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Quantum entanglement and nonlocality properties of two-mode Gaussian squeezed states*

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Quantum entanglement and nonlocality properties of a family of two-mode Gaussian pure states have been investigated. The results show that the entanglement of these states is determined by both the two-mode squeezing parameter and the difference of the two single-mode squeezing parameters. For the same two-mode squeezing parameter, these states show larger entanglement than the usual two-mode squeezed vacuum state. The violation of Bell inequality depends strongly on all the squeezing parameters of these states and disappears completely in the limit of large squeezing. In particular, these states can exhibit much stronger violation of local realism than two-mode squeezed vacuum state in the range of experimentally available squeezing values.

Keywords: entanglement, nonlocality, Gaussian state
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1. Introduction

Being the simplest nontrivial instance of multipartite entangled Gaussian states, the two-mode squeezed vacuum state (TMSVS) has attracted more and more interest in recent years, because of its crucial role in quantum information processing (QIP) with continuous variables (CV) including quantum teleportation,[1−3] quantum entanglement swapping,[4] quantum key distribution (QKD).[5] It has been shown that the performance of the above schemes depends critically upon how squeezed the state can be made; the larger the degree of squeezing, the higher the quality of these schemes. Unfortunately, the available values of squeezing are so far in the range of $0 \sim 1$ and higher levels of squeezing are very hard to achieve.[6,7] Therefore, finding a Gaussian state with higher entanglement than TMSVS is of particular importance in QIP with CV.

Quantum nonlocality is another significant feature of quantum mechanics. Starting from the pioneering work of Einstein *et al*,[8] the problem of quantum nonlocality versus local realism has been extensively investigated.[9−11] In the last decade, a large number of CV versions of Bell's inequalities, based on Wigner function measurements,[12−14] construction of direct analogue to the Pauli spin operators,[15] and Schwinger spin operators in infinite-dimensional Hilbert space,[16,17] have been proposed. Among them, tests of Bell's inequality with Wigner distribution function have been of increasing interest, due to the fact that the Wigner functions for some quantum states can be reconstructed and measured in experiment.[18−20]

Although its experimental realization is seldom investigated, the theoretical scheme of testing Bell's inequality related to two-mode Wigner function has been proposed. Banaszek *et al*[12] and Wu[14] have individually discussed the violation of Bell's inequalities by using the Wigner function of two-mode running fields. They found that TMSVS can violate Bell's inequality. The larger the initial squeezing parameter r, the more rapidly the maximal violation of Bell's inequalities decreases. Wu *et al*[21] have also investigated the nonlocality of N-mode squeezed state and pointed out that the violation is enhanced with increasing dimension N. Inspired by these investigations, we here present another class of two-mode Gaussian pure states. We shall study the entanglement properties of these states by means of the logarith-

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mic negativity and discuss the influence of the three squeezing parameters on the maximal violation of the Bell's inequalities by using the two-mode Wigner function measurement.

The paper is organized as follows. In the next section we recall the concept and the basic properties of two-mode Gaussian state. Then, in Section 3 we introduce a class of two-mode Gaussian pure states and evaluate their entanglement by means of the logarithmic negativity. In Section 4 we analyse quantum nonlocality of these states. Our conclusion is presented in Section 5.

2. General overview of two-mode Gaussian state

Let us now set the notation and review the known results about two-mode Gaussian state of CV systems that we need in the following section. The Hilbert space of a CV system of two canonical bosonic modes is a tensor product of the subsystems' Hilbert spaces, i.e., $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$. Each mode is described by dimensionless quadrature operators defined as $\hat{x}_j = (\hat{a}_j^\dagger + \hat{a}_j)/\sqrt{2}$ and $\hat{p}_j = \mathrm{i}(\hat{a}_j^\dagger - \hat{a}_j)/\sqrt{2}$ $(j = 1, 2)$. These operators satisfy the bosonic commutation relation $[\hat{x}_i, \hat{p}_j] = \mathrm{i}\delta_{ij}$, which is equivalent to $[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij}$. Introducing the vector $\hat{X}^{\mathrm{T}} := (\hat{x}_1, \hat{p}_1, \hat{x}_2, \hat{p}_2)$, we can write the canonical commutation relations for the X_i's in the compact form as

$$[\hat{X}_i, \hat{X}_j] = 2\mathrm{i}\Omega_{ij}, \tag{1}$$

where Ω is the standard symplectic form

$$\Omega = \bigoplus_{k=1,2} \omega_k, \qquad \omega_k = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{2}$$

