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A quantum network for implementation of the optimal quantum cloning*

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This paper presents a quantum network to implement the optimal 1→2 quantum cloning in 2 dimensions, including the optimal asymmetric universal, the optimal symmetric phase-covariant, and the asymmetric real state cloning. By only choosing different angles of the single-qubit rotations, the quantum network can implement three optimal quantum cloning.

Keywords: quantum network, quantum cloning
PACC: 0367

1. Introduction

No-cloning theorem prohibits cloning an arbitrary unknown quantum state.[1,2] The theorem is a basic one in quantum mechanics, and guarantees the absolute security of quantum cryptography.[3] In recent years, quantum cloning theory has been extensively investigated both in theory and experiment.[4] Quantum cloning is not only a fundamental of quantum mechanics, but also has many promising applications to quantum information processing.[5−9]

Though perfect copies cannot be obtained by the restriction of no-cloning theorem, imperfect copies can be acquired in an approximate way. To clone an unknown input state, the simplest but important case is to copy an unknown pure state $|\psi\rangle^{(\mathrm{in})} = \alpha|0\rangle + \beta e^{i\varphi}|1\rangle$ ($\alpha,\ \beta \in R$ satisfying $\alpha^2+\beta^2=1$, and $\varphi \in [0, 2\pi)$). From the point of view of the form of input states to be cloned, therefore, the cloner can be classified into three kinds: the universal quantum cloning[10−16] if α, β and φ are unknown; the phase-covariant cloning[17−22] with $\alpha = \beta = 1/\sqrt{2}$ and $\varphi \in [0, 2\pi)$ being unknown; and the real state cloning[23−25] with $\varphi = 0$ and $\alpha, \beta \in [-1, 1]$. Recently, some schemes have been proposed for implementation of different optimal cloners.[26−29] In experiment, by exploring different physical systems, some of three cloners have been realized.[30−33]

In this paper, we present a quantum network to implement the optimal $1 \to 2$ quantum cloning in 2 dimensions, including the optimal asymmetric universal, the optimal symmetric phase-covariant, and the optimal asymmetric real state cloning. The quantum network we use is similar to the one in Ref.[34] where the authors exploited it to implement the optimal symmetric $1 \to 2$ universal quantum cloning. By using the slightly modified quantum network, we can implement three optimal cloners. In the process, only by modulating different angles of the single-qubit rotations, different cloners can be implemented.

The paper is organized as follows. In Section 2, we briefly review some previous contributions. Then in Section 3, we present a quantum network to implement three cloners. The paper ends with a summary.

2. Three optimal cloners

In 1996, Bužek and Hillery proposed the optimal symmetric universal quantum cloning in 2 dimensions.[10] To clone the input state $|\psi_1\rangle_1^{(\mathrm{in})} = \alpha|0\rangle + \beta e^{i\varphi}|1\rangle$ with α, β and $\varphi \in [0, 2\pi)$ being unknown, the optimal asymmetric $1 \to 2$ universal quantum cloning in 2 dimensions is defined as[14,15,21]

$$\begin{aligned} |0\rangle_1|00\rangle_{2,\mathrm{A}} &\to (p+q)|000\rangle_{1,2,\mathrm{A}} \\ &\quad + (p|01\rangle + q|10\rangle)_{1,2}|1\rangle_{\mathrm{A}}, \\ |1\rangle_1|00\rangle_{2,\mathrm{A}} &\to (p+q)|111\rangle_{1,2,\mathrm{A}} \\ &\quad + (p|10\rangle + q|01\rangle)_{1,2}|0\rangle_{\mathrm{A}}, \end{aligned} \tag{1}$$

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where the particles 1, 2 , and A denote the input state to be cloned, the blank copy, and the ancillary. The cloning coefficients are under the constraint

$$p^2 + q^2 + pq = \frac{1}{2} \tag{2}$$

with $p, q \geq 0$. The fidelities of two clones are given as

$$F_1^{(\mathrm{AU})} = 1 - q^2, \quad F_2^{(\mathrm{AU})} = 1 - p^2. \tag{3}$$

When $p = q = \sqrt{1/6}$, the optimal asymmetric universal quantum cloner can be reduced to the optimal symmetric one with fidelity $F_{1(2)}^{(\mathrm{SU})} = 5/6$.[10] For convenience in Section 3, we choose the parameters of the cloning coefficients as

$$p = \frac{\cos 2\theta}{\sqrt{2 + \sin 4\theta}}, \quad q = \frac{\sin 2\theta}{\sqrt{2 + \sin 4\theta}} \tag{4}$$

