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# Information entropy for static spherically symmetric black holes*

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By using the new equation of state density derived from the generalized uncertainty relation, the number of the quantum states near event horizon is obtained, with which then the information entropy of static spherically symmetric black holes has been discussed. It is found that the divergent integral of quantum states near the event horizon can be naturally avoided if using the new equation of state density without introducing the ultraviolet cut-off. The information entropy of black holes can be obtained precisely by the residue theorem, which is shown to be proportional to the horizon area. The information entropy of black holes obtained agrees with the Bechenstein–Hawking entropy when the suitable cutoff factor is adopted.

**Keywords:** black hole, event horizon, generalized uncertainty relation, information entropy
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## 1. Introduction

Since Hawking proved that black holes generate thermal radiation, the thermodynamic properties of black holes have been always the important researching field in theoretic physics. Especially, the research on the black hole entropy becomes a hotspot.[1−4] 't Hooft first studied the statistical entropy in scalar field in 1985 by using the WKB approximation method. In order to avoid the divergent integral of the state density near the event horizon and at infinity, he put forward the brick-wall model.[1] Li and Zhao,[5] as well as Liu and Zhao[6] further improved the brick-wall model to the thin film model. This thin film model provides an effective method to study black holes entropy under the condition of non-equilibrium state. Using this thin film model, the statistical entropy for black holes in the scalar field and Dirac field is successfully calculated;[7−10] and adopting this model, the integral of the state density at infinity need not be calculated. Therefore, when we calculate black holes entropy, the divergence of the infrared integration is naturally avoided. In order to avoid the divergent integral near the event horizon, the ultraviolet cut-off is again introduced. Recently, researchers have obtained the non-commutative quantum field theory by combining the non-commutative geometry and the string theory, and then obtained the generalized uncertainty relation. It is found that the black hole entropy is still proportional to the horizon area if using the thin film model and the generalized uncertainty relation and the divergent integral near the event horizon can be avoided too.[11,12] According to the theory of holography, the whole information inside the black holes can be regarded as being concentrated on the event horizon, and thus the quantum states on the horizon should be equal to that inside the black holes and contain the whole information of them. Therefore, the entropy of the quantum states near event horizon is, in essence, the information entropy of black holes. By using this characteristic of the information entropy for black holes, the information of black holes can be indirectly obtained by studying the quantum states near the event horizon, although one cannot directly obtain this information. In this paper, based on the above viewpoint and using the four-dimensional invariant line element, we study the information entropy of the static spherically symmetric black holes through calculating the quantum states by using the new equation of state density derived from the generalized uncertainty relation. It is found that the information entropy of black holes is proportional to the horizon area too. When the suitable cut-off factor is adopted, the information entropy of black holes will be consistent with the Bechenstein–Hawking entropy.

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## 2. The generalized uncertainty relation and the new equation of state density

The coordinate and momentum of microcosmic particles satisfy the Heisenberg uncertainty relation in quantum mechanics

$$\Delta x \Delta p \geq \hbar/2, \tag{1}$$

where $\hbar = h/2\pi$, $h$ is Planck constant. Based on statistical physics, the number of quantum states in the volume element at infinity in the phase space can be expressed as

$$\mathrm{d}N = (\mathrm{d}^3x\mathrm{d}^3p)/(2\pi\hbar)^3. \tag{2}$$

On the Planck scale, the uncertainty relation of Eq.(1) is not satisfied. Under such a condition, equation (1) should be improved to the generalized uncertainty relation

$$\Delta x \Delta p \geq \hbar/2 \left[1 + \lambda\langle p^2\rangle + \beta\langle x^2\rangle + \ldots\right]. \tag{3}$$

According to the generalized uncertainty principle, the uncertainty relation of generalized coordinate and momentum can be expressed as follows:[13,14]

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left[1 + \lambda\langle p^2\rangle\right] = \frac{1}{2}\left[\hbar + \frac{\lambda}{\hbar}(\Delta p)^2\right], \tag{4}$$

where $\sqrt{\lambda}$ is of an order of the Planck length,[15] and $\lambda \propto l_{\mathrm{p}}^2$. Therefore, according to the generalized uncertainty principle, the length of the cell in the phase space should be $2\pi\hbar\left(1 + \lambda p^2\right)$. From Eq.(4), the smallest length of the $\Delta x$ is

$$\Delta x_{\min} = \sqrt{\lambda}. \tag{5}$$

Based on the generalized uncertainty relation, the number of quantum states in the infinite volume element in the phase space should be expressed as

$$\mathrm{d}N = (\mathrm{d}^3x\mathrm{d}^3p)/[(2\pi\hbar)^3\left(1 + \lambda p^2\right)^3]. \tag{6}$$