A state is called Gaussian if and only if its characteristic function is a Gaussian function in phase space, i.e., $\phi(X) = \exp\left(-\frac{1}{2}X^{\mathrm{T}}MX + \mathrm{i}\overline{X}X\right)$ with $\overline{X} = (\langle\hat{X}_1\rangle, \langle\hat{X}_2\rangle, \langle\hat{X}_3\rangle, \langle\hat{X}_4\rangle)$ being its first statistical moment and $\boldsymbol{M}$ its covariance matrix (CM), whose generic elements are defined as

$$M_{ij} = \frac{1}{2}\langle\hat{X}_i\hat{X}_j + \hat{X}_j\hat{X}_i\rangle - \langle\hat{X}_i\rangle\langle\hat{X}_j\rangle. \tag{3}$$

First moments can be arbitrarily adjusted by local unitary operations in phase space, which cannot affect any quantity related to entanglement or mixedness of Gaussian state. Therefore, we will adjust the first moments to zero without any loss of generality for the scopes of our analysis. On the other hand, the covariance matrix M contains all the information about the quantum correlations between two CV subsystems and can be written, in terms of the three 2×2 matrices $\boldsymbol{L}_1$, $\boldsymbol{L}_2$ and $\boldsymbol{G}$, as

$$M = \begin{pmatrix} \boldsymbol{L}_1 & \boldsymbol{G} \\ \boldsymbol{G}^{\mathrm{T}} & \boldsymbol{L}_2 \end{pmatrix}, \tag{4}$$

where $\boldsymbol{L}_1$ and $\boldsymbol{L}_2$ are Hermitian matrices containing only local elements, while $\boldsymbol{G}$ is the correlation between two subsystems. For any physically acceptable Gaussian state,[22,23] its covariance matrix must be positive semidefinite and satisfies the uncertainty principle

$$M + \mathrm{i}\Omega \geq 0. \tag{5}$$

In terms of the constraint on the symplectic invariants $\det M$ and $\Delta(M) = \det\boldsymbol{L}_1 + \det\boldsymbol{L}_2 + 2\det\boldsymbol{G}$, the uncertainty relation (5) can be rewritten as

$$\Delta(M) \leq 1 + \det M. \tag{6}$$

The necessary and sufficient condition for separability of a $1 \times N$ Gaussian state is the positivity of the partial transposed density matrix (PPT criterion). The partial transpose operation on Gaussian state with M corresponds to flipping the sign of $\det\boldsymbol{G}$. If we set $\widetilde{\Delta}(M) = \det\boldsymbol{L}_1 + \det\boldsymbol{L}_2 - 2\det\boldsymbol{G}$, then the PPT criterion can be expressed as

$$\widetilde{\Delta}(M) \leq 1 + \det M. \tag{7}$$

The partial transposed matrix of the covariance matrix M has the symplectic eigenvalues

$$\widetilde{\nu}_\pm = \frac{1}{\sqrt{2}} \times \sqrt{\widetilde{\Delta}(M) \pm \sqrt{\widetilde{\Delta}(M)^2 - 4\det(M)}}. \tag{8}$$

If $\widetilde{\nu}_-$ is the smallest eigenvalue, the PPT criterion for two-mode Gaussian state becomes the very simple inequality

$$\widetilde{\nu}_- \geq \frac{1}{2}. \tag{9}$$

We now turn our attention to quantifying the entanglement of bipartite Gaussian states. An important class of entanglement monotones is the negativity, $\mathcal{N}(\rho)$, which is defined as[24]

$$\mathcal{N}(\rho) = \frac{1}{2}\left(||\rho^{\mathrm{T}}||_1 - 1\right), \tag{10}$$

where ρ^{T} denotes the partially transposed density matrix and $||\hat{W}||_1 = \mathrm{Tr}\sqrt{\hat{W}^\dagger\hat{W}}$ is the trace norm of the operator $\hat{W}$. Very closely related to the negativity,

another entanglement measure is the logarithmic negativity

$$E_N(\rho) := \log_2 \left(||\rho^{\mathrm{T}}||_1 \right), \tag{11}$$

which constitutes an upper bound to the distillable entanglement of the quantum state. Both the negativity and the logarithmic negativity have been proven to possess the crucial property of being monotonic under local operations and classical communication (LOCC). In terms of the smallest symplectic eigenvalue (8), the logarithmic negativity of the two-mode Gaussian state can be written as

$$E_N(\rho) = \max\Big\{0, -\log_2 2\widetilde{\nu}_-\Big\}. \tag{12}$$

This expression quantifies the amount by which PPT inequality (9) is violated. The symplectic eigenvalue, $\widetilde{\nu}_-$, thus completely qualifies and quantifies the quantum entanglement of a $1 \times N$ Gaussian state. In the next section, we will use the logarithmic negativity to measure the amount of entanglement of the introduced two-mode Gaussian states.