with $\theta \in [0, \pi/4]$. One can easily check that the cloning coefficients we choose are satisfied by the constraint defined by Eq.(2). To clone the input state $|\psi_2\rangle_1^{(\mathrm{in})} = \alpha\,|0\rangle + \beta \mathrm{e}^{\mathrm{i}\varphi}\,|1\rangle$ with $\alpha = \beta = 1/\sqrt{2}$, and $\varphi \in [0, 2\pi)$ being unknown, the optimal symmetric $1 \to 2$ phase-covariant cloning in 2 dimensions is defined as[18,21]

$$\begin{aligned} |0\rangle_1 |00\rangle_{2,\mathrm{A}} &\to \frac{1}{\sqrt{2}} \left[|000\rangle_{1,2,\mathrm{A}} + \frac{1}{\sqrt{2}} \left(|01\rangle + |10\rangle \right)_{1,2} |1\rangle_{\mathrm{A}} \right], \\ |1\rangle_1 |00\rangle_{2,\mathrm{A}} &\to \frac{1}{\sqrt{2}} \left[|111\rangle_{1,2,\mathrm{A}} + \frac{1}{\sqrt{2}} \left(|10\rangle + |01\rangle \right)_{1,2} |0\rangle_{\mathrm{A}} \right] \end{aligned} \tag{5}$$

with the fidelity $F_{1(2)}^{(\mathrm{SP})} = \frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right)$. To clone the input state $|\psi_3\rangle_1^{(\mathrm{in})} = \alpha\,|0\rangle + \beta \mathrm{e}^{\mathrm{i}\varphi}\,|1\rangle$ with $\varphi = 0$ and $\alpha,\ \beta \in [-1,\ 1]$ being unknown, we present a simplified transformation of the optimal asymmetric $1 \to 2$ real state cloning in 2 dimensions as

$$\begin{aligned} |000\rangle_{1,2,\mathrm{A}} &\to a\,|000\rangle_{1,2,\mathrm{A}} + \left(b\,|01\rangle + c\,|10\rangle\right)_{1,2} |1\rangle_{\mathrm{A}} + d\,|110\rangle_{1,2,\mathrm{A}}\,, \\ |100\rangle_{1,2,\mathrm{A}} &\to a\,|111\rangle_{1,2,\mathrm{A}} + \left(b\,|10\rangle + c\,|01\rangle\right)_{1,2} |0\rangle_{\mathrm{A}} + d\,|001\rangle_{1,2,\mathrm{A}}\,, \end{aligned} \tag{6}$$

where the cloning coefficients can be determined by

$$\begin{aligned} a &= \frac{1 + \cos 2\theta + \sin 2\theta}{2\sqrt{2}}, \\ b &= \frac{1 + \cos 2\theta - \sin 2\theta}{2\sqrt{2}}, \\ c &= \frac{1 - \cos 2\theta + \sin 2\theta}{2\sqrt{2}}, \\ d &= \frac{-1 + \cos 2\theta + \sin 2\theta}{2\sqrt{2}} \end{aligned} \tag{7}$$

with $\theta \in [0, \pi/4]$. The fidelities of two clones are

$$\begin{aligned} F_1^{(\mathrm{AR})} &= (1 + \cos 2\theta)/2, \\ F_2^{(\mathrm{AR})} &= (1 + \sin 2\theta)/2. \end{aligned} \tag{8}$$

The fidelity given in Eq.(8) is coincident with our previous result.[24,25] When $\theta = \pi/8$, the optimal asymmetric real state cloner can be reduced to the optimal symmetric one with the fidelity $F_{1(2)}^{(\mathrm{SR})} = \frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right)$ under the cloning coefficients $a = \frac{1}{2} + \frac{1}{\sqrt{8}}$, $b = c = \frac{1}{\sqrt{8}}$, and $d = \frac{1}{2} - \frac{1}{\sqrt{8}}$. In the following section, we present a quantum network to implement three optimal cloners.

