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Diffraction of terahertz waves after passing through a Fresnel lens*

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The spatiotemporal and spectral characteristics of ultrawide-band terahertz pulses after passing through a Fresnel lens are studied by using the scalar diffraction theory. The simulation shows that the transmitted terahertz waveforms compress with increasing propagation distance, and the multi-frequency focusing phenomenon at different focal points is observed. Additionally, the distribution of terahertz fields in a plane perpendicular to the axis is also discussed, and it is found that the diffraction not only induces focusing on-axis but also inhibits focusing at off-axis positions. Therefore, the Fresnel lens may be a useful alternative approach to being a terahertz filter. Moreover, the terahertz pulses travelling as a basic mode of a Gaussian beam are discussed in detail.

Keywords: terahertz, Fresnel lens, diffraction field
PACC: 7830, 4278, 4225F

1. Introduction

The advent of optoelectronic sources and receivers of subpicosencond pulses of terahertz electromagnetic radiation has aroused much interest in the propagation properties of these pulses.[1−4] In terahertz time-domain spectroscopy and imaging technologies, the focusing and the collimating of terahertz waves have relied mainly on parabolic mirrors, silicon lenses and polyethylene lenses. However, for a terahertz beam, it is impossible to fabricate lenses with short focal lengths and large numerical apertures by using silicon or polyethylene, and for two-dimensional CCD terahertz imaging, it is very difficult to obtain a high-quality terahertz image on a ZnTe sensor by using parabolic mirrors because of their aberrations and the difficulty of alignment. However, fabricating a large numerical aperture terahertz Fresnel lens with a short focal length is possible by using modern planar microfabrication technology, which makes such lenses attractive to terahertz focusing, filtering, and imaging applications. The focal length of a Fresnel lens is linearly proportional to the frequency of incident beam; this property allows terahertz pulses, which have broad spectral contents, to be focused on the axis at various positions with different corresponding frequencies.[5] Compared with conventional lenses, Fresnel lenses have several attractive features for terahertz applications. These include their flexibility in design, capacity of integration, and freedom in the choice of substrate.[6] Also, the linearly frequency-dependent