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A new local-world evolving network model*

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In some real complex networks, only a few nodes can obtain the global information about the entire networks, but most of the nodes own only local connections therefore own only local information of the networks. A new local-world evolving network model is proposed in this paper. In the model, not all the nodes obtain local network information, which is different from the local world network model proposed by Li and Chen (LC model). In the LC model, each node has only the local connections therefore owns only local information about the entire networks. Theoretical analysis and numerical simulation show that adjusting the ratio of the number of nodes obtaining the global information of the network to the total number of nodes can effectively control the valuing range for the power-law exponent of the new network. Therefore, if the topological structure of a complex network, especially its exponent of power-law degree distribution, needs controlling, we just add or take away a few nodes which own the global information of the network.

Keywords: complex networks, local-world evolving network, power-law
PACC: 0250, 0200

1. Introduction

In recent years, complex network theories have attracted much attention due to the gradual understanding of the topological structures of various natural, social, biological and communication networks,[1−5] including the internet,[6] World Wide Web (WWW),[7] citation network,[8] collaboration network,[9] word web,[10] etc. Studies of their structures show that most of them exhibit the scale-free degree distribution, that is to say, the degree distributions of those networks obey a power-law distribution with the exponent γ:

$$P(k) \sim k^{-\gamma}, \tag{1}$$

where $\gamma > 0$ is the most important parameter that describes how densely a network can be connected together. And the exponents of these networks mostly range from 2 to 3 according to the existing results.

In order to explain the emergence of the scale-free networks, Barabási *et al* [11] have demonstrated by using the mean-field theory that a scale-free network can be formed through two fundamental evolving mechanisms—growth and preferential attachment. Network growth means that the number of nodes increases with time as a whole, which is in accord with the fact that the scale of most natural networks grows gradually in their evolving process. The new nodes tend to be connected with old nodes with a large degree, which is the mechanism of preferential attachment. And this important evolving rule describes the edge-attachment characteristic of the evolving process of many natural networks in principle and explains the so-called 'rich get richer' phenomenon existing in natural societies.

In the preferential attachment mechanism of the scale-free network proposed by Barabási and Albert (BA), it is necessary that the new node must own the information of all existing nodes' degrees in the network. It means that all new nodes obtain the global information of the entire network. On the other hand, Li and Chen (LC)[12] have suggested that there should exist a local world of each node in various real-world complex networks. And the preferential attachment mechanism of a scale-free network does not work in the global network, but does work in a local world of each node. They have proposed a local-world evolving network model with the local-world preferential attachment mechanism. Moreover, when the selection approach of the local world in an evolving network model is changed, it is proved that the model represents a transition between power-law and exponential scaling and the BA scale-free model is only one of its

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special cases.

However, not all of the networks that follow a power-law degree distribution hold one of the above-mentioned two properties, that is to say, in some complex networks, there are a few nodes obtaining the global information about the entire networks, but most nodes own only local connections therefore own only local information of the networks.[12−16] For example, in the World Trade Web, the global preferential attachment mechanism is not applicable to those countries that have trade relationship with only a handful of countries. But as far as USA, China and Japan are concerned, they are promoting their economic cooperation in the world. These nodes can access to the overall business and trade information and satisfy the mechanism of preferential attachment in the web. In the Internet, computers are organized and managed on the basis of the same routers or domains. A mainframe is usually associated with the same area of the other mainframes only. So a new computer entering into the internet can obtain only the connected information of the computers in the local routers.[6] On the contrary, the internet backbone nodes of a country can own almost all of the connection information. Although the nodes obtaining the global information of the entire network are usually small in number, they are very important to the entire topological structure and act as hub nodes in the network. Meanwhile, the new node does not necessarily understand the global information of the entire network according to the specific characteristics of the real network topologies.