3. Implementation of three optimal quantum cloners

The quantum network in the present paper is similar to the one in Ref.[34], but is just slightly modified. In the structure of the quantum network, only two operators are needed. One is a single-qubit rotation $R_j(\theta)$ acting on the basis vectors of qubits, defined by

$$\begin{aligned} R_j(\theta)\,|0\rangle_j &= \cos\theta\,|0\rangle_j + \sin\theta\,|1\rangle_j\,, \\ R_j(\theta)\,|1\rangle_j &= -\sin\theta\,|0\rangle_j + \cos\theta\,|1\rangle_j\,. \end{aligned} \tag{9}$$

The other is a C-Not operation $P_{k,l}$ performing on the target qubit (k denotes the control qubit and l

the target), defined as

$$P_{k,l}\left|0\right\rangle_k\left|0\right\rangle_l=\left|0\right\rangle_k\left|0\right\rangle_l,$$
$$P_{k,l}\left|0\right\rangle_k\left|1\right\rangle_l=\left|0\right\rangle_k\left|1\right\rangle_l,$$
$$P_{k,l}\left|1\right\rangle_k\left|0\right\rangle_l=\left|1\right\rangle_k\left|1\right\rangle_l,$$
$$P_{k,l}\left|1\right\rangle_k\left|1\right\rangle_l=\left|1\right\rangle_k\left|0\right\rangle_l. \tag{10}$$

In Fig.1, we give the quantum network. The optimal cloning can be divided into two processes: the preparation and the cloning.

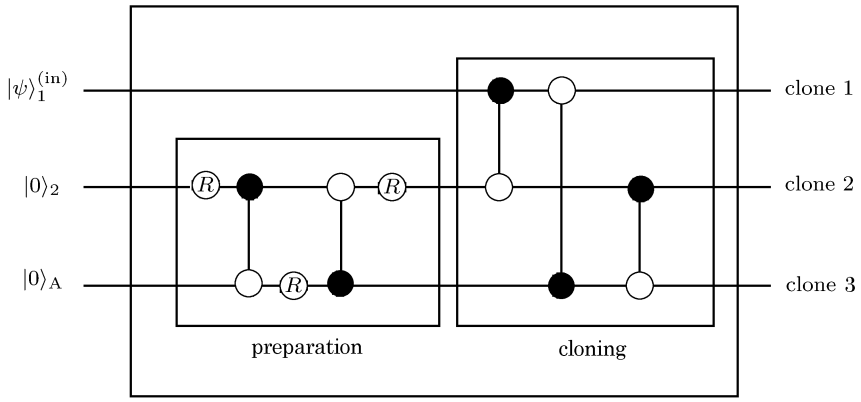


Fig.1. Optimal quantum cloning network.

The process of the preparation can be expressed as

$$\begin{aligned}|\psi\rangle_{2,\mathrm{A}}^{(\mathrm{prep})}=&R_2\left(\theta_1\right)P_{2,\mathrm{A}}R_{\mathrm{A}}\left(\theta_2\right)P_{A,2}R_2\left(\theta_3\right)\left|00\right\rangle_{2,\mathrm{A}}\\=&(C_1\left|00\right\rangle+C_2\left|10\right\rangle+C_3\left|01\right\rangle\\&+C_4\left|11\right\rangle)_{2,\mathrm{A}},\end{aligned} \tag{11}$$

and the process of the cloning can then be determined by

$$\begin{aligned}|\psi\rangle_{1,2,\mathrm{A}}^{(\mathrm{out})}=&P_{2,\mathrm{A}}P_{\mathrm{A},1}P_{1,2}\left|\psi\right\rangle_1^{(\mathrm{in})}\left|\psi\right\rangle_{2,\mathrm{A}}^{(\mathrm{prep})}\\=&\alpha[C_1\left|000\right\rangle+(C_2\left|01\right\rangle+C_3\left|10\right\rangle)\left|1\right\rangle\\&+C_4\left|110\right\rangle]_{1,2,\mathrm{A}}\\&+\beta\mathrm{e}^{\mathrm{i}\varphi}[C_1\left|111\right\rangle+(C_2\left|10\right\rangle+C_3\left|01\right\rangle)\left|0\right\rangle\\&+C_4\left|001\right\rangle]_{1,2,\mathrm{A}}.\end{aligned} \tag{12}$$

After the whole process, the cloning coefficients of the output state are given as

$$\begin{aligned}&\cos\theta_1\cos\theta_2\cos\theta_3+\sin\theta_1\sin\theta_2\sin\theta_3=C_1,\\&\cos\theta_1\cos\theta_2\sin\theta_3-\sin\theta_1\sin\theta_2\cos\theta_3=C_2,\\&-\cos\theta_1\sin\theta_2\sin\theta_3+\sin\theta_1\cos\theta_2\cos\theta_3=C_3,\\&\cos\theta_1\sin\theta_2\cos\theta_3+\sin\theta_1\cos\theta_2\sin\theta_3=C_4,\end{aligned} \tag{13}$$

Let us determine the angles of the single-qubit rotation $R_j\left(\theta\right)$ for different optimal cloners.

