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The influence of electric field on a parabolic quantum dot qubit *

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This paper calculates the time evolution of the quantum mechanical state of an electron by using variational method of Pekar type on the condition of electric–LO-phonon strong coupling in a parabolic quantum dot. It obtains the eigenenergies of the ground state and the first-excited state, the eigenfunctions of the ground state and the first-excited state This system in a quantum dot may be employed as a two-level quantum system qubit. The superposition state electron density oscillates in the quantum dot with a period when the electron is in the superposition state of the ground and the first-excited state. It studies the influence of the electric field on the eigenenergies of the ground state, the first-excited state and the period of oscillation at the different electron–LO-phonon coupling constant and the different confinement length.

Keywords: quantum dot, quantum information, qubit
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1. Introduction

During the past decade, the study of quantum computing and quantum information processing has generated widespread interest among physicists from areas ranging from atomic physics and optics, to various branches of condensed matter physics.[1,2] The key thrust behind the rush toward a working quantum computer (QC) is the development of a quantum algorithm that can factorize large numbers exponentially faster than any available classical algorithm.[3] This exponential speedup is due to the intrinsic quantum parallelism in the superposition principle and the unitary evolution of quantum mechanics. It implies that a computer made up of entirely by quantum mechanical parts (qubit), whose evolution is governed by quantum mechanics. The two-level system is usually employed as the elementary unit for storing information. Quantum computation will be realized by the laws of quantum mechanics. Several schemes have been proposed for realizing QC in recent years.[4−9] In order to show the advantage of QC over the most classical computer, QC need to be composed of at least thousands of qubits to be feasible. Consequently, it is clear that QC with a large significant number of qubits would be more realizable in solid-state systems. However, self-assembled quantum dots (QDs) have been attracted substantial attention due to their perfect crystal structures. Therefore, it is one of the most popular solid-state quantum information research fields that qubits can be realized by solid-state devices. Many schemes have been proposed for researching into quantum dot and have many kinds of contents, but they are in the initial research stage at present.

In this article, we obtained the eigenenergies and their relevant eigenwavefunctions of the ground and the first-excited state of an electron in a parabolic quantum dot (QD) using the Pekar variational method considering the parabolic potential in the electron–LO-phonon strong-coupling region. A single qubit can be envisaged as this kind of two-level quantum system in a QD. For this single-electron QD qubit, Li *et al* presented a kind of parameter-phase diagram schemes and defined the parameters region for the use of the an InAs/GaAs as a two-level quantum system[8−9]. We have obtained the probability density of electron oscillating with a period when electron was in a superposition state of the ground and first-excited state. The relations of the period of oscillation on the electric

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field, the electron–LO-phonon coupling constant and the confinement length are derived. Our results should be meaningful for designing the solid-state implementation of quantum computing both theoretically and experimentally.

2. Theoretical model

We consider the system which the electrons are bounded by the parabolic potential. The electrons are much more confined in one direction (taken as the Z direction) than in other two directions. Therefore, we shall confine ourselves to consider only the effect of electron and LO-phonon, and the electron moving on the $X-Y$ plane. We assume that the confining potential in a single QD is parabolic.

$$V_{(\rho)} = \frac{1}{2} m^* \omega_0^2 \rho^2, \tag{1}$$

where m^* is the band mass of electron, ρ is the two-dimensional coordinate vector and ω_0 is the confinement strength. The Hamiltonian of electron-phonon system in presence of an electric field F along the x axis can be written as

$$\begin{aligned} H =& -\frac{\hbar^2}{2m^*}\nabla_\rho^2 + \frac{1}{2} m^* \omega_0^2 \rho^2 \\ &+ \sum_q \hbar\omega_{\mathrm{LO}} b_q^+ b_q \\ &+ \sum_q (V_q \mathrm{e}^{\mathrm{i}q\cdot r} b_q + \mathrm{h\cdot c}) - e^* F x, \end{aligned} \tag{2}$$

where $b_q^+(b_q)$ is the creation (annihilation) operator of bulk LO-phonon with the wave vector $q(q = q_{//}, q_\perp)$, $r = (\rho, z)$ is the coordinate of the electron, e^* is the electron charge.

