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Two simple schemes for implementing Toffoli gate via atom–cavity field interaction in cavity quantum electrodynamics*

Shao Xiao-Qiang(邵晓强)[a], Chen Li(陈 丽)[b], and Zhang Shou(张 寿)[a)b)†]

[a)] *Center for the Condensed-Matter Science and Technology, Harbin Institute of Technology, Harbin* 150001, *China*

[b)] *Department of Physics, College of Science, Yanbian University, Yanji* 133002, *China*

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This paper proposes two schemes for implementing three-qubit Toffoli gate with an atom (as target qubit) sent through a two-mode cavity (as control qubits). The first scheme is based on the large-detuning atom–cavity field interaction and the second scheme is based on the resonant atom-field interaction. Both the situations with and without cavity decay and atomic spontaneous emission are considered. The advantages and the experimental feasibility of these two schemes are discussed.

Keywords: Toffoli gate, cavity quantum electrodynamics
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1. Introduction

Quantum computers would perform certain tasks, such as factoring a number and searching a data in an array, faster than a classical computer.[1,2] This matter of fact has triggered in the past years a lot of studies on the theoretical and practical aspects of quantum computing. The physical models of quantum computation have been proposed in different systems such as ion trap,[3,4] cavity quantum electrodynamics (QED),[5,6] nuclear magnetic resonance,[7] linear optics[8] and quantum dot,[9] etc. The developing technology of microwaves and optical cavities provides a way to coherently exchange quantum information between material qubits (atoms or ions), so cavity QED can be considered as an ideal system to implement quantum information processing. Up to now, many schemes for realizing two-qubit quantum gate have been proposed in cavity QED system.[10−16]

Multiqubit gate can be decomposed into conditional z gate and one-qubit gate,[17] but the procedure will get more complicated as the number of the qubits increases. Hence, researchers are trying to find effective methods to implement multiqubit gate directly. A series of schemes for realizing multiqubit quantum phase gate have been proposed in cavity QED system.[18−21] It is noticed that there are two important kinds of three-qubit gate known as Toffoli gate (controlled–controlled-not gate)[22] and Fredkin gate (controlled-exchange gate)[23] which have given a simple implementation of quantum computer to solve Grover's algorithm[24] and Deutsch's problem,[25] respectively. Three-atom-qubit Toffoli gate and multi-atom-qubit Toffoli gate have been realized in cavity QED.[26,27] It should be noted that in many physical carriers of qubit, photon is best for quantum information process due to its weak interaction with environment, and some schemes for phase gate with photon as qubit have been proposed.[28−31] However, it is easier to perform a single qubit operation on an atom than on a photon in cavity. By combining the manageable characteristic of atom in cavity and the stability of photon, several schemes for implementing quantum logic gate with an atom and the quantum states of cavity mode have been proposed, i.e., Biswas *et al*[32] realized a two-qubit phase gate involving the atomic ground-state basis and the Fock state basis in one mode of a bimodel optical cavity. In Ref.[24], a three-qubit gate is proposed whose operation involves the atomic ground-state basis and the Fock-state basis in two modes. Very recently, we became aware of that in many protocols, an ancillary ground state

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†Corresponding author. E-mail: szhang@ybu.edu.cn

plays an important role in the implementation of multiqubit quantum gate.[18−21,33−36] Enlightened by the above works, we propose two schemes for implementing Toffoli gate through different atom–cavity field interaction with the zero- and one-photon Fock states of two different intracavity modes $|0_j\rangle$, $|1_j\rangle$ $(j = 1, 2)$ as control qubits and the atom as target qubit.

The paper is organized as follows. In Section 2 we first propose a scheme to implement a three-qubit Toffoli gate based on atom–cavity field large detuning interaction and investigate the decoherence on the system. In Section 3 we propose another scheme to implement a three-qubit Toffoli gate based on atom–cavity field resonant interaction, study the effect of cavity decay and the atomic emission on the fidelity of the three-qubit Toffoli gate and on the corresponding probability of success. Finally, in Section 4 we discuss the experimental feasibility of both the schemes and give a conclusion.