For the optimal asymmetric universal quantum cloning, we determined the angles of the single-qubit rotation $R_j\left(\theta\right)$

$$\theta_1+\theta_3=\frac{\pi}{4},$$

$$\begin{aligned}\sin\theta_2&=\frac{\sqrt{2}\sin\theta_1\left(\sin\theta_1-\cos\theta_1\right)}{\sqrt{2+\sin 4\theta_1}},\\\cos\theta_2&=\frac{\sqrt{2}\cos\theta_1\left(\sin\theta_1+\cos\theta_1\right)}{\sqrt{2+\sin 4\theta_1}},\\\sin\theta_3&=\frac{\cos\theta_1-\sin\theta_1}{\sqrt{2}},\\\cos\theta_3&=\frac{\cos\theta_1+\sin\theta_1}{\sqrt{2}},\end{aligned} \tag{14}$$

where $\theta_1,\theta_3\in[0,\pi/4]$. Under our choice of the angles of the single-qubit rotation $R_j\left(\theta\right)$, the cloning coefficients of the optimal asymmetric universal quantum cloning can be expressed as

$$\begin{aligned}C_1=p+q&=\frac{\cos 2\theta_1+\sin 2\theta_1}{\sqrt{2+\sin 4\theta_1}},\\C_2=p&=\frac{\cos 2\theta_1}{\sqrt{2+\sin 4\theta_1}},\\C_3=q&=\frac{\sin 2\theta_1}{\sqrt{2+\sin 4\theta_1}},\quad C_4=0.\end{aligned} \tag{15}$$

When $\theta_1=\theta_3=\pi/8$ and $\theta_2=$

$-\arcsin\left(\frac{1}{2}-\frac{\sqrt{2}}{3}\right)^{1/2}$, the optimal asymmetric universal quantum cloning can be reduced to the symmetric one with the cloning coefficients $p=q=\sqrt{1/6}$.

For the optimal asymmetric real state cloning, we determined the angles of the single-qubit rotation $R_j(\theta)$ as

$$\theta_1+\theta_3=\pi/4, \quad \theta_2=0, \tag{16}$$

where $\theta_1,\theta_3 \in [0,\pi/4]$. Under the relation $(\theta_3=\pi/4-\theta_1)$, it is easily calculated that

$$\begin{aligned} C_1=a&=\frac{1+\cos 2\theta_1+\sin 2\theta_1}{2\sqrt{2}},\\ C_2=b&=\frac{1+\cos 2\theta_1-\sin 2\theta_1}{2\sqrt{2}}\\ C_3=c&=\frac{1-\cos 2\theta_1+\sin 2\theta_1}{2\sqrt{2}},\\ C_4=d&=\frac{-1+\cos 2\theta_1+\sin 2\theta_1}{2\sqrt{2}}. \end{aligned} \tag{17}$$

When $\theta_1=\theta_2=\pi/8$, the optimal asymmetric real state cloner can be reduced to the symmetric one with the cloning coefficients $a=\frac{1}{2}+\frac{1}{\sqrt{8}}$, $b=c=\frac{1}{\sqrt{8}}$, $d=\frac{1}{2}-\frac{1}{\sqrt{8}}$.

For the optimal symmetric phase-covariant cloning, we determined the angles of the single-qubit rotation $R_j(\theta)$ as

$$\theta_1=\theta_3=\frac{1}{2}\left(\pi+\arccos\frac{1}{\sqrt{3}}\right), \quad \theta_2=\frac{7\pi}{12}. \tag{18}$$

With the explicit angles of the single-qubit rotation $R_j(\theta)$ given by Eq.(18), one can readily calculate

$$C_1=\frac{1}{\sqrt{2}}, \quad C_2=C_3=\frac{1}{2}, \quad C_4=0. \tag{19}$$

Here, we complete our goal. One can observe that the same quantum network can implement three different optimal cloners by only choosing different angles of the single-qubit rotation $R_j(\theta)$.

4. Summary

In this paper, we present a quantum network to implement the optimal quantum cloning. By choosing different angles of the single-qubit rotation $R_j(\theta)$, the quantum network can implement three different cloners. As known, the single-qubit rotation is a mature technique at present. If C-Not operation can be realized in different physical systems, the present quantum network can be utilized to realize three different cloners easily.

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