Let δ denote the ratio of the number of nodes obtaining the global information of the network to the total number of nodes. In this paper, a new local-world evolving network model is proposed, which introduces the above-mentioned ratio to distinguish the abilities for new nodes each to own the network connection information. Since the ratio of nodes obtaining the global information of the network is δ, the probability that a new node belongs to the global node set is δ. The nodes obtaining the global information of the network belong in the set of 'global nodes', while nodes owning only local information of their local worlds can be classified as the set of 'local nodes'. For the former, select M nodes from their existing network randomly as the 'local world' of the new node. For the latter, adopt the mechanism of the global preferential attachment to connect the new edges. The evolving processes of the LC model and the new local-world evolving network model are shown in Fig.1.

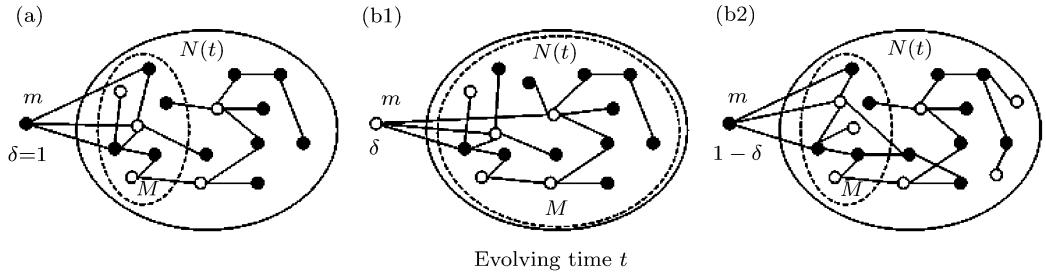


Fig.1. The evolving processes of the LC model and the new local-world network model. Panel (a) shows the evolving process of the LC model. At each time step, a new node which owns only local-world information of the network (denoted by solid circle •) is added to the network with m new edges. And the local-world of the new node which includes M nodes is selected from the existing network randomly. The new edges are connected with the old nodes with local preferential attachment mechanism. Panel (b) shows the corresponding process of the new local-world network model, if the new node belongs to the set of global nodes (denoted by open circle ◯), i.e. the node can obtain the global nodes. With the probability δ, the new node with m edges is added to the network and here its 'local world' is the entire network, which is shown in panel (b1). And the corresponding case of the local node is shown in panel (b2). In the case, with the probability $1-\delta$, m new edges are connected with the existing nodes according to the local preferential attachment rule.

We adopt the same selection approach of the local world as that introduced in Ref.[12] for our analysis. Let t be the evolving time and m_0 denote the number of the nodes in the original network. It is obvious that the number of the local world nodes, M, ranges from m to $t+m_0$, i.e. $m \leq M \leq t+m_0$. Hence, there are three possible cases for the values of M, i.e. $M = t+m_0$, $M = m$ and $m \ll M \ll t+m_0$. We study the degree distributions of the three cases and prove that the new local-world evolving network model ex-

hibits a scale-free topological structure in the latter two cases. Moreover, we present a functional relationship between the exponent of the scale-free network and the ratio δ. Theoretical analysis and numerical simulation indicate that the valuing scope of the power-law exponent for the new network can be effectively changed just by adjusting the value of ratio δ.

The remainder of this paper is devoted to the scaling properties of the LC local-world network model and the new local-world network model. In Section 2, the LC local-world network model is discussed. The degree distribution in three cases, along with the average shortest path length and the clustering coefficient of the new local-world evolving network model, is studied in Section. 3. Finally, the conclusions drawn from the present study is given in Section 4.

2. The LC local-world network model

In the LC local-world evolving network model, each node has only the local connection information and so owns only local information about the entire network. And M nodes selected randomly from the existing network are referred to as the "local world" of the new coming node. The network generation algorithm of the LC local-world network model is as follows:

(i) To start with, node number m_0 and edge number e_0 both are taken as small values.