2. Toffoli gate based on atom–cavity field large detuning interaction

We first consider a four-level atom interaction with a two-mode cavity field. The atomic states are denoted by $|g\rangle$, $|e\rangle$, $|f\rangle$, and $|i\rangle$, as illustrated in Fig.1.

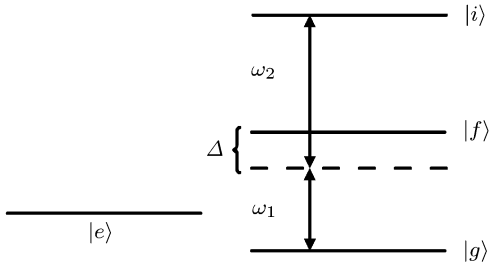


Fig.1. Atomic level structure of four-level atom.

The state $|e\rangle$ is an auxiliary state which is not affected by the atom–cavity field interaction. The atomic transition between levels $|f\rangle \leftrightarrow |g\rangle$ is coupled to the cavity mode 1 with coupling strength g_1, detuning parameter Δ, and the transition $|i\rangle \leftrightarrow |f\rangle$ is coupled to the cavity mode 2 with coupling strength g_2, detuning parameter $-\Delta$. The interaction Hamiltonian (assuming $\hbar = 1$) in the interaction picture is

$$\begin{aligned} H_{\rm i} =& g_1(a_1|f\rangle\langle g|{\rm e}^{{\rm i}\Delta t} + a_1^\dagger|g\rangle\langle f|{\rm e}^{-{\rm i}\Delta t}) \\ &+ g_2(a_2|i\rangle\langle f|{\rm e}^{i\Delta t} + a_2^\dagger|f\rangle\langle i|{\rm e}^{{\rm i}\Delta t}), \end{aligned} \tag{1}$$

where g_j is the coupling constant between the atom and the jth cavity mode. $a_j^\dagger$ (a_j) is the creation and (annihilation) operator of the jth cavity mode. Under the condition $|\Delta| \gg |g_j|$, we can obtain the effective Hamiltonian of the system

$$\begin{aligned} H_{\rm eff} =& \lambda_{gi}|g\rangle\langle i| + \lambda_{ig}|i\rangle\langle g| \\ &+ \lambda_f|f\rangle\langle f| + \lambda_i|i\rangle\langle i| + \lambda_g|g\rangle\langle g|, \end{aligned} \tag{2}$$

where $\lambda_{gi} = -g_1g_2a_1^\dagger a_2^\dagger/\Delta$, $\lambda_{ig} = -g_1g_2a_1a_2/\Delta$, $\lambda_f = (g_2^2a_2^\dagger a_2 + g_1^2a_1a_1^\dagger)/\Delta$, $\lambda_i = -g_2^2a_2a_2^\dagger/\Delta$, and $\lambda_g = -g_1^2a_1^\dagger a_1/\Delta$. In Eq.(2), the first two parts of the effective Hamiltonian represent the two-photon transition between $|g\rangle \leftrightarrow |i\rangle$ mediated by the level $|f\rangle$ due to the large-detuning atom–cavity field interaction, and the rest three parts denote the Stark shift of levels $|g\rangle$, $|f\rangle$, and $|i\rangle$. We encode the target logic qubit on the atomic states $|e\rangle$ and $|g\rangle$ as $|0\rangle$ and $|1\rangle$, respectively. The other two logic qubits will consist of the zero- and one-photon Fock states of two different intracavity modes $|0_j\rangle, |1_j\rangle$ $(j = 1, 2)$. So in the subspace $\{|0_1\rangle, |1_1\rangle, |0_2\rangle, |1_2\rangle, |e\rangle, |g\rangle\}$, the Hamiltonian Eq.(2) can be rewritten as

$$\begin{aligned} H_{\rm eff} =& -\frac{g_1g_2}{\Delta}(|g,1_1,1_2\rangle\langle i,0_1,0_2| \\ &+ |i,0_1,0_2\rangle\langle g,1_1,1_2|) \\ &- \frac{g_1^2}{\Delta}(|g,1_1,0_2\rangle\langle g,1_1,0_2| \\ &+ |g,1_1,1_2\rangle\langle g,1_1,1_2|). \end{aligned} \tag{3}$$