The four-dimensional invariant line element of non-extreme static spherically symmetric black holes in four-dimensional spacetime is

$$\mathrm{d}s^2 = -g_{tt}\mathrm{d}t^2 + g_{tt}^{-1}\mathrm{d}r^2 + R^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right), \tag{7}$$

where the component of the metric is generally expressed as $g_{tt} = G(r)(r - r_H)$, $R = R(r)$, $r_{\mathrm{H}}$ is the position of the external event horizon. According to Eq.(7), the corresponding determinant and contravariant form of the metric are

$$g = -R^4\sin^2\theta, \tag{8}$$

$$g^{tt} = -g_{tt}^{-1}, \quad g^{rr} = g_{tt},$$
$$g^{\theta\theta} = R^{-2} \quad g^{\varphi\varphi} = R^{-2}\sin^{-2}\theta. \tag{9}$$

Substituting Eqs.(8) and (9) into the massless Klein–Gordon equation, we can obtain

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial\Phi}{\partial x^\nu}\right) = 0, \tag{10}$$

Using the separation of variables and setting $\Phi = \mathrm{e}^{\mathrm{i}\omega t}\Psi(r,\theta,\varphi)$, then we can rewrite Eq.(10) as

$$\frac{\partial^2\Psi}{\partial r^2} + \left[\frac{1}{\sqrt{-g}}\frac{\partial\sqrt{-g}}{\partial r} + \frac{1}{g^{rr}}\frac{\partial g^{rr}}{\partial r}\right]\frac{\partial\Psi}{\partial r}$$
$$+ \frac{1}{g^{rr}}\left[-g^{tt}\omega^2 + g^{\theta\theta}\frac{\partial}{\partial\theta^2} + \left(\frac{1}{\sqrt{-g}}g^{\theta\theta}\frac{\partial\sqrt{-g}}{\partial\theta} + \frac{\partial g^{\theta\theta}}{\partial\theta}\right)\frac{\partial}{\partial\theta} + g^{\varphi\varphi}\frac{\partial^2}{\partial\varphi^2}\right]\Psi = 0. \tag{11}$$

Adopting the WKB approximation method and setting $\Psi = \mathrm{e}^{\mathrm{i}S(r,\theta,\varphi)}$, we can obtain the following equation from Eq.(11),

$$p_r^2 = \frac{1}{g^{rr}}\left(g_{tt}^{-1}\omega^2 - g^{\theta\theta}p_\theta^2 - g^{\varphi\varphi}p_\varphi^2\right), \tag{12}$$

where $p_r = \partial S/\partial r$, $p_\theta = \partial S/\partial\theta$, $p_\varphi = \partial S/\partial\varphi$. From Eq.(12), we can easily obtain

$$p^2 = g_{tt}^{-1}\omega^2. \tag{13}$$

By using Eq.(6), the number of quantum states whose energy is less than $\omega$ can be obtained:

$$\begin{aligned}\Gamma(\omega)=&\frac{1}{(2\pi)^3}\int\frac{\mathrm{d}r\mathrm{d}\theta\mathrm{d}\varphi\mathrm{d}p_r\mathrm{d}p_\theta\mathrm{d}p_\varphi}{\left(1+\lambda g_{tt}^{-1}\omega^2\right)^3}\\=&\frac{1}{(2\pi)^3}\int\frac{\mathrm{d}r\mathrm{d}\theta\mathrm{d}\varphi}{\left(1+\lambda g_{tt}^{-1}\omega^2\right)^3}\int\frac{2}{\sqrt{g^{rr}}}\left(g_{tt}^{-1}\omega^2-g^{\theta\theta}p_\theta^2-g^{\varphi\varphi}p_\varphi^2\right)^{1/2}\mathrm{d}p_\theta\mathrm{d}p_\varphi\\=&\frac{4\pi\omega^3}{3\,(2\pi)^3}\int\frac{R^2\mathrm{d}r}{g_{tt}^2\left(1+\lambda\omega^2/g_{tt}\right)^3}\int\sin\theta\mathrm{d}\theta\mathrm{d}\varphi=\frac{2\omega^3}{3\pi}\int\frac{R^2\mathrm{d}r}{g_{tt}^2\left(1+\lambda g_{tt}^{-1}\omega^2\right)^3}.\end{aligned}\tag{14}$$