(ii) Add a new node with m edges, connecting m nodes in its local world, using a local preferential attachment with probability $\prod_{\text{Local}}(k_i)$ determined at every time step t by

$$\prod\nolimits_{\text{Local}}(k_i) = \prod{}'(i \in \text{Local} - \text{world})\frac{k_i}{\sum_{j\ \text{Local}} k_j}. \tag{2}$$

At each time step t, there are m new edges, and they each attach a new node to m different nodes existing in the network. And the total number of edges in the network is m_0+t. Hence, the value of M is in a range $m \le M \le t+m_0$, and the probability that the node i is selected into the M-node local-world network is

$$\prod{}'(i \in \text{Local} - \text{world}) = \frac{M}{m_0+t}. \tag{3}$$

According to the mean-field theory,[11] the degree k_i of node i is assumed to be a continuous real variable, and the rate change of k_i is expected to be proportional to $\prod_{\text{Local}-\text{world}}(k_i)$. Hence, k_i satisfies the following dynamical equation:[17−21]

$$\frac{\partial k_i}{\partial t} = m\prod\nolimits_{\text{Local}}(k_i) = \frac{mM}{m_0+t}\frac{k_i}{\sum_{j\ \text{Local}} k_j}. \tag{4}$$

Since the random selecting of the M nodes contributes to a local world connection at each time step t, the cumulative degree of the local world depends on the random selection. To simplify the following analysis, Li and Chen[12] have assumed that

$$\frac{k_i}{\sum_{j\ \text{Local}} k_j} = \langle k_i\rangle\, M, \tag{5}$$

where the average degree $\langle k_i\rangle = (2mt+2e_0)/(m_0+t)$. Substituting expression (5) into Eq.(4) leads to

$$\begin{aligned}\frac{\partial k_i}{\partial t} =& \frac{mM}{m_0+t}\frac{k_i}{\dfrac{(2mt+2e_0)M}{m_0+t}}\\ =& \frac{mk_i}{2mt+2e_0} \approx \frac{k_i}{2t},\end{aligned} \tag{6}$$

which means, with the assumption (5), that the distribution follows the same power-law as the BA scale-free model, that is, $P(k) \sim 2m^2/k^3$.

The LC local-world network model successfully simulates the complex networks of this kind, in which the nodes own only the local information of the networks. However, the mechanism of constructing a node depends on the real local connection condition in different networks.

3. The new local-world network model

3.1. Network topology structure

In the LC local-world network model, each node owns only the information in its local world network. And in the three cases of the local-world scale range, that is, $M = m$, $M = t+m_0$, and $m \ll M \ll t+m_0$, for each new node its local world is undistinguishable. However, in some real complex networks, there are a few nodes which can own the global information of the entire networks. For example, although most nodes of local area network obtain only the inner connection information, the central node of the skeleton network in the national internet must master the global connection information of the whole country. Hence, it is rational that for each node its local world is differently selected. On the other hand, if we can control the local world of each node in some important complex

networks, especially the networks joined by people, we can adjust the evolving processes of the networks and the topological structures artificially. It is the macro-control method based on the evolving process.

Let the ratio of the number of nodes obtaining the global information of the network to the total number of nodes be δ, which is a certain number in a range of [0, 1]. When $\delta = 0$, it means that there is no node owning the global information, which is the same as the assumption of the LC local-world network model. When $\delta = 1$, it means that each node can own the global connection information of the network, which degenerates into the BA model. The evolving algorithm of the new local-world evolving network model, which introduces the node ability to own the information of the network into the local world selection, is as follows:

(i) To start with, node number m_0 and edge number e_0 both are taken to be the same as those in the LC model.

(ii) Add a new node with m edges. If the new node belongs to the global node set, which means that the node can obtain the global information of the network, the probability that the new m edges will be connected with node i depends on the degree k_i of that node, such that

$$\prod\nolimits_{\text{Global}}(k_i) = \frac{k_i}{\sum_j k_j}. \tag{7}$$

If the new node belongs to the local node set, which means that the node owns only the information of its local world network, the new m edges will be connected with the node i in its local world through a preferential attachment with probability $\prod_{\text{Local}}(k_i)$ determined at every time step t by

$$\prod\nolimits_{\text{Local}}(k_i) = \prod{}'(i \in \text{Local} - \text{world})\frac{k_i}{\sum_{j\ \text{Local}} k_j}. \tag{8}$$

(iii) The probability that a new node belongs to the 'global' node set is δ according to the above assumption.