Through the unitary transformation

$$\begin{aligned} U_0 = \exp\Big[&\frac{{\rm i}g_1^2}{\Delta}(|g,1_1,0_2\rangle\langle g,1_1,0_2| \\ &+ |g,1_1,1_2\rangle\langle g,1_1,1_2|)t\Big], \end{aligned} \tag{4}$$

the Hamiltonian in Eq.(3) becomes[29]

$$\begin{aligned} H_{\rm e}' =& -\frac{g_1g_2}{\Delta}(|g,1_1,1_2\rangle\langle i,0_1,0_2| \\ &+ |i,0_1,0_2\rangle\langle g,1_1,1_2|). \end{aligned} \tag{5}$$

Thus the dynamics of the system is reduced to an effective two-level model. It should be made clear that in order to obtain Eq.(5), we perform a unitary transformation to exclude the Stark-shift terms of Eq.(3), which is different from the methods in Refs.[29,37],

where an additional field has been utilized to cancel the unwanted Stark shifts. In the subspace, only the state $|g,1_1,1_2\rangle$ suffers the Rabi oscillations with the effective frequency $-g_1g_2/\Delta$, the other states including $|e,0_1,0_2\rangle$, $|e,0_1,1_2\rangle$, $|e,1_1,0_2\rangle$, $|e,1_1,1_2\rangle$, $|g,0_1,0_2\rangle$, $|g,0_1,1_2\rangle$, and $|g,1_1,0_2\rangle$ will not evolve in the process of atom–cavity field interaction,

$$\begin{aligned}|g,1_1,1_2\rangle \rightarrow & \cos(\lambda t)|g,1_1,1_2\rangle \\ & + \mathrm{i}\sin(\lambda t)|i,0_1,0_2\rangle, \end{aligned} \tag{6}$$

where $\lambda = g_1g_2/\Delta$, after selecting the interaction time $t = \pi/\lambda$, equation (6) becomes $-|g,1_1,1_2\rangle$. Applying the classical field to induce the atomic transition

$$\begin{aligned}|g\rangle &\rightarrow \frac{1}{\sqrt{2}}(|e\rangle - |g\rangle), \\ |e\rangle &\rightarrow \frac{1}{\sqrt{2}}(|e\rangle + |g\rangle), \end{aligned} \tag{7}$$

we can obtain a Toffoli gate in our computational subspace

$$T_{1,1} = H^{\dagger}T_{\mathrm{p}}H = \begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0 \end{pmatrix}, \tag{8}$$

where H is the Hadamard gate on the atom with $|g\rangle \rightarrow (|e\rangle - |g\rangle)/\sqrt{2}$, $|e\rangle \rightarrow (|e\rangle + |g\rangle)/\sqrt{2}$, T_{p} is the CPHASE operation in Eq.(6).

From Eq.(8), we can see that the key step for realizing a Toffoli gate is the three-qubit phase gate, thus in this part, we mainly investigate the effect of decoherence on the three-qubit phase gate. In the scheme, we suppose that no photon is detected either by the atomic spontaneous emission or by the leakage of a photon through the cavity mirror. Thus the effective Hamiltonian governing the system is

$$\begin{aligned}H^1_{\mathrm{con}} = & -\frac{g_1g_2}{\Delta}(|g,1_1,1_2\rangle\langle i,0_1,0_2| \\ & + |i,0_1,0_2\rangle\langle g,1_1,1_2|) \\ & - \frac{\mathrm{i}}{2}\gamma|i\rangle\langle i| - \sum_{n=1}^{2}\frac{\mathrm{i}}{2}\kappa_n a_n^{\dagger}a_n. \end{aligned} \tag{9}$$