3. Entanglement properties of two-mode Gaussian squeezed states

Two decades ago, Schumaker[25] introduced the most general two-mode Gaussian pure states (GPSs), namely $|z_1, z_2, z_{1,2}, \alpha_1, \alpha_2\rangle := \hat{S}_1(z_1)\hat{S}_2(z_2)\hat{S}_{12}(z_{1,2})|\alpha_1, \alpha_2\rangle$, where $\hat{S}_j(z_j) = \exp\left[-\frac{r_j}{2}(\mathrm{e}^{\mathrm{i}\theta_j}\hat{a}_j^{+2} - \mathrm{e}^{-\mathrm{i}\theta_j}\hat{a}_j^2)\right]$ is the single-mode squeeze operator for the jth mode ($j = 1, 2$), while $\hat{S}_{12}(z_{12}) = \exp\left[-r_3\left(\mathrm{e}^{\mathrm{i}\theta_3}\hat{a}_1^\dagger\hat{a}_2^\dagger - \mathrm{e}^{\mathrm{i}\theta_3}\hat{a}_1\hat{a}_2\right)\right]$ is the two-mode squeeze operator. In this paper, we present another class of two-mode GPSs of the form

$$|\psi\rangle = |z_{12}, z_2, z_1\rangle := \hat{S}_{12}(z_{12})\hat{S}_2(z_2)\hat{S}_1(z_1)|0, 0\rangle. \tag{13}$$

For convenience, we call the states (13) two-mode Gaussian squeezed states (TGSSs). Our choice is based on the following two reasons. One is that the single- and two-mode squeezers have been experimentally realized so far.[26] The other is that any TGSSs can be obtained by first acting with two single-mode squeeze operators on the vacuum and then squeezing it through a two-mode squeeze operator. This implementation leads to the fact that TGSSs can exhibit some interesting and appealing properties of quantum entanglement and nonlocality, as we will see below. We also note that such an analogous implementation has been applied to produce light source with sub-Poissonian statistics,[27] three-photon entangled state[28] and squeezed Bell-like states.[29] While for Schumaker's state, it is in fact equivalent locally to two-mode squeezed vacuum state. In what follows, we study entanglement properties of these states by means of the logarithmic negativity.

For the sake of simplicity, we consider a case of $\theta_j = 0$ ($j = 1, 2, 3$). The characteristic function for the two-mode state ρ in the Wigner representation is defined as $\chi(\beta_1, \beta_2) = \mathrm{Tr}\left[\rho \exp\left(-\beta_1^*\hat{a}_1 + \beta_1\hat{a}_1^\dagger - \beta_2^*\hat{a}_2 + \beta_2\hat{a}_2^\dagger\right)\right]$. Then after some algebraic manipulation, we arrive at the characteristic function of the states (13) as $\chi(\beta_1, \beta_2) = \exp\left\{-\frac{1}{2}\Lambda^{\mathrm{T}}M\Lambda\right\}$ with $\Lambda = (\beta_1^*, \beta_1, \beta_2^*, \beta_2)^{\mathrm{T}}$, where M is a 4×4 covariance matrix and has the form

$$M = \frac{1}{2} \times \begin{pmatrix} A_{11} & 2A_{12} & C & D \\ 2A_{12} & A_{11} & D & C \\ C & D & A_{21} & 2A_{22} \\ D & C & 2A_{22} & A_{21} \end{pmatrix}; \tag{14}$$

the elements of the matrix M are given by

$$\begin{aligned} A_{11} &= \cosh(2r_1)\cosh^2(r_3) + \cosh(2r_2)\sinh^2(r_3), \\ A_{12} &= \frac{1}{2} \times [\sinh(2r_1)\cosh^2(r_3) \\ &\quad + \sinh(2r_2)\sinh^2(r_3)], \\ A_{21} &= \cosh(2r_2)\cosh^2(r_3) + \cosh(2r_1)\sinh^2(r_3), \\ A_{22} &= \frac{1}{2} \times [\sinh(2r_2)\cosh^2(r_3) \\ &\quad + \sinh(2r_1)\sinh^2(r_3)], \\ C &= \frac{1}{2} \times \sinh(2r_3)\left[\sinh(2r_1) + \sinh(2r_2)\right], \\ D &= \frac{1}{2} \times \sinh(2r_3)\left[\cosh(2r_1) + \cosh(2r_2)\right]. \end{aligned} \tag{15}$$