In the same way that the mean-field theory is introduced into the new local-world network model, we can suppose that the degree k of a node is continuous, and the probability $\prod_{\text{All}}(k_i)$ can be interpreted as a continuous rate of change of k_i. So the degree k_i of the node i satisfies the following dynamical equation:

$$\begin{aligned}\frac{\partial k_i}{\partial t} =& m\prod\nolimits_{\text{All}}(k_i) = m\Big[\delta\prod\nolimits_{\text{Global}}(k_i) \\ &+ (1-\delta)\prod\nolimits_{\text{Local}}(k_i)\Big] \\ =& m\left[\delta\frac{k_i}{\sum_j k_j} + (1-\delta)\frac{M}{m_0+t}\frac{k_i}{\sum_{j\ \text{Local}} k_j}\right].\end{aligned} \tag{9}$$

For a local-world evolving network model, the selection method of the node local-world is very important and very difficult to the study or simulation of the network. With the same method as that of the LC model, we discuss three cases of local-world selection: $M = t + m_0$, $M = m$ and $m \ll M \ll t + m_0$.

Case A $M = t + m_0$

This limiting case means that the local world is the same as the whole network. So for the node j, the total of the degrees of its local world is $\sum\limits_{j\ \text{Local}} k_j = 2mt + 2e_0$. Then Eq.(9) can be written as

$$\begin{aligned}\frac{\partial k_i}{\partial t} =& m\left[\delta\frac{k_i}{2mt+2e_0} + (1-\delta)\frac{M}{m_0+t}\frac{k_i}{2mt+2e_0}\right] \\ =& \frac{mk_i}{2mt+2e_0} \approx \frac{k_i}{2t}.\end{aligned} \tag{10}$$

According to the same approach as that introduced in Ref.[11], the degree distribution of the local-world network in the case, which can be approximated by $P(k) \sim 2m^2/k^3$, follows a power-low distribution and is unrelated to the proportion of the nodes obtaining the global information of the network. And it is obvious since the local world degenerates into the global world.

Case B $M = m$

At the evolving time step t, the m nodes selected randomly are referred to as the new coming node. Hence, in the limiting case, only a small percentage of the nodes in the whole network satisfy the mechanism of the holistic preferential attachment and most of them connect a new edge randomly. Moreover, the cumulative degree of the local world depends on the random selection. Therefore, we can assume that

$$\sum\nolimits_{j\ \text{Local}} k_j = \langle k_i\rangle M = \langle k_i\rangle m \approx k_i m, \tag{11}$$

where $\langle k_i\rangle$ is the average degree of the network in the case. Then Eq.(9) can be written as

$$\begin{aligned}\frac{\partial k_i}{\partial t} =& m\left[\delta\frac{k_i}{2mt+2e_0} + \frac{1-\delta}{m_0+t}\right] \\ \approx& \frac{\delta k_i}{2t} + \frac{(1-\delta)m}{t} \\ =& \frac{\delta k_i + 2(1-\delta)m}{2t}.\end{aligned} \tag{12}$$

With the boundary condition of the above equation of $k_i(t_i)$, we can obtain the solution of the dynamic equation

$$k_i(t) = \frac{2-\delta}{\delta} m \left(\frac{t}{t_i}\right)^{\delta/2} - \frac{2(1-\delta)}{\delta} m. \tag{13}$$

Expression (13) indicates that the functional relationship between degree $k_i(t)$ and t is expressed as $k_i(t) \sim t^{\beta}$, where $\beta = \delta/2$. According to the same analysis as that in Ref.[22], we can prove that the degree distribution of the networks in the case follows a power-low distribution, and the exponent γ of the scale-free network can be obtained from the functional relationship $\beta(\gamma - 1) = 1$ between β and γ as

$$\gamma = 1 + \frac{2}{\delta}. \tag{14}$$

As mentioned above, δ denotes the ratio of the number of nodes owning the global information of the network to the total number of nodes. Hence, the value of δ is very small in most complex networks. It means that the power-low exponent of the network in this case has a bigger value, maybe much bigger than the actual values of the real scale-free networks. Although the range of γ is $[3, +\infty)$ for a rational range of ratio δ, it is of little practical significance to control macroscopically the power-law exponent of the network by adjusting the ratio δ.