The state $|g,\ 1_1,\ 1_2\rangle$ governed by Eq.(9) will evolve to

$$\begin{aligned}|g,1_1,1_2\rangle \rightarrow & \frac{\mathrm{e}^{-\frac{1}{4}\lambda t(\eta+\mu+\nu+\xi)}}{2\eta}[(\mathrm{e}^{\frac{1}{2}\lambda t\eta}+1)\eta \\ & + (\mathrm{e}^{\frac{1}{2}\lambda t\eta}-1) \\ & (\mu-\nu-\xi)]|g,1_1,1_2\rangle \\ & + \frac{2\mathrm{i}\mathrm{e}^{-\frac{1}{4}\lambda t(\eta+\mu+\nu+\xi)}}{\eta} \\ & (\mathrm{e}^{\frac{1}{2}\lambda t\eta}-1)|i,0_1,0_2\rangle, \end{aligned} \tag{10}$$

where $\eta = \sqrt{(\mu+\nu+\xi)^2 - 4(4+\mu\nu+\mu\xi)}$, $\mu = \gamma/\lambda$, $\nu = \kappa_1/\lambda$, and $\xi = \kappa_2/\lambda$.

We can calculate the fidelity according to the following relation:[38]

$$F = \overline{\langle \Psi_0|U^{\dagger}\rho_t U|\Psi_0\rangle}, \tag{11}$$

where we have assumed $\kappa_1 = \kappa_2 = \gamma$ for simplicity. The over line indicates average overall possible input state $|\Psi_0\rangle$, U is the ideal three-qubit phase gate, and $\rho_t = |\Psi_t\rangle\langle\Psi_t|$, $|\Psi_t\rangle$ is the final state after the phase gate operation in our scheme. Suppose that the system is initially in the state

$$|\Psi_0\rangle = \frac{1}{2\sqrt{2}}(|e\rangle + |g\rangle)(|0_1\rangle + |1_1\rangle)(|0_2\rangle + |1_2\rangle), \tag{12}$$

we plot the fidelity and the success possibility of three-qubit gate vs the cavity decay and the atomic emission in Fig.2.

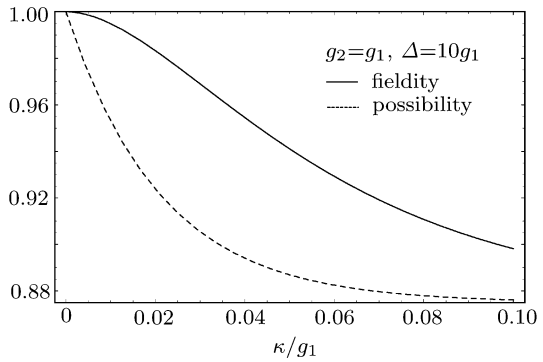


Fig.2. The evolutions of fidelity (solid line) and the probability of success (dashed line) versus κ/g_1.

3. Toffoli gate based on atom–cavity field resonant interaction

Now we consider another model of a four-level atom interaction with a two-mode cavity field. The atomic states are denoted by $|e\rangle$, $|g\rangle$, $|f\rangle$ and $|i\rangle$. The state $|e\rangle$ is not affected during the atom–cavity

interaction. As illustrated in Fig.3, the transitions

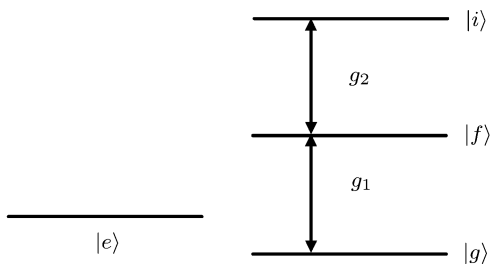


Fig.3. Atomic level structure of four-level atom.