$$V_q = \mathrm{i}(\hbar\omega_{\mathrm{LO}}/q)(\hbar/2m^*\omega_{\mathrm{LO}})^{1/4}(4\pi\alpha/V)^{1/2}, \tag{3}$$

$$\alpha = (e^2/2\hbar\omega_{\mathrm{LO}})(2m^*\omega_{\mathrm{LO}}/\hbar)^{1/2}(1/\varepsilon_\infty - 1/\varepsilon_0). \tag{4}$$

Using the following Lee–Low–Pines transformation to Eq.(2):

$$U = \exp\left[\sum_q (f_q b_q^+ - f_q^* b_q)\right], \tag{5}$$

where f_q will be treated as a variational function, we have

$$H' = U^{-1} H U. \tag{6}$$

Suppose that the Gaussian function approximation is valid in the ground-state of electron-phonon system by variational method of Pekar type:

$$\left|\varphi_{\mathrm{e-p}}\right\rangle = \frac{\lambda}{\sqrt{\pi}} \exp\left(-\frac{\lambda^2\rho^2}{2}\right) \left|\xi(z)\right\rangle \left|0_{\mathrm{ph}}\right\rangle, \tag{7}$$

where λ is the variational parameter. Since the electrons are much more strongly confined in Z direction than in other two directions and considered to be confined in an infinitesimally narrow layer, so$\langle\xi(z)|\,\xi(z)\rangle = \delta(z)$. $|0_{\mathrm{ph}}\rangle$is unperturbed zero phonon state which satisfies $b_q\,|0_q\rangle = 0$. We then obtain the electron ground-state energy

$$\begin{aligned} E_0(\lambda) =& \langle\,\Psi_{\mathrm{e-p}}|\,H'\,|\,\Psi_{\mathrm{e-p}}\rangle \\ =& \frac{\hbar^2\lambda^2}{2m^*} + \frac{\hbar^2}{2m^*\lambda^2 l_0^4} - \frac{1}{2}(2\pi)^{1/2}\alpha\hbar\omega_{\mathrm{LO}} r_0 \lambda \\ &- \frac{e^* F}{2\sqrt{\pi}\lambda}. \end{aligned} \tag{8}$$

The ground state energy of electron in a parabolic QD can be written as

$$\begin{aligned} E_0(\lambda_0) =& \frac{\hbar^2\lambda_0^2}{2m^*} + \frac{\hbar^2}{2m^*\lambda_0^2 l_0^4} - \frac{1}{2}(2\pi)^{1/2}\alpha\hbar\omega_{\mathrm{LO}} r_0 \lambda_0 \\ &- \frac{e^* F}{2\sqrt{\pi}\lambda_0}, \end{aligned} \tag{9}$$

where $l_0 = (\hbar/m^*\omega_0)^{1/2}$, $r_0 = (\hbar/2m^*\omega_{\mathrm{LO}})^{1/2}$, l_0, r_0 is the confinement length and the polaron radius, respectively.

Similarly, the trial wave-function of electron-phonon system in the first-excited state may be chosen as

$$\begin{aligned} |\varphi_{\mathrm{e-p}}\rangle' =& \frac{\lambda^2}{\sqrt{\pi}}\rho\exp\left(-\frac{\lambda^2\rho^2}{2}\right) \\ &\times \exp(\pm\mathrm{i}\phi)\,|\xi(z)\rangle\,|0_{\mathrm{ph}}\rangle\,. \end{aligned} \tag{10}$$

This satisfies the following relations:

$$\begin{aligned} \langle\varphi_{\mathrm{e-p}}|\,\varphi_{\mathrm{e-p}}\rangle' &= 0, \\ {}'\langle\varphi_{\mathrm{e-p}}|\,\varphi_{\mathrm{e-p}}\rangle' &= 1. \end{aligned} \tag{11}$$