Case C $m \ll M \ll t + m_0$

At the evolving step time t, there are $t + m_0$ nodes in the network. When the new node belongs to the local node set, M nodes randomly selected in the network construct its local world, so the probability that the node i is selected into the local world is $M/(m_0 + t)$. On the one hand, since the probability that the node i belongs to the global node set is δ, we can assume that the average degree is $\langle k_i \rangle = (2mt + 2e_0)/(m_0 + t)$. On the other hand, we can assume without lost of generality that the average degree is m if the node i belongs to the local node set. So the total of degrees of the local world is

$$\sum\nolimits_{j\ \mathrm{Local}} k_j = [\delta \langle k_i \rangle + (1-\delta)m]M. \tag{15}$$

Then, Eq.(9) can be transformed into

$$\begin{aligned}\frac{\partial k_i}{\partial t} =& m\left[\delta\frac{k_i}{2mt+2e_0} + (1-\delta)\frac{M}{m_0+t}\frac{k_i}{[\delta\langle k_i\rangle + (1-\delta)m]M}\right] \\ =& mk_i\left[\frac{\delta}{2mt+2e_0} + \frac{1-\delta}{\delta(2mt+2e_0)+(m_0+t)(1-\delta)m}\right] \\ \approx& k_i\left[\frac{\delta}{2t} + \frac{1-\delta}{2\delta t+(1-\delta)t}\right] = \left[\frac{\delta}{2} + \frac{1-\delta}{1+\delta}\right]\frac{k_i}{t}.\end{aligned} \tag{16}$$

So we obtain that

$$\beta = \frac{\delta}{2} + \frac{1-\delta}{1+\delta}. \tag{17}$$

It means that the evolving network in the case has a power-law degree distribution[22] and its exponent is

$$\gamma = 1 + \frac{1}{\beta} = \frac{4+\delta+\delta^2}{2-\delta+\delta^2}. \tag{18}$$

According to expression (18) and the range of ratio δ, the exponent of the network γ can be adjusted from 2 to 3, which is shown in Fig.2. The larger the value of δ gradually becomes, the smaller the deviation between the theoretical values and simulation values of the exponent γ will be. This phenomenon can be explained as follows. When the value of ratio δ is small, most of the nodes in the evolving network belong to the set of 'local nodes' and the local world

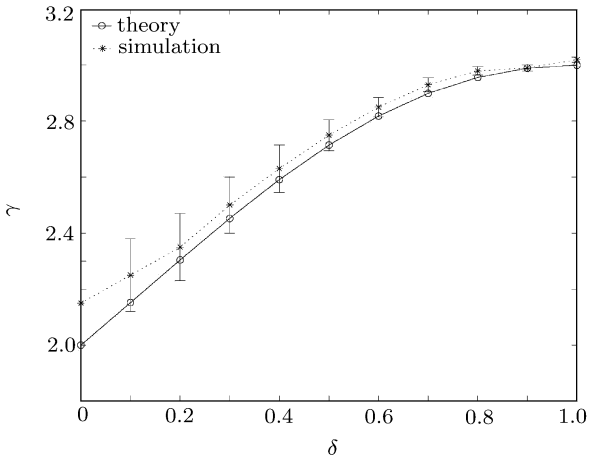


Fig.2. Power-law exponent γ of the network versus ratio δ in the case of $m \ll M \ll t + m_0$, with the $N = m_0 + t = 10000$, $m_0 = 6$, $e_0 = 7$, $m = 5$ and $M = 50$. For all values of the ratio δ, averages are taken over 100 runs.

of each new node is selected randomly. Hence, in the evolving process, the global preferential attachment mechanism almost does not come to play and only network growth can exert an influence on the topological structure of the network. In this case, the simulation values of the exponent γ will be larger than the theoretical values and so there appears a big difference between the two values.