## 3. The information entropy for the static spherically symmetric black hole

Based on canonical ensemble, the free energy of a system can be obtained from Eq.(14):

$$\begin{aligned}F=&\frac{1}{\beta}\int\mathrm{d}\Gamma(\omega)\ln\left(1-\mathrm{e}^{-\beta\omega}\right)\\=&-\int_0^\infty\frac{\Gamma(\omega)\,\mathrm{d}\omega}{\mathrm{e}^{\beta\omega}-1}\\=&-\frac{2}{3\pi}\int\frac{R^2\mathrm{d}r}{g_{tt}^2}\int_0^\infty\frac{\omega^3\mathrm{d}\omega}{\left(\mathrm{e}^{\beta\omega}-1\right)\left(1+\lambda\omega^2/g_{tt}\right)^3}.\end{aligned}\tag{15}$$

Then the information of black holes can be obtained using the relation between the entropy and the free energy

$$\begin{aligned}S=&\beta^2\frac{\partial F}{\partial\beta}\\=&\frac{2\beta^2}{3\pi}\int\frac{R^2\mathrm{d}r}{g_{tt}^2}\int_0^\infty\frac{\mathrm{e}^{\beta\omega}\omega^4\mathrm{d}\omega}{\left(\mathrm{e}^{\beta\omega}-1\right)^2\left(1+\lambda\omega^2/g_{tt}\right)^3}\\=&\frac{2}{3\pi\lambda^{3/2}}\int R^2g_{rr}^{1/2}\mathrm{d}r\\&\times\int_0^\infty\frac{b^2x^4\mathrm{d}x}{\left(\mathrm{e}^{bx/2}-\mathrm{e}^{-bx/2}\right)^2\left(x^2+1\right)^3},\end{aligned}\tag{16}$$

where we have used the variable substitution $x^2=\lambda\omega^2/g_{tt}$, $b=\beta\sqrt{g_{tt}/\lambda}$. Near the event horizon, $g_{tt}\to0$, so $b\to0$. From Eq.(16), it can be seen that there is no divergent integral of the entropy on the event horizon $r=r_{\mathrm{H}}$. Therefore, the ultraviolet cut-off can be avoided and the lower limit of the integral can be set to the position of the event horizon. On the other hand, because black holes entropy should have a determined value and not change with the integral limit, it is unreasonable that the information entropy changes with the integral upper limit and associated with volume[1,10] from Eq.(16). According to holography theory, the whole information in the inside of black holes can be regarded as concentrated on the event horizon, thus the contribution of the quantum states near the event horizon to black holes entropy is most significant, especially the contribution from the quantum states on the event horizon. Therefore, the integral lower limit should be set at position of the event horizon. Since the whole information in the inside of black holes can be regarded as concentrated on the event horizon, the information changing with the integral limit and associated with the volume should not be black hole information. When we adopt the thin film model to calculate the quantum states, the thickness of the thin film should not be too big. We think that when the obtained information entropy is equal to the Bekenstein–Hawking entropy, the integral upper limit is the most suitable choice. Based on the above consideration, when we calculate the information entropy of black holes, the integral limit should be $[r_{\mathrm{H}},\ r_{\mathrm{H}}+\alpha]$, where $\alpha$ is a small quantity, namely the cut-off of the thickness of thin film. From Eq.(16), the information entropy of black holes can be expressed as

$$\begin{aligned}S=&\frac{2}{3\pi\lambda^{3/2}}\int_{r_{\mathrm{H}}}^{r_{\mathrm{H}}+\alpha}R^2g_{rr}^{1/2}\mathrm{d}r\\&\times\int_0^\infty\frac{b^2x^4\mathrm{d}x}{\left(\mathrm{e}^{bx/2}-e^{-bx/2}\right)^2\left(x^2+1\right)^3}.\end{aligned}\tag{17}$$

Set

$$f(x)=2\int_0^\infty\frac{b^2x^4\mathrm{d}x}{\left(\mathrm{e}^{bx/2}-\mathrm{e}^{-bx/2}\right)^2\left(x^2+1\right)^3}.\tag{18}$$

Since the integral function of $f(x)$ is an even function, we integrate Eq.(18) for $x$ and obtain

$$f(x)=\int_{-\infty}^\infty\frac{b^2x^4\mathrm{d}x}{\left(\mathrm{e}^{bx/2}-\mathrm{e}^{-bx/2}\right)^2\left(x^2+1\right)^3}.\tag{19}$$