We can obtain the energy in the first-excited state by using the $E_1(\lambda) = {}'\langle\varphi_{\mathrm{e-p}}|\,H'\,|\varphi_{\mathrm{e-p}}\rangle$,

$$\begin{aligned} E_1(\lambda) =& \frac{\hbar^2\lambda^2}{m^*} + \frac{\hbar^2}{m^*\lambda^2 l_0^4} - \frac{11}{32}(2\pi)^{1/2}\alpha\hbar\omega_{\mathrm{LO}} r_0 \lambda \\ &- \frac{e^* F}{4\lambda}. \end{aligned} \tag{12}$$

The first-excited state energy of electron in a parabolic QD can be written as

$$\begin{aligned} E_1(\lambda_0) =& \frac{\hbar^2\lambda_0^2}{m^*} + \frac{\hbar^2}{m^*\lambda_0^2 l_0^4} - \frac{11}{32}(2\pi)^{1/2}\alpha\hbar\omega_{\mathrm{LO}} r_0 \lambda_0 \\ &- \frac{e^* F}{4\lambda_0}. \end{aligned} \tag{13}$$

We can obtain λ_0 by using the variational method. Then we can get the eigenlevel and the eigenwavefunction. Then, we obtain the two-level system needed by

a single qubit. The superposition state of electron can be expressed as

$$|\psi_{01}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \tag{14}$$

where

$$|0\rangle = \varphi_0(\rho) = \frac{\lambda_0}{\sqrt{\pi}}\exp\left(-\frac{\lambda_0^2\rho^2}{2}\right), \tag{15}$$

$$|1\rangle = \varphi_1(\rho) = \frac{\lambda_0^2}{\sqrt{\pi}}\rho\exp\left(-\frac{\lambda_0^2\rho^2}{2}\right)\exp(\pm \mathrm{i}\phi). \tag{16}$$

The time evolution of the quantum state of the electron can be written as

$$\begin{aligned}\psi_{01}(t,\rho) =& \frac{1}{\sqrt{2}}\varphi_0(\rho)\exp\left(-\frac{\mathrm{i}E_0 t}{\hbar}\right) \\ &+ \frac{1}{\sqrt{2}}\varphi_1(\rho)\exp\left(-\frac{\mathrm{i}E_1 t}{\hbar}\right).\end{aligned} \tag{17}$$

The probability density is

$$\begin{aligned}Q(\rho,t) =& |\psi_{01}(t,\rho)|^2 \\ =& \frac{1}{2}[|\varphi_0(\rho)|^2 + |\varphi_1(\rho)|^2 \\ &+ \varphi_0^*(\rho)\varphi_1(\rho)\exp(\mathrm{i}\omega_{01}t) \\ &+ \varphi_0(\rho)\varphi_1^*(\rho)\exp(-\mathrm{i}\omega_{01}t)],\end{aligned} \tag{18}$$

where

$$\omega_{01} = \frac{(E_1 - E_0)}{\hbar}. \tag{19}$$

3. Results and discussion

The numerical results of the probability density of electron and the period of oscillation versus the electric field, the electron–LO-phonon coupling constant and the confinement length in a parabolic QD are presented in Fig.1. Throughout this study, the length and energy are taken in units of the polaron radius r_0 and the phonon energy constant $R^* = \hbar\omega_{\mathrm{LO}}$.