At the same value of ratio δ, by increasing the local-world scale M from 20 to 100 with a fixed value of $m = 3$, the degree distribution always displays a scale-free characteristic and its exponent is nearly unchanged too, which is shown in Fig.3. Hence, the new local-world evolving network has a scale-free feature if a constant local-world scale has $m \ll M$, and the exponent is not correlated with the local-world scale M, which is indicated in Eq.(18). The ratio δ is one and only parameter of the scale-free exponent of the new model in Case C. Hence, we can macro-control the topology of the network in the case by changing the value of the ratio δ.

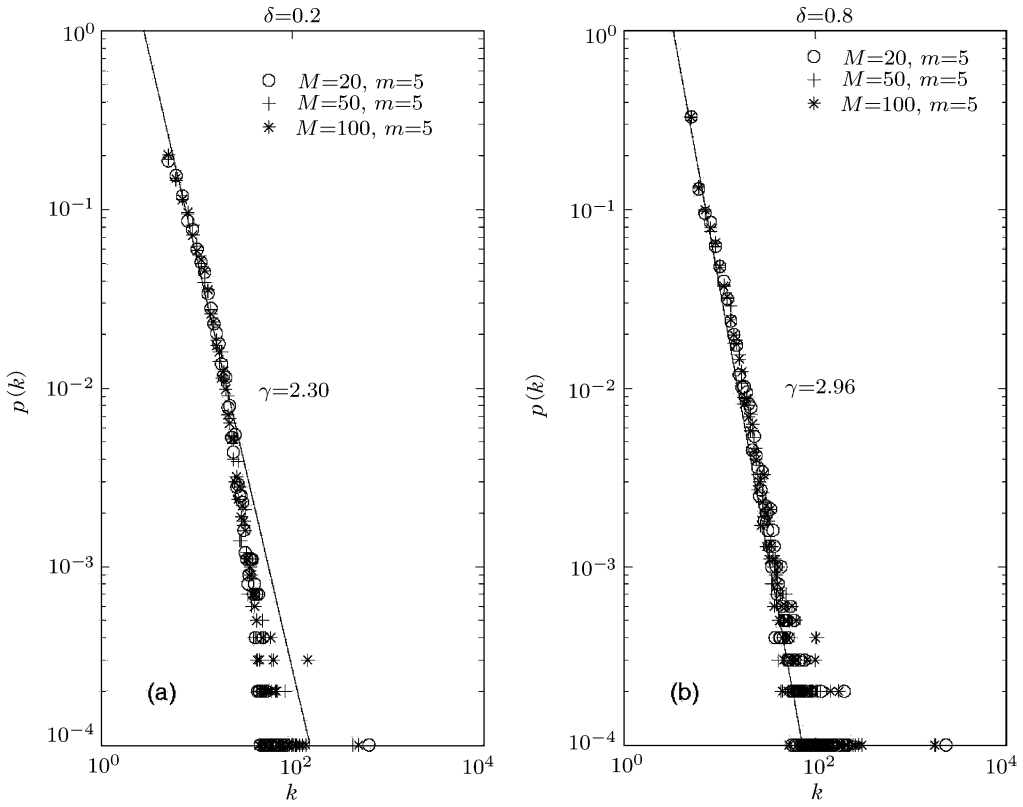


Fig.3. The degree distribution comparison in the log-log scale, for Case C with $N = 10000$, $m = 5$, and $M = 20, 50, 100$, separately at ratio $\delta = 0.2$ and the theoretical value of the exponent of the scale-free network $\gamma = 2.30$ (a) and at ratio $\delta = 0.8$ and the corresponding value of the exponent $\gamma = 2.96$ (b).