Using local linear unitary Bogoliubov operations,[30] we can change the covariance matrix (14) into the following standard form:

$$M^{Sf} = \begin{pmatrix} n & 0 & c_+ & 0 \\ 0 & n & 0 & c_- \\ c_+ & 0 & n & 0 \\ 0 & c_- & 0 & n \end{pmatrix}, \tag{16}$$

where the elements of the matrix (16) are given by

$$n^2 = \frac{1}{4}\Big[\cosh^4(r_3) + \sinh^4(r_3)$$

$$+\frac{1}{2}\sinh^2(2r_3)\cosh 4(r_1-r_2)\Big], \quad (17)$$

$$c_+c_- = -\frac{1}{8}\sinh^2(2r_3)\Big[1+\cosh 4(r_1-r_2)\Big]. \quad (18)$$

This parametrization is particularly useful for the evaluation of entanglement of the two-mode Gaussian state. From Eq.(16), we see that TGSSs are symmetric bipartite Gaussian states and find that if the single-mode squeezing vanishes, then the matrix M is simplified to that of the two-mode squeezed vacuum state.[31] Based on Eq.(83) in Ref.[32], we easily obtain the necessary condition for TGSSs to be entangled as $r_3 \neq 0$. This is because in the limit of vanishing of two-mode squeezing, we have $c_+c_- = 0$. As a result, quantum correlation of TGSSs disappears completely.

After some tedious but straightforward algebraic calculation, we can obtain the symplectic eigenvalues of the partial transposed matrix of the covariance matrix M as

$$\widetilde{\nu}_\pm = \sqrt{(n \pm c_+)(n \mp c_-)}. \quad (19)$$

Furthermore, in the case of an entangled state, the smallest eigenvalue is expressed as $\widetilde{\nu}_- = \sqrt{(n-|c_+|)(n+|c_-|)}$. It is obvious that for $\widetilde{\nu}_\pm \geq 1/2$, the two-mode Gaussian state is separable, otherwise it is entangled. It is not difficult to verify that the symplectic eigenvalue $\widetilde{\nu}_+$ in Eq.(19) is always greater than $1/2$ in any condition, and has no effect on establishing the nonseparability of these states. The eigenvalue $\widetilde{\nu}_-$ may be less than $1/2$ and determines the nonseparability of TGSSs. The smaller the $\widetilde{\nu}_-$, the more entangled the corresponding Gaussian state. Note that for a special case of $r_1 = r_2$, the smallest eigenvalue $\widetilde{\nu}_-$ reduces to $\mathrm{e}^{-2r_3}/2$, and then we can compute the logarithmic negativity as

$$E_N(\rho)|_{r_1=r_2} = \frac{2}{\ln 2} r_3, \quad (20)$$

which implies that in this case, the entanglement of TGSSs is uniquely determined by the two-mode squeezing parameter r_3 regardless of the initial amount of single-mode squeezing and equals the amount of entanglement of two-mode squeezed vacuum state. This can be explained as follows: We see from Eqs.(17) and (18) that if $r_1 = r_2$ such that one has $\cosh 4(r_1-r_2) = 1$, then the entanglement of these states is only determined by the two-mode squeezing parameter r_3 and irrelative to the two single-mode squeezing parameters. But for $r_1 - r_2 \neq 0$, we see that the smallest symplectic eigenvalue is related to both the two-mode squeezing parameter r_3 and the difference, $\Delta r = |r_1 - r_2|$, of the two single-mode squeezing parameters. In the following, we investigate the influence of these parameters on the entanglement of TGSSs.