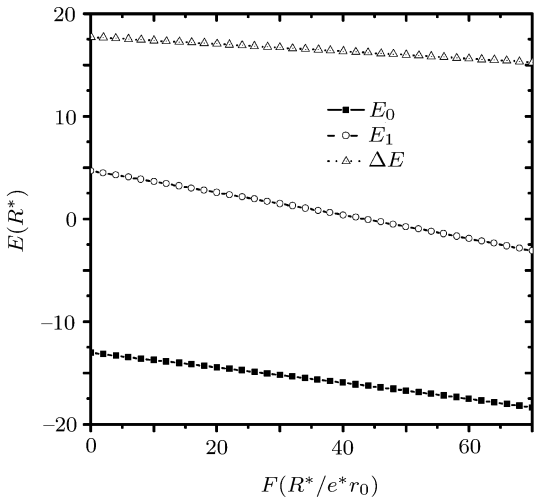
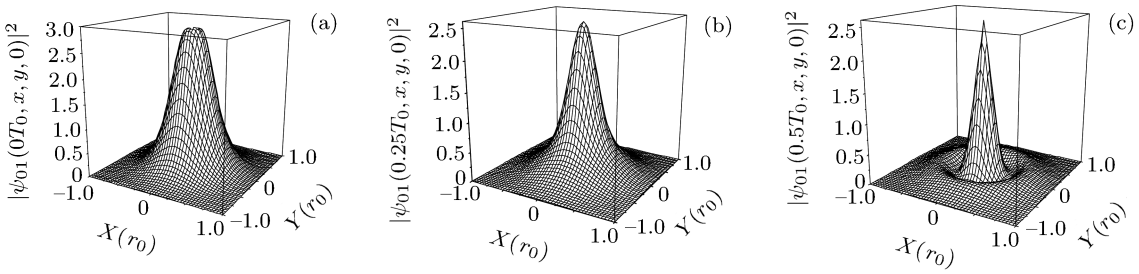


Fig.1. Energy spacing between the ground state and the first-excited state and energy levels of ground state and the first-excited state vs parallel electric field along the x direction.

Figure 1 shows the energy spacing ΔE and energy levels of the ground state (E_0) and the first-excited state (E_1) as a function of the electric field along the x direction for the coupling constant α=6, the confinement length l_0=0.5. It turns out that the energy of ground state E_0 and the first-excited state E_1 decrease with the increase of the electric field. But the influence of electric field on the energy of first-excited state E_1 is greater than the energy of ground state E_0. So the energy spacing ΔE decreases with the increase of the electric field.

Figure 2 shows that the time evolution of the electron probability density $|\psi_{01}(t,x,y,0)|^2$ when electrons exist in the superposition state of $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ for electron–LO-phonon coupling constant α=6, the confinement length l_0=0.5 and the phase difference $\phi = 2\pi$. The probability density of electron oscillated with the period of oscillation $T_0 = \frac{h}{(E_1 - E_0)}$, the time t in Figs.2(a), (b), (c), (d), (e) are 0, 0.25, 0.5, 0.75, and $1T_0$, respectively.



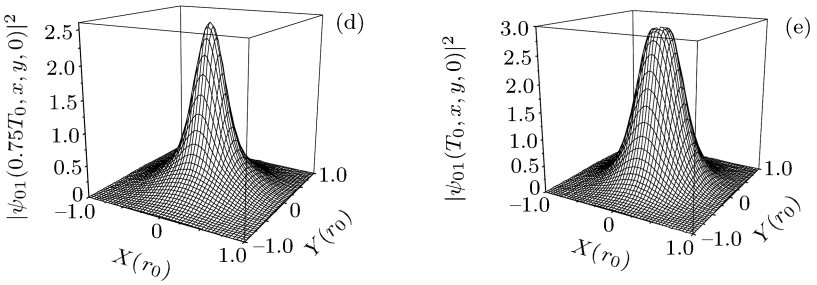


Fig.2. The time evolution of the electron probability density. Time t in (a), (b), (c), (d), and (e) are 0, 0.25, 0.5, 0.75, and $1T_0$, respectively.