$|g\rangle \leftrightarrow |f\rangle$, $|f\rangle \leftrightarrow |i\rangle$ are coupled to cavity 1 mode and 2 mode, respectively. Similar to the scheme I, the target logic qubit is encoded on the atomic states $|e\rangle$ and $|g\rangle$ as $|0\rangle$ and $|1\rangle$, respectively. The other two logic qubits include the zero- and one-photon Fock states of two different intracavity modes $|0_j\rangle, |1_j\rangle$ $(j = 1, 2)$. The interaction Hamiltonian (assuming $\hbar = 1$) of the system in the interaction picture is

$$\begin{aligned} H_{\rm I} =& g_1\big(a_1|f\rangle\langle g| + a_1^\dagger|g\rangle\langle f|\big) \\ &+ g_2\big(a_2|i\rangle\langle f| + a_2^\dagger|f\rangle\langle i|\big), \end{aligned} \tag{13}$$

if the initial state of atom–cavity field is $|g, 1_1, 0_2\rangle$, the exact evolution of this system is obtained as

$$\begin{aligned} |g, 1_1, 0_2\rangle \rightarrow& \cos(g_1 t)|g, 1_1, 0_2\rangle \\ &- \mathrm{i}\sin(g_1 t)|f, 0_1, 0_2\rangle, \end{aligned} \tag{14}$$

if the total system is in the initial state $|g, 1_1, 1_2\rangle$, after the interaction with the cavity, the state evolves to

$$\begin{aligned} |g, 1_1, 1_2\rangle \rightarrow& \frac{g_1^2\cos(\alpha t) + g_2^2}{\alpha^2}|g, 1_1, 1_2\rangle \\ &- \mathrm{i}\frac{g_1\sin(\alpha t)}{\alpha}|f, 0_1, 1_2\rangle \\ &+ \frac{[\cos(\alpha t) - 1]g_1 g_2}{\alpha^2}|i, 0_1, 0_2\rangle, \end{aligned} \tag{15}$$

where $\alpha = \sqrt{g_1^2 + g_2^2}$, and we assume the coupling strength between the atom and the two-mode cavity satisfying the condition $g_2 = \sqrt{3}g_1$. In our system, the other states including $|e, 0_1, 0_2\rangle$, $|e, 0_1, 1_2\rangle$, $|e, 1_1, 0_2\rangle$, $|e, 1_1, 1_2\rangle$, $|g, 0_1, 0_2\rangle$ and $|g, 0_1, 1_2\rangle$ remain unchanged in the evolution, thus we can obtain a three-qubit phase gate via choosing the interaction time $t = \pi/g_1$,

$$\begin{aligned} &|e, 0_1, 1_2\rangle \rightarrow |e, 0_1, 1_2\rangle, \\ &|e, 0_1, 0_2\rangle \rightarrow |e, 0_1, 0_2\rangle, \\ &|e, 1_1, 1_2\rangle \rightarrow |e, 1_1, 1_2\rangle, \\ &|e, 1_1, 0_2\rangle \rightarrow |e, 1_1, 0_2\rangle, \\ &|g, 0_1, 1_2\rangle \rightarrow |g, 0_1, 1_2\rangle, \\ &|g, 0_1, 0_2\rangle \rightarrow |g, 0_1, 0_2\rangle, \\ &|g, 1_1, 1_2\rangle \rightarrow |g, 1_1, 1_2\rangle, \\ &|g, 1_1, 0_2\rangle \rightarrow -|g, 1_1, 0_2\rangle, \end{aligned} \tag{16}$$

after we performing the Hadamard gate on the atom (target qubit), we obtain another form of Toffoli gate as

$$T_{1,0} = \begin{pmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1 \end{pmatrix}. \tag{17}$$

Equation (17) can be considered as a kind of generalized Toffoli gate, where only the control qubits are '1' and '0', the target qubit flips.