3.2. Average shortest path length and clustering coefficient of the network

We analyse two characteristics of the new local-world network model, i.e. the average shortest path length and the clustering coefficient.[3] The path length L_{ij} is the number of edges in a shortest path between two nodes v_i and v_j. And the average shortest-path length of node v_i is $L_{v_i} = \sum_j L_{ij}/N$. Then the average shortest-path length of a network is

$$L = \frac{1}{N}\sum_{i=1}^{N} L_{v_i}. \tag{19}$$

Let $\langle k\rangle$ be the mean degree of a random network, then its average shortest-path length[23] will be

$$L_{\text{rand}} = \frac{\log(N)}{\log(\langle k\rangle)}. \tag{20}$$

For a selected node v_i of a complex network with its degree k, if there actually exist z_{v_i} edges among these k neighbours, the clustering coefficient of this node can be obtained from the formula $C_{v_i} = 2v_{v_i}/[k(k-1)]$. It is the ratio of the actual number of edges, which are among the given node neighbours, to theoretical number of edges. And the clustering

coefficient C of network is

$$C = \frac{1}{N\sum\limits_{i} C_{v_i}}. \tag{21}$$

The clustering coefficient of a random network[23] can be calculated from the formula

$$C_{\rm rand} = \frac{\langle k\rangle}{N-1}. \tag{22}$$

The relationship between the average shortest-path length L and the clustering coefficient C of the model and the ratio δ are shown in Fig.4. From this figure, it can be seen that the value of L decreases but the value of C increases as the value of ratio δ increases. So the new local-world network has a small-world behaviour that emerges gradually with the value of C increasing and the value of L decreasing.

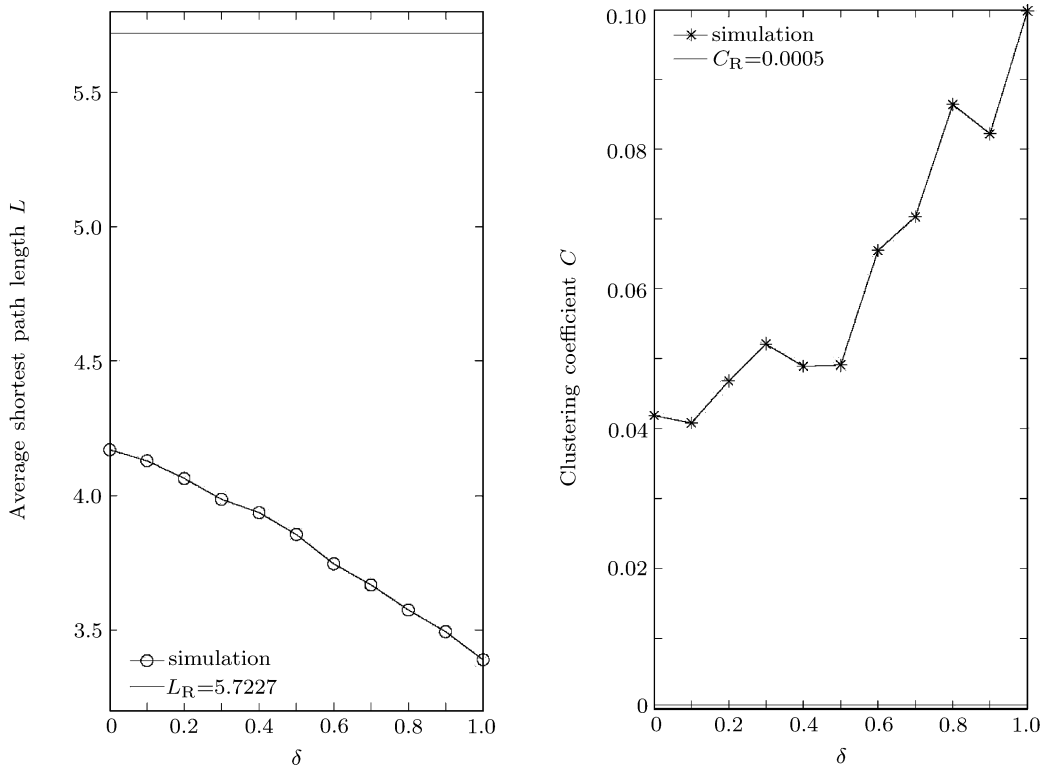


Fig.4. The average shortest path length L and the clustering coefficient C of the new model versus ratio δ. All the networks have $N = 10000$, $M = 50$ and $m = 5$. And $L_{\rm R}$ and $C_{\rm R}$ can be calculated from expressions (20) and (22), respectively.