Figure 1 illustrates the dependence of the logarithmic negativity $E_N(\rho)$ of TGSSs on the two-mode squeezing parameter r_3 for different Δr. We see that $E_N(\rho)$ is always greater than zero as long as $r_3 \neq 0$. That is, the TGSSs become entangled only if the two-mode squeezing does not vanish. High two-mode squeezing means high entanglement. On the other hand, we note that the entanglement is not affected by the two individual single-mode squeezing parameters but by their difference, $\Delta r = |r_1 - r_2|$. For a fixed value of r_3, the larger the difference Δr, the more entangled the states (13). We also see an interesting phenomenon in Fig.1. When both modes are equally squeezed, their degree of entanglement is the smallest for the same two-mode squeezing. Namely, we always observe $E_N(\rho)|_{r_1=r_2} \leq E_N(\rho)$. That is to say, at given two-mode squeezing, the two-mode squeezed vacuum state (TMSVS) that has been frequently used as entanglement source for existing CV QIP has the smallest degree of entanglement among all TGSSs. This, in turn, shows that one can prepare a Gaussian state with larger entanglement than TMSVS by unequal squeezing on the two modes at the initial stage even if the squeezing is weak. For example, if we choose r_3 to be 0.8, then we have an entanglement measure $E_N(\rho) = 2.9002$ for $\Delta r = 1$,

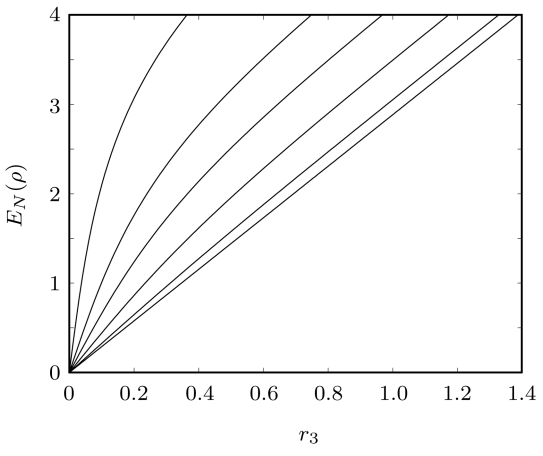


Fig.1. Logarithmic negativity $E_N(\rho)$ of TGSSs as a function of the two-mode squeezing parameter r_3 for different values of Δr. The curves from the bottom to top correspond to Δr = 0, 0.5, 1, 1.5, 2, and 3. The amount of $E(\rho)$ is shown in unit of bits.

while for $\Delta r = 0$, an entanglement measure is given by $E_N(\rho) = 2.3083$. Thus, we expect that the states (13) are more suitable for CV quantum communication and computation than TMSVS.

4. Bell-inequality violations of TGSSs

To date, there have been developed three strategies to reveal quantum nonlocality in the framework of CV system. In this section, we will use the formalism of parity operator correlations introduced in Refs.[12,13] to investigate the nonlocal property of TGSSs. A two-mode parity operator is defined as $\hat{\Pi}_{ab}(\alpha,\beta) = \hat{\Pi}_a(\alpha)\otimes\hat{\Pi}_b(\beta) = \hat{D}(\alpha)\hat{D}(\beta)(-1)^{\hat{a}^\dagger\hat{a}+\hat{b}^\dagger\hat{b}}\hat{D}^\dagger(\beta)\hat{D}^\dagger(\alpha)$. The expectation value of this operator is proportional to the two-mode Wigner function

$$\Pi_{ab}(\alpha,\beta) = \langle\hat{\Pi}_{ab}(\alpha,\beta)\rangle = \frac{1}{4}W(\alpha,\beta). \tag{21}$$

As is well known, any two-mode Gaussian state may be written, in terms of the Wigner representation, as

$$W(\alpha,\beta) = \sqrt{\det\boldsymbol{W}}\exp\Big(-\frac{1}{2}\Lambda^{\mathrm{T}}\boldsymbol{W}\Lambda\Big), \tag{22}$$

where the 4×4 Hermitian matrix $\boldsymbol{W}$ can be obtained from $\boldsymbol{M}$ using the relation between the Weyl–Wigner characteristic function and the Wigner function. They are related by a Fourier transform which yields

$$\boldsymbol{W} = \boldsymbol{E}\boldsymbol{M}^{-1}\boldsymbol{E}, \quad \boldsymbol{M} = \boldsymbol{E}\boldsymbol{W}^{-1}\boldsymbol{E}, \tag{23}$$

where the matrix $\boldsymbol{E} = \mathrm{diag}[1,-1,1,-1]$. It is easy to verify that for any pure two-mode Gaussian state, one has $|\boldsymbol{W}| = 16$.