In order to obtain the accurate value of Eq.(19), we adopt the complex function and integrate for the upper half complex plane.

$$f(Z)=\frac{b^2Z^4}{\left(\mathrm{e}^{bZ/2}-\mathrm{e}^{-bZ/2}\right)^2\left(Z^2+1\right)^3}.\tag{20}$$

Since $Z = 0$ is a removable singularity and $Z = 2k\pi \mathrm{i}/b$, $(k = 1, 2\ldots)$ is a series of second-order singularity, the residue of the upper half complex plane is

$$\begin{aligned}&\operatorname{Res} f\left(2k\pi \mathrm{i}/b\right)\\ =&-2\mathrm{i}\sum_{k}\left(b/2k\pi\right)^{3}\frac{\left(b/2k\pi\right)^{2}+2}{\left[\left(b/2k\pi\right)^{2}-1\right]^{4}}.\end{aligned}\tag{21}$$

When $b\left(r_{\mathrm{H}}\right)=\beta\sqrt{g_{tt}/\lambda}\to 0$ near the event horizon, the residue has no contribution to the integration of the upper half complex plane. Therefore, the contribution to the integration is $Z=\mathrm{i}$ under the condition $b\neq 2\pi k$, $(k=1,2\ldots)$, the corresponding residue is

$$\begin{aligned}\operatorname{Res} f\left(\mathrm{i}\right)=&-\mathrm{i}\frac{1}{2^{7}\sin^{4}\dfrac{b}{2}}\left[b^{4}\left(1+2\cos^{2}\frac{b}{2}\right)\right.\\ &\left.-5b^{3}\sin b+6b^{2}\sin^{2}\frac{b}{2}\right].\end{aligned}\tag{22}$$

Near the event horizon, $b\to 0$, the sum residue of all the singularities of the $f(Z)$ on the upper half complex plane can be obtained

$$\operatorname{Res} f\left(Z\right)=-\frac{1}{16}\mathrm{i}.\tag{23}$$

Integrating Eq.(19) by residue theorem, we can obtain

$$f\left(x\right)=\int_{-\infty}^{\infty}\frac{b^{2}x^{4}\mathrm{d}x}{\left(\mathrm{e}^{bx/2}-\mathrm{e}^{-bx/2}\right)^{2}\left(x^{2}+1\right)^{3}}=\frac{\pi}{8}.\tag{24}$$

Substituting Eq.(24) into Eq.(17),the information entropy of spherically symmetric black holes is written as

$$\begin{aligned}S\approx&\frac{1}{24\lambda^{3/2}}\int_{r_{\mathrm{H}}}^{r_{\mathrm{H}}+\alpha}R^{2}\left(r_{\mathrm{H}}\right)g_{rr}^{1/2}\mathrm{d}r\\ =&\frac{A_{\mathrm{H}}}{96\pi\lambda^{3/2}}\delta,\end{aligned}\tag{25}$$

where $A_{\mathrm{H}}=4\pi R^{2}\left(r_{\mathrm{H}}\right)$ is the event horizon area, $\delta$ is the distance between the integral upper limit and the event horizon. It can be expressed as

$$\delta=\int_{r_{\mathrm{H}}}^{r_{\mathrm{H}}+\alpha}\sqrt{g_{rr}}\mathrm{d}r.\tag{26}$$

It is evident that the information entropy of spherically symmetric black holes is proportional to the horizon area. Equation (25) is in good agreement with the result of Refs.[16,17], which represents the black hole entropy related with cut-off factor by using the statistical method. When suitable cut-off factor is adopted, equation (25) will represent the Bekenstein–Hawing entropy. Therefore, equation (25) has a more profound significance and most universality.

# 4. Conclusion

We firstly discuss the universality of Eq.(25). It can be seen that Eq.(25) is universally true for static spherically symmetric black holes. In order to make out its physical meaning clearly, the smallest length can be expressed as

$$\begin{aligned}\sqrt{\lambda}=&\int_{r_{\mathrm{H}}}^{r_{\mathrm{H}}+\varepsilon}\sqrt{g_{rr}}\mathrm{d}r\\ \approx&\int_{r_{\mathrm{H}}}^{r_{\mathrm{H}}+\varepsilon}\frac{\mathrm{d}r}{\sqrt{2\kappa\left(r-r_{\mathrm{H}}\right)}}\\ =&\sqrt{\frac{2\varepsilon}{\kappa}},\end{aligned}\tag{27}$$

