm{t}}\right]. \tag{33}$$

To further get the value of e_{0}, we take into account the covariant anomalous equation with $\nu = t$ component outside the EH and find

$$\begin{aligned}\partial_{\mathrm{r}}\tilde{T}^{\mathrm{r}}_{\mathrm{t}} =&a_{0}\partial_{\mathrm{r}}\tilde{A}_{\mathrm{t}}(r) + \left[\partial_{\mathrm{r}}(\frac{Q_{\mathrm{h}}^{2}}{4\pi}\tilde{A}_{\mathrm{t}}^{2}(r) + \tilde{N}^{\mathrm{r}}_{\mathrm{t}})H\right]\\ &+\left(\tilde{T}^{\mathrm{r}}_{t(0)} - \tilde{T}^{\mu}_{\mathrm{t}(\mathrm{H})} + \frac{Q_{\mathrm{h}}^{2}}{4\pi}\tilde{A}_{\mathrm{t}}^{2} + \tilde{N}^{\mathrm{r}}_{\mathrm{t}}\right)\\ &\times\delta\left(r - r_{+} - \varepsilon\right).\end{aligned} \tag{34}$$

In the right-hand side of Eq.(34), the first term is the classical effect arising from the Lorenz force, the second term has to be cancelled by the quantum effects of the classically irrelevant ingoing modes that we have omitted previously. Therefore, only the coefficient of the delta function, which satisfies

$$e_{0} = e_{\mathrm{H}} + \frac{Q_{\mathrm{h}}^{2}}{4\pi}\tilde{A}_{\mathrm{t}}^{2}(r_{+}) - \tilde{N}^{\mathrm{r}}_{\mathrm{t}}(r_{+}), \tag{35}$$

can save the underlying general coordinate invariance at the quantum level. However this condition is still insufficient to fix the value of energy–momentum tensor flux at infinity, thus we impose the vanishing condition that requires the covariant flux to vanish at the EH. In this case, the total compensating flux of

energy-momentum tensor, which can cancel the gravitational anomaly at the EH to save the general coordinate covariance at the quantum level, can be written as

$$e_0 = \frac{Q_{\rm h}^2}{4\pi}\tilde{A}_{\rm t}^2(r_+) - \tilde{N}_{\rm t}^{\rm r}(r_+) = \frac{Q_{\rm h}^2}{4\pi}\tilde{A}_{\rm t}^2(r_+) + \frac{\pi T_+^2}{12}, \quad (36)$$

where

$$T_+ = \frac{\kappa}{2\pi} = \frac{f'(r)\big|_{r_+}}{4\pi} = \frac{2r_+^3 - l^4 Q_{\rm h}^2}{4\pi l^2 r_+^2} \quad (37)$$

is the Hawking temperature of the charged and magnetized BTZ black hole.

Next, we shall explore the relation between the compensating fluxes and those of Hawking radiation. At the EH, the Planck distribution of the charged and magnetized BTZ black hole with chemical potential takes the form as

$$N_{\pm Q_{\rm h}}(\omega) = 1 \Big/ \left(\exp\left(\frac{\omega \mp Q_{\rm h}\tilde{A}_{\rm t}(r_+)}{T_+}\right) \pm 1\right). \quad (38)$$

Integrating it, we find

$$\int_0^\infty \frac{Q_{\rm h}}{2\pi}[N_{Q_{\rm h}}(\omega) - N_{-Q_{\rm h}}(\omega)]{\rm d}\omega = \frac{Q_{\rm h}^2}{2\pi}Q_{\rm h}\ln\frac{r_+}{l}, \quad (39)$$

$$\int_0^\infty \frac{\omega}{2\pi}[N_{Q_{\rm h}}(\omega) + N_{-Q_{\rm h}}(\omega)]{\rm d}\omega = \frac{Q_{\rm h}^2}{4\pi}\tilde{A}_{\rm t}^2(r_+) + \frac{\pi T_+^2}{12}. \quad (40)$$

Comparing Eqs.(27) and (36) with Eqs.(39) and (40), we find that the compensating fluxes of charge flow and energy-momentum tensor, which are required to cancel the gauge and gravitational anomalies at the EH to restore the gauge and general covariance at the quantum level, precisely equal those of a (1+1)-dimensional blackbody radiation at the Hawking temperature.