We now consider the atomic spontaneous emission and cavity decay. Under the condition that no photon is detected by the leakage of a photon by the cavity mirrors, the evolution of the system is governed by the conditional Hamiltonian

$$\begin{aligned} H_{\rm con}^2 =& g_1\big(a_1|f\rangle\langle g| + a_1^\dagger|g\rangle\langle f|\big) \\ &+ g_2\big(a_2|i\rangle\langle f| + a_2^\dagger|f\rangle\langle i|\big) \\ &- \frac{\mathrm{i}}{2}\gamma_f|f\rangle\langle f| - \frac{\mathrm{i}}{2}\gamma_i|i\rangle\langle i| \\ &- \sum_{n=1}^{2}\frac{\mathrm{i}}{2}\kappa_n a_n^\dagger a_n, \end{aligned} \tag{18}$$

we obtain corresponding evolution of the states:

$$\begin{aligned} |g, 1_1, 1_2\rangle \rightarrow& \frac{1}{\alpha^2}\big[g_1^2\mathrm{e}^{-\kappa t/2}\cos(\alpha t) + g_2^2\mathrm{e}^{-\kappa t/2}\big]|g, 1_1, 1_2\rangle \\ &+ \frac{g_1 g_2}{\alpha^2}\big[\mathrm{e}^{-\kappa t/2}\cos(\alpha t) - \mathrm{e}^{-\kappa t/2}\big]|i, 0_1, 0_2\rangle \\ &- \mathrm{i}\frac{g_1}{\alpha}\mathrm{e}^{-\kappa t/2}\sin(\alpha t)|f, 0_1, 1_2\rangle, \end{aligned} \tag{19}$$

$$\begin{aligned} |g, 1_1, 0_2\rangle \rightarrow& \mathrm{e}^{-\kappa t/4}\big[\cos(g_1 t)|g, 1_1, 0_2\rangle \\ &- \mathrm{i}\sin(g_1 t)|f, 0_1, 0_2\rangle\big], \end{aligned} \tag{20}$$

and

$$|g,0_1,1_2\rangle \to \mathrm{e}^{-\kappa t/2}|g,0_1,1_2\rangle, \tag{21a}$$
$$|e,0_1,1_2\rangle \to \mathrm{e}^{-\kappa t/2}|e,0_1,1_2\rangle, \tag{21b}$$
$$|e,1_1,0_2\rangle \to \mathrm{e}^{-\kappa t/2}|e,1_1,0_2\rangle, \tag{21c}$$
$$|e,1_1,1_2\rangle \to \mathrm{e}^{-\kappa t}|e,1_1,1_2\rangle, \tag{21d}$$

where we have assumed that $\kappa_1 = \kappa_2 = \gamma_f = \kappa/2$, and $r_i = \kappa$ for sake of convenience. Similar to Eqs.(16) and (17), we plot the fidelity and the success possibility of three-qubit gate vs the cavity decay and the atomic spontaneous emission of this scheme in Fig.4.

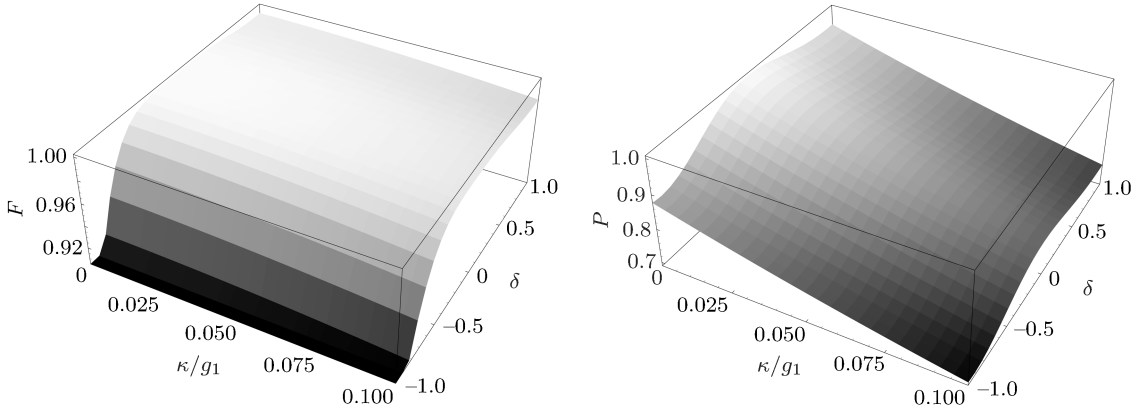


Fig.4. The evolutions of the fidelity and the probability of success versus both κ/g_1 and δ, respectively. Other parameters: $g_2 = \sqrt{3}g_1$, $\kappa_1 = \kappa_2 = \gamma_f = \kappa/2$, $r_i = \kappa$.