where $\varepsilon$ is a small quantity, $\kappa=\frac{1}{2}G\left(r_{\mathrm{H}}\right)$ is the Hawking temperature on the event horizon. It is evident that a smallest cut-off factor will be defined by Eq.(27), which is determined by the constant $\lambda$. We consider that the $\varepsilon$ corresponding to the smallest length of $\sqrt{\lambda}$ is just the smallest cut-off factor. For an arbitrary truncation length, it will be integral multiple of the $\varepsilon$. So the distance between the integral upper limit and the event horizon (truncation distance) can be expressed as

$$\delta=\int_{r_{\mathrm{H}}}^{r_{\mathrm{H}}+n\varepsilon}\sqrt{g_{rr}}\mathrm{d}r.\tag{28}$$

It is found that equation (27) has a certain limitation. However, equation (28) containing the parameter $n$ has most extensive universality as compared with Eq.(27). When $n=1$, Eq.(28) naturally becomes Eq.(27).

## 4.1. The information entropy of the Schwarzschild black hole

In the Schwarzschild spacetime, the four-dimensional invariant line element is

$$\begin{aligned}\mathrm{d}s^{2}=&-\left(1-\frac{2M}{r}\right)\mathrm{d}t^{2}+\left(1-\frac{2M}{r}\right)^{-1}\mathrm{d}r^{2}\\ &+r^{2}\mathrm{d}\theta^{2}+r^{2}\sin^{2}\theta\mathrm{d}\varphi^{2}.\end{aligned}\tag{29}$$

By using Eq.(28), the distance between the truncation and the event horizon can be expressed as

$$\begin{aligned}\delta =& \int_{r_{\rm H}}^{r_{\rm H}+\alpha} \sqrt{g_{rr}}{\rm d}r \\ =& \int_{r_{\rm H}}^{r_{\rm H}+n\varepsilon} \frac{{\rm d}r}{\sqrt{1-2M/r}} \\ =& \int_{r_{\rm H}}^{r_{\rm H}+n\varepsilon} \frac{r{\rm d}r}{\sqrt{r^2-2Mr}} \\ \approx& \sqrt{\frac{2n\varepsilon}{\kappa}}\end{aligned} \tag{30}$$

where $r_{\rm H}=2M$ and $\kappa=2r_{\rm H}$. Substituting Eq.(30) into Eq.(25), the information entropy of the Schwarzschild black hole is

$$S=\frac{\sqrt{n}A_{\rm H}}{96\pi\lambda}, \tag{31}$$

where $A_{\rm H}=4\pi r_{\rm H}^2$ is the event horizon area of the Schwarzschild black hole.

## 4.2. The information entropy of the Reissner–Nordstorm black hole

In the Reissner–Nordstorm spacetime, the four-dimensional invariant line element is

$$\begin{aligned}{\rm d}s^2 =& -\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right){\rm d}t^2 \\ &+\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-1}{\rm d}r^2 \\ &+r^2{\rm d}\theta^2+r^2\sin^2\theta{\rm d}\varphi^2.\end{aligned} \tag{32}$$

By using Eq.(28), the distance between the truncation and the event horizon can be expressed as

$$\begin{aligned}\delta =& \int_{r_{\rm H}}^{r_{\rm H}+\alpha} \sqrt{g_{rr}}{\rm d}r \\ =& \int_{r_{\rm H}}^{r_{\rm H}+n\varepsilon} \frac{{\rm d}r}{\sqrt{1-2M/r+Q^2/r^2}} \\ =& \int_{r_{\rm H}}^{r_{\rm H}+n\varepsilon} \frac{r{\rm d}r}{\sqrt{r^2-2Mr+Q^2}} \\ \approx& \sqrt{\frac{2n\varepsilon}{\kappa}},\end{aligned} \tag{33}$$

where $r_{\rm H}=M+\sqrt{M^2-Q^2}$, $\kappa=(r_{\rm H}-M)/r_{\rm H}^2$. Substituting Eq.(33) into Eq.(25), the information entropy of the Reissner–Nordstorm black hole is

$$S=\frac{\sqrt{n}A_{\rm H}}{96\pi\lambda}, \tag{34}$$

where $A_{\rm H}=4\pi r_{\rm H}^2$ is the event horizon area of the Reissner–Nordstorm black hole.