4. Conclusions

We have proposed a new local-world evolving network model by considering a few nodes that obtain the global information of the entire network in this paper. The new local-world network model represents a transition between exponential scaling and power-law scaling, which is the same as that in the LC model. Moreover, under the reasonable assumption of the selection approach for the local world, we use the mean-field theory to prove that the new local-world network has a power-law degree distribution when the local-world scale M satisfies the requirements in Case C. In addition, the two important topological characteristics of the new model, i.e. the average shortest path length and the clustering coefficient, are simulated numerically, indicating that the network has a small-world property.

The main difference between the new model and the LC model is that there exist a few global nodes that can connect the new edges in the overall network and the global preferential attachment mechanism is satisfied in the former network. The ratio δ between the number of nodes containing the global information of the network to the total number of nodes is introduced into the study of the new local-world network. The important parameter can exclusively control the scale-free exponent of the new network when an appropriate local-world scale is given. On the other hand, owing to the assumption that all nodes have their own local-worlds in the LC model, for the new local-world

network model it is obvious that the LC model is only one of its special cases. When $\delta = 0$, the new local-world network model degenerates into the LC model, which is shown in Fig.1.

Therefore, the new local-world not only can remain the main merit of the LC network, i.e. the transition between exponential scaling and power-law scaling, but also can easily adjust the scale-free exponent of the network in a range from 2 to 3, which accords with the existing results of the exponents for many real complex networks. Since the ratio δ is the one and only parameter of the scale-free exponent of the new network, if the topological structure of a complex network needs controlling, especially its exponent of power-law degree distribution, needs controlling, we just add or take away a few nodes that own the global information of the network.

References

[1] Barabási A L and Albert R 1999 *Science* **286** 509

[2] Dorogovtsev S N and Mendes J F F 2001 cond-mat/0106144

[3] Albert R and Barabási A L 2002 *Rev. Mod. Phys.* **74** 47

[4] Newman M E J 2003 *SIAM Rev.* **45** 167

[5] Guo J L 2007 *Chin. Phys.* **16** 1239

[6] Faloutsos M, Faloutsos P and Faloutsos C 1999 *Proc. ACM SIGComm.* **29** 251

[7] Huberman B A and Adamic L A 1999 *Nature* (London) **401** 131

[8] Redner S 1998 *Eur. Phys. J.* B **4** 131

[9] Barabási A L, Jeong H, Néda Z, Ravasza E, Schubert A and Vicsek T 2002 *Physica* A **311** 590

[10] Dorogovtsev S N and Mendes J F F 2001 *Proc. R. Soc. London Ser.* B **268** 2603

[11] Barabási A L, Albert R and Jeong H 1999 *Physica* A **272** 173

[12] Li X and Chen G R 2003 *Physica* A **328** 274

[13] Serrano M A and Boguna M 2003 cond-mat/0301015

[14] Dai S and Guo Y J 2004 *Chin. Phys.* **13** 423

[15] Garlaschelli D and Loffredo M I 2005 *Physica* A **355** 138

[16] Li X, Jin Y Y and Chen G R 2003 *Physica* A **328** 287

[17] Dorogovtsev S N and Mendes J F F 2001 *Phys. Rev.* E **63** 056125

[18] Sen P 2004 *Phys. Rev.* E **69** 046107

[19] Sarshar N and Roychowdhury V 2004 *Phys. Rev.* E **69** 026101

[20] Wu J J, Gao Z Y and Sun H J 2007 *Chin. Phys.* **16** 47

[21] Sun H J and Wu J J 2007 *Chin. Phys.* **16** 1581

[22] Dorogovtsev S N and Mendes J F F 2000 *Phys. Rev. Lett.* **85** 4633

[23] Davidsen J, Ebel H and Bornholdt S 2002 *Phys. Rev. Lett.* **88** 128701