4. Discussion and conclusion

In order to obtain a good approximation, we choose the detuning parameter between the frequency of atomic transition and the one of cavity mode satisfying the condition $|\Delta| = 10g_1$ for the first scheme, and we can see from Fig.2 that the fidelity is 96.90% and the successful probability is 90.54% even under the condition $\kappa/g_1 = 0.03$. Besides, this scheme does not need to locate the position of the atom precisely in the bimodel cavity, which may simplify the complexity of operations experimentally. In the scheme of atom–cavity resonant interaction, we still obtain a higher fidelity for the Toffoli gate when $\kappa/g_1 = 0.1$ than the first scheme. Nevertheless, it should be mentioned that in order to acquire the π phase on the state $|g_1,1_1,0_2\rangle$, we must satisfy the condition $g_2 = \sqrt{3}g_1$, which may not be complemented in experiment accurately, so we assume that there should be a deviation when selecting the value of g_2. Assuming $g_2 = (\sqrt{3}+\delta)g_1$, we also plot the fidelity and corresponding probability of success vs the deviation δ in Fig. 4. From which we can see that the fidelity is more sensitive to δ when $\delta < 0$, i.e., under the condition $\kappa/g_1 = 0.03$, the fidelity drops to 92.85% when $\delta = -0.8$ whereas the fidelity is still as high as 99.37% when $\delta = 0.8$, but the successful probability is not as much obviously sensitive to the value of δ as the fidelity. Thus, when we realize the Toffoli gate via atom–cavity field interaction, we must choose the value of g_2 as accurately as possible, which depends on the advancement of the technology in experiment.

We now give a brief discussion of the experimental feasibility of the proposed scheme within cavity QED. Both schemes suit well under the condition of superconducting cavity. We may employ ^{87}Rb atom with higher Rydberg atomic states for our proposal, the ground states $|e\rangle$ and $|g\rangle$ correspond to Zeeman levels $|F = 1, m = 1\rangle$ and $|F = 1, m = 0\rangle$ of $49S_{1/2}$ electronic excited state respectively, while the excited states $|f\rangle$ and $|i\rangle$ correspond to Zeeman levels $|F = 2, m = 1\rangle$ of $50P_{1/2}$ and $|F = 2, m = 2\rangle$ of $51P_{3/2}$ electronic excited states respectively. In cavity the coupling rate between the atomic internal levels and the cavity mode depends on the atomic position r through the relation $g_{1(2)} = g_0 \sin(k_{1(2)}z)\exp[-(x^2+y^2)/\omega^2_{0,1(2)}]$, where g_0 is the peak coupling rate, $\omega_{0,1(2)}$

and $k_{1(2)} = 2\pi/\lambda_{1(2)}$ are, respectively, the width and the wave vector of the Gaussian cavity mode, and z is assumed to be along the axis of the cavity.[39] Thus we can locate the position of atom in the two-mode cavity to satisfy the condition $g_1 = g_2 = 7 \times 10^5\,\mathrm{s}^{-1}$ for the first scheme and $\sqrt{3}g_1 = g_2 = 7 \times 10^5\,\mathrm{s}^{-1}$ for the second scheme. The gating times for implementing the two schemes are $t_1 \approx 4.5 \times 10\,\mathrm{s}^{-4}$ and $t_2 \approx 7.8 \times 10\,\mathrm{s}^{-6}$, respectively, which are much smaller than the lifetime of the atom 0.03 s and the lifetime 0.01 s for the high-Q cavity,[40] in this sense we have completely neglected cavity loss and atomic spontaneous decay in our model.

In conclusion, we have proposed two schemes to implement Toffoli gate via different atom–cavity field interaction. The results show that both the fidelity and the probability of success in our schemes are relatively high, and it is easy to be implemented. We believe that our schemes are more meaningful for quantum logic gate both theoretically and experimentally.

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