## 4.3. The information entropy of the G-M dilaton black hole

In the static G-M dilaton spacetime, the four-dimensional invariant line element is[19]

$$\begin{aligned}{\rm d}s^2 =& -\frac{(r-r_+)(r-r_-)}{R^2}{\rm d}t^2 \\ &+\frac{R^2}{(r-r_+)(R-r_-)}{\rm d}r^2 \\ &+R^2\left({\rm d}\theta^2+\sin^2\theta{\rm d}\varphi^2\right),\end{aligned} \tag{35}$$

where $R^2=r^2-D^2$, $D=\left({\rm e}^2-Q^2\right)/2M$. The truncation distance can be obtained from Eq.(28)

$$\begin{aligned}\delta =& \int_{r_{\rm H}}^{r_{\rm H}+n\varepsilon} \sqrt{g_{rr}}{\rm d}r \\ \approx& \int_{r_{\rm H}}^{r_{\rm H}+n\varepsilon} \frac{{\rm d}r}{\sqrt{2\kappa(r-r_{\rm H})}} \\ =& \sqrt{\frac{2n\varepsilon}{\kappa}},\end{aligned} \tag{36}$$

where

$$\kappa=\frac{(r_{\rm H}-r_-)}{r_{\rm H}^2-D^2},$$

$$r_{\rm H}=M+\sqrt{M^2+D^2-{\rm e}^2-Q^2},$$

$$r_-=M-\sqrt{M^2+D^2-{\rm e}^2-Q^2}.$$

Substituting Eq.(36) into Eq.(25), the information entropy of the static G-M dilaton black hole is

$$S=\frac{\sqrt{n}A_{\rm H}}{96\pi\lambda}, \tag{37}$$

where $A_{\rm H}=4\pi\left(r^2+D^2\right)$ is the event horizon area of the static G-M dilaton black hole.

## 4.4. The information entropy of the Garfinkle–Horowitz–Strominger dilaton black hole

In the static Garfinkle–Horowitz–Strominger dilaton spacetime, the four dimensional invariant line element is[20]

$$\begin{aligned}{\rm d}s^2 =& -\left(1-\frac{2M}{r}\right){\rm d}t^2+\left(1-\frac{2M}{r}\right)^{-1}{\rm d}r^2 \\ &+r(r-a)({\rm d}\theta^2+\sin^2\theta{\rm d}\varphi^2),\end{aligned} \tag{38}$$

where $a=Q^2/2M{\rm e}^{-2\Phi_0}$. The truncation distance can

be obtained from Eq.(28)

$$\begin{aligned}\delta &= \int_{r_{\rm H}}^{r_{\rm H}+\alpha} \sqrt{g_{rr}}{\rm d}r \\ &= \int_{r_{\rm H}}^{r_{\rm H}+n\varepsilon} \frac{{\rm d}r}{\sqrt{1-2M/r}} \\ &= \int_{r_{\rm H}}^{r_{\rm H}+n\varepsilon} \frac{r{\rm d}r}{\sqrt{r^2-2Mr}} \\ &\approx \sqrt{\frac{2n\varepsilon}{\kappa}}, \end{aligned} \tag{39}$$

where $r_{\rm H} = 2M$ and $\kappa = 2r_{\rm H}$. Substituting Eq.(39) into Eq.(25), the information entropy of the static G-H-S dilaton black hole is

$$S = \frac{\sqrt{n}A_{\rm H}}{96\pi\lambda}, \tag{40}$$

where $A_{\rm H} = 4\pi r_{\rm H}\left(r_{\rm H} - a\right)$ is the event horizon area of static Garfinkle–Horowitz–Strominger dilaton black hole.

From the above discussion, it can be seen that the information entropy of spherically symmetric black holes has the same form and is proportional to the event horizon area. When we set $\lambda = l_{\rm p}^2$ in Eq.(40) and choose a suitable truncation which can satisfy $n = 576\pi^2$, the information entropy of spherically symmetric black holes can be expressed as

$$S = \frac{A_{\rm H}}{4l_{\rm p}^2}. \tag{41}$$

Therefore, if the suitable cut-off factor is adopted, the information entropy of spherically symmetric black holes will be consistent with the Bekenstein–Hawking entropy. In the natural unit, the information entropy is equal to one quarter of the event horizon area

$$S = \frac{A_{\rm H}}{4}. \tag{42}$$

At present in many researches on the black hole entropy, all obtained results indicate that the black hole entropy is proportional to the event horizon area. Our results further indicate that the black hole entropy mainly comes from the contribution of the quantum states near the event horizon, and is a kind of quantum effect near the event horizon. In a word, the black hole entropy essentially reflects the influence of the information in the inside of black holes on the outside, and further reveals that the information in the inside of black holes can be reflected to the outside through the event horizon.

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