Thus, according to Eqs.(14), (22) and (23), we obtain the Wigner function of the TGSSs as

$$\begin{aligned} &W(\alpha,\beta)\\ &= 4\exp\Big\{-2\Big[A_{11}|\alpha|^2 + A_{12}(\alpha^2+\alpha^{*2})\\ &\quad +A_{21}|\beta|^2 + A_{22}(\beta^2+\beta^{*2})\\ &\quad +C(\alpha^*\beta+\alpha\beta^*) + D(\alpha\beta+\alpha^*\beta^*)\Big]\Big\}, \end{aligned} \tag{24}$$

where A_{ij}, C and D are given by Eq.(15). We find that if $r_1 = r_2 = 0$, then equation (24) reduces to the Wigner function for TMSVS, i.e., $W_{\mathrm{TMSVS}}(\alpha,\beta) = 4\exp\{-2[\cosh(2r_3)(|\alpha|^2+|\beta|^2) + \sinh(2r_3)(\alpha\beta+\alpha^*\beta^*)]\}$.[33]

We can now consider the Clauser–Horne combination[10,11]

$$\begin{aligned} B =\ &\Pi_{ab}(0,0) + \Pi_{ab}(\alpha,0) + \Pi_{ab}(0,\beta)\\ &-\Pi_{ab}(\alpha,\beta). \end{aligned} \tag{25}$$

Since the parity operators $\hat{\Pi}_a(\alpha)$ and $\hat{\Pi}_b(\beta)$ have their expectation values bounded between $+1$ and -1, the prediction of any local hidden variable theory is $|B| \le 2$.

Substituting Eq.(21) into Eq.(25) gives

$$B = \frac{1}{4}\times\Big[W(0,0) + W(\alpha,0) + W(0,\beta) - W(\alpha,\beta)\Big]. \tag{26}$$

Then we can test Bell's inequality $-2 \le B \le 2$ by means of the two-mode Wigner function measurement.

Using Eq.(24) and setting $\alpha = \sqrt{J}\mathrm{e}^{\mathrm{i}\theta_\alpha}$ and $\beta = \sqrt{J}\mathrm{e}^{\mathrm{i}\theta_\beta}$, we can write the Bell combination as

$$\begin{aligned} B =\ &1 + \exp\Big\{-2J\Big[A_{11} + 2A_{12}\cos2\theta_\alpha\Big]\Big\}\\ &+\exp\Big\{-2J\Big[A_{21} + 2A_{22}\cos2\theta_\beta\Big]\Big\}\\ &-\exp\Big\{-2J\Big[A_{11} + A_{21} + 2\Big(A_{12}\cos2\theta_\alpha\\ &+A_{22}\cos2\theta_\beta + C\cos\theta_- + D\cos\theta_+\Big)\Big]\Big\}, \end{aligned} \tag{27}$$

where $\theta_\pm = \theta_\alpha \pm \theta_\beta$. From the above expression, we see that B is determined by both the initial squeezing parameters of TGSSs and the individual phases, θ_α and θ_β. But it is a formidable task to work out directly the maximal violation of Bell inequalities for TGSSs. Then we start to seek out the individual phases that maximize the violation of Bell inequality by means of numerical calculation. We have plotted the Bell combination B as a function of the phases α and β, with fixed $r_1 = 0.4, r_2 = 0.6, r_3 = 0.5$ and $J = 0.01$, as shown in Fig.2.

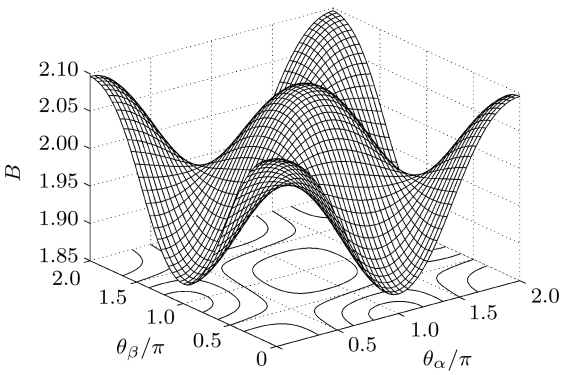


Fig.2. The Bell combination for TGSSs as a function of the phases $(\theta_\alpha,\theta_\beta)$ of α and β with the parameters $r_1 = 0.4$, $r_2 = 0.6$, $r_3 = 0.5$ and $J = 0.01$.