Figure 3 presents the period of oscillation as a function of the electric field for the different electron–LO-phonon coupling constant α=5, 6, 7 and the confinement length l_0=0.5. From Fig.4, we can see that the period of oscillation increases with the increase of the electric field along the x direction. As a result of existence of the electric field, the ground and the first-excited state energies are reduced and the influence on the first-excited state energy is greater than the ground state energy. For this reason, the energy spacing between the ground and the first-excited states decreases and the period of oscillation increases. Meanwhile, we also see that the period of oscillation decreases with the increase of the electron–LO-phonon coupling constant. This is because the electron–LO-phonon coupling strength in the first-excited state is weaker than that in the ground-state while increasing the coupling strength.[10] So the increase of the energy spacing between the first-excited and ground states causes the reduction of the period of oscillation.

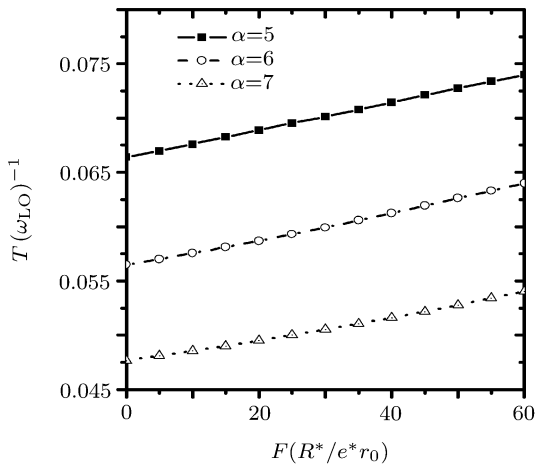


Fig.3. The period of oscillation as a function of the electric field for different electron–LO-phonon coupling constants in a parabolic quantum dot.

Figure 4 plots the period of oscillation T_0 as a function of the electric field for the different confinement lengths l_0=0.4, 0.5, 0.6 and the electron–LO-phonon coupling constant α=6. From Fig.3, one finds that the period of oscillation increases with the increase of the electric field and the same reason as that in the Fig.3. It also can be seen that the period of oscillation T_0 increases with the increase of the confinement length. The influence of electric field on the period of oscillation is greater with increasing the confinement length, which indicates the effect of quantum size.

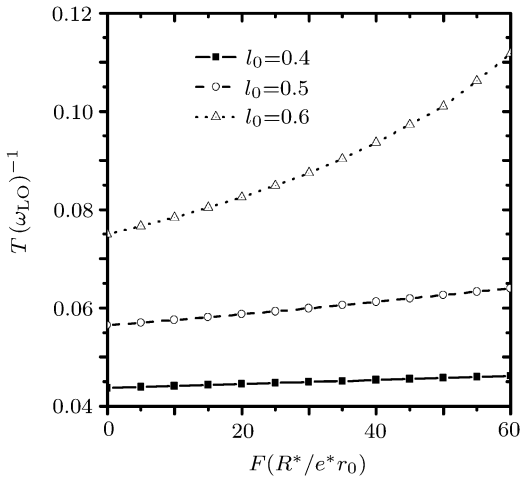


Fig.4. The period of oscillation in a parabolic quantum dot as a function of the electric field for different confinement lengths.

The above discussions indicate that the period of oscillation increases in superposition state of electron in a QD due to the existence of the electric field. However, a qubit cannot be independent of environment and must be interacted with the heat bath. As a result, the interaction destroys the superposition state of a qubit, which is decoherence.[11−13] The period of oscillation T_0 increases, that is the lifetime of a qubit increases. So the process of decoherence is slower. It is very useful to store information which makes the

QD as its elementary unit. This is in agreement with discussions in Refs.[8, 9].

4. Conclusion

The energies and the relevant eigenwavefunctions of the ground and the first-excited states of electron have been obtained in a parabolic QD using the Pekar variational method considering the parabolic potential in the electron–LO-phonon strong coupling region. The single qubit can be envisaged as this kind of two-level quantum system in a QD. The probability density of electron oscillates with a period when the electron is in the superposition state of the ground and the first-excited states. The results indicate that the period of oscillation increases with the decrease of the electron–LO-phonon coupling constant, and the increase of the electric field as well as the confinement length.

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