4. Concluding remarks

We have extended the anomaly cancellation method to the charged and magnetized BTZ black hole and found that Hawking radiation from its EH also can be determined by the compensating fluxes. The core of our work is the definition of an equivalent charge and gauge potential corresponding to the source with electric and magnetic charges by reconstructing the electromagnetic field tensor. Actually, we can verify our work from the first law of black hole thermodynamics. It is well known that the general differential form of the first law of black hole thermodynamics is

$${\rm d}S_{\rm BH} = \frac{1}{T}{\rm d}M - \frac{V_{\rm e}}{T}{\rm d}Q_{\rm e} - \frac{V_{\rm g}}{T}{\rm d}Q_{\rm g}. \quad (41)$$

As for the charged and magnetized BTZ black hole

$$V_{\rm e} = -Q_{\rm e}\ln\frac{r}{l}, \quad V_{\rm g} = -Q_{\rm g}\ln\frac{r}{l}. \quad (42)$$

Considering the relations between the electric charge and magnetic charge in Eqs.(9) and (10), we can easily rewrite it as

$${\rm d}S_{\rm BH} = \frac{1}{T}({\rm d}M - V_0{\rm d}Q_{\rm h}). \quad (43)$$

where

$$V_0 = -Q_{\rm h}\ln\frac{r}{l}. \quad (44)$$

This is similar to that only with electric charge. That is to say, the substitution of an equivalent charge to the electric and magnetic charges is reasonable. Thus our work can be extended to the other black holes with electric charge and magnetic charge.

References

[1] Hawking S W 1974 *Nature* **248** 30
Hawking S W 1975 *Comm. Math. Phys.* **43** 199

[2] Gibbons G and Hawking S W 1977 *Phys. Rev.* D **15** 2752

[3] Strominger A and Vafa C 1996 *Phys. Lett.* B **379** 99

[4] Gibbons G and Perry M J 1978 *Proc. R. Soc. Lond.* A **358** 467

[5] Parikh M K and Wiclzek F 2000 *Phys. Rev. Lett.* **85** 5042
Zhang J Y and Zhao Z 2005 *Phys. Lett.* B **618** 14
Han Y W 2005 *Acta Phys. Sin.* **54** 5018 (in Chinese)
Yang S Z, Jiang Q Q and Li H L 2005 *Chin. Phys.* **14** 2411
Han Y W 2007 *Chin. Phys.* **16** 923
Chen D Y, Jiang Q Q and Yang S Z 2006 *Chin. Phys.* **15** 1709
Jiang Q Q and Wu S Q 2006 *Acta Phys. Sin.* **55** 4428 (in Chinese)
Jiang Q Q, Li H L and Yang S Z 2005 *Chin. Phys.* **14** 1736
Cao J L, Ren J and Zhao Z 2006 *Chin. Phys.* **15** 2256

[6] Robinson S P and Wilczek F 2005 *Phys. Rev. Lett.* **95** 011303

[7] Iso S, Umetsu H and Wilczek F 2006 *Phys. Rev. Lett.* **96**

151302
[8] Iso S, Umetsu H and Wilczek F 2006 *Phys. Rev.* D **74** 044017
[9] Murata K and Soda J 2006 *Phys. Rev.* D **74** 044018
[10] Jiang Q Q, Wu S Q and Cai X 2007 *Phys. Lett.* B **651** 65
Jiang Q Q, Wu S Q and Cai X 2007 *Phys. Lett.* B **651** 58
Zeng X X, Chen D Y and Yang S Z 2008 *Chin. Phys.* **17** 1629
[11] Shin H and Kim W 2007 *High Energy Phys.* **06** 012
[12] Lin K and Zeng X X 2008 *Chin. Phys. Lett.* **25** 309
[13] Xiao K, Liu W B and Zhang H B 2007 *Phys. Lett.* B **647** 482
[14] Setare M R 2006 *Euro. Phys J.* C **49** 865
[15] Jiang Q Q, Wu S Q and Cai X 2007 *Phys. Rev.* D **75** 064029
[16] Xu Z and Chen B 2007 *Phys Rev.* D **75** 024041
[17] Zhang J Y and Fan J H 2007 *Phys. Lett.* B **648** 133
[18] Medved A J M and Kunstatter G 2001 *Phys. Rev.* D **63**104005
[19] Buric M, Dimitrijevic M and Radovanovic V 2002 *Phys. Rev.* D **65** 064022
[20] Aharony O, Gubser S S, Maldacena J M, Ooguri H and Oz Y 2000 *Phys. Rep.* **323** 183
[21] Sfetsos K and Skenderis K 1998 *Nucl. Phys.* B **517** 179
[22] Bañados M, Teitelboim C and Zanelli J 1992 *Phys. Rev. Lett.* **69** 1849
[23] Banerjee R and Kulkarni S 2008 *Phys. Rev.* D **77** 024018
[24] Bardeen W A and Zumino B 1984 *Nucl. Phys.* B **244** 421
Alvarez-Gaume L and Witten E 1984 *Nucl. Phys.* B **234** 269
Bertlmann R and Kohlprath E 2001 *Ann. Phys.* (N.Y.) **288** 137