We can easily see from Fig.2 that when $(\theta_\alpha, \theta_\beta) = (0,0), (0,2\pi), (\pi,\pi), (2\pi,0)$, or $(2\pi,2\pi)$, B reaches its maximum

$$\begin{aligned} B_{\max} = {} & 1+\exp\Big[-2J\Big(\mathrm{e}^{2r_1}+\mathrm{e}^{2r_2}\Big)\sinh^2(r_3)\Big] \\ & \times\Big[\exp\Big(-2J\mathrm{e}^{2r_1}\Big)+\exp\Big(-2J\mathrm{e}^{2r_2}\Big)\Big] \\ & -\exp\Big\{-2\Big[\mathrm{e}^{2r_1}+\mathrm{e}^{2r_2}\Big]\mathrm{e}^{2r_3}J\Big\}. \end{aligned} \tag{28}$$

Thus, the states (13) are said to be quantum mechanically nonlocal as $B_{\max}$ is larger than 2 and the nonlocality is stronger as $B_{\max}$ gets larger. From the expression (28), one sees that the Bell-inequality violation depends on the two-mode squeezing parameter r_3 of and the two single-mode squeezing parameters $r_{1,2}$ of TGSSs, but we find that the parameter r_3 plays a central role in testing the nonlocality of the quantum state. This is because for the case of $r_3 = 0$, $B_{\max}$ is not greater than 2, which implies that the nonlocality of these states never occurs regardless of the values of $r_{1,2}$. Therefore, the necessary condition for violating quantum nonlocality for the states (13) is that the two-mode squeezing does not vanish. In what follows, we study the influence of all the squeezing parameters of such states on the $B_{\max}$.

Figure 3(a) shows the curves of the Bell combination, $B_{\max}$, as a function of r_3 and J with fixed $r_1 = 0.2$ and $r_2 = 0.6$, while figure 3(b) shows the curves of the Bell combination, $B_{\max}$, as a function of r_3 for different values of r_1 and r_2 with fixed $J = 0.01$. From Fig.3 it is easy to see that violation of the upper bound of Bell's inequalities is clear. In Fig.3(a), we find that the nonlocality for TGSSs can easily be destroyed as the coherent amplitude of the two modes increases. It shows that the inequality is violated only when the coherent amplitude is small enough. Also note that in the realistic range $r_3 \in [0,1]$,[6,7] the maximal Bell combination increases with the increase of the squeezing parameter r_3. However, it is always not true when the value of r_3 is out of this range. As can be seen in Fig.3(b), when the squeezing parameter r_3 increases, the maximal Bell combination initially increases, reaches a maximum, and then decreases to 1. This shows that there exists a value of r_3 that maximizes the Bell combination $B_{\max}$ at fixed r_1, r_2 and J. The optimal value of r_3, in principle, can be calculated by means of the expression $\partial B_{\max}/\partial r_3 = 0$. In addition, it is interesting to note that in the range of experimentally realistic values $r_j \in [0,1]$ of the squeezing, TGSSs can show much stronger in quantum nonlocality than the TMSVS, which has been extensively used to test the problem of quantum nonlocality versus local realism. Therefore, we think that TGSSs are potential candidates for testing the Bell-inequality violation of CV systems with current technology.

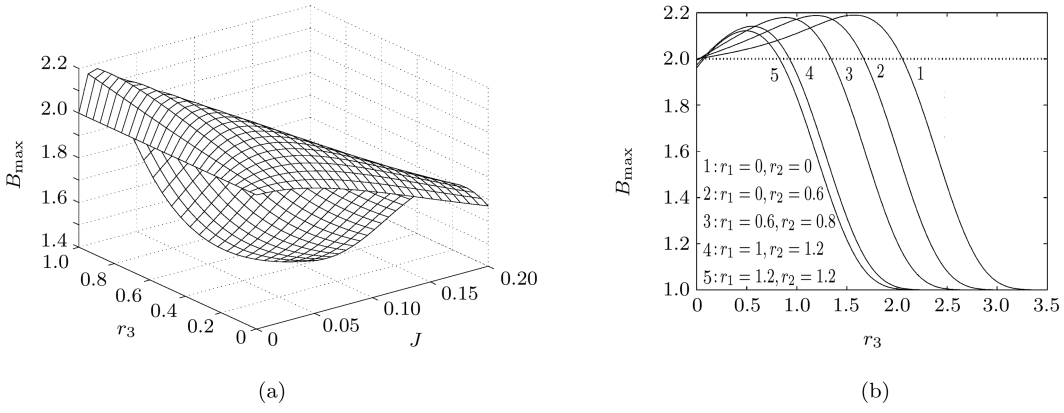


Fig.3. The Bell combination, $B_{\max}$, for TGSSs versus (a) the coherent amplitude J and the squeezing parameter r_3 with fixed $r_1 = 0.2$, $r_2 = 0.6$; (b) the squeezing parameter r_3 with various values of $r_{1,2}$ and fixed $J = 0.01$. When $r_1 = r_2 = 0$, TGSSs reduce to TMSVS.

The results of the numerical analysis of the Bell combination, $B_{\max}$, for TGSSs as a function of the squeezing parameters of r_1 and r_2 are shown in Fig.4. Clearly, we see that, for $r_3 \leq 1$, as the two single-mode squeezing parameters r_i $(i = 1,2)$ increase, the Bell combination, $B_{\max}$, appears to increase and approach its maximum, and then reduces to below 2. It suggests that in the case of $r_3 \leq 1$, TGSSs can exhibit the strongest violation of local realism if the parameters r_1 and r_2 are suitably chosen and indeed can be computed by means of the expressions $\partial B_{\max}/\partial r_1 = 0$ and $\partial B_{\max}/\partial r_2 = 0$. On the other hand, we see that

when the initial two-mode squeezing parameter r_3 is greater than 1, $B_{\max}$ decreases monotonically with the increase of squeezing parameters r_1 and r_2. The larger the initial degrees of the two single-mode squeezing parameters, the more rapidly the $B_{\max}$ decreases, and then the weaker the nonlocality becomes. Therefore, through previous analyses, we come to a conclusion that the violation of the Bell-inequality for TGSSs can occur only when the squeezing is weak. If the squeezing is chosen to be too large, quantum nonlocality can decrease much faster and further.

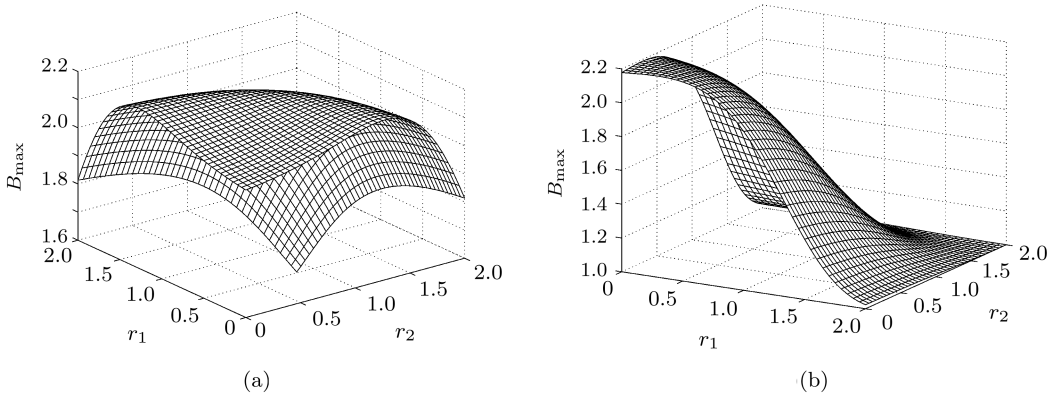


Fig.4. The Bell combination, $B_{\max}$, for TGSSs versus the squeezing parameters r_1 and r_2 with fixed $J = 0.01$ and (a) $r_3 = 0.6$, (b) $r_3 = 1.4$.

5. Conclusions

In summary, we have introduced a simple but interesting class of two-mode Gaussian pure states and studied their entanglement and nonlocality properties. We found that the necessary condition for TGSSs to be entangled is that the two-mode squeezing parameter r_3 is not zero and that the amount of entanglement is determined not only by the degree of the two-mode squeezing but also by the difference between the degrees of the two single-mode squeezing parameters. For quantum nonlocality of the states under consideration, our results showed that the violation of local realism can occur only when the squeezing parameters are small. Especially, for realistic values of these parameters, the TGSSs can violate the Bell inequality stronger than the TMSVS. Comparing with the usual TMSVS, we believe that the introduced states may be one of the promising candidates to implement the high-quality protocols of quantum information processing and to test the fundamental problem of quantum theory with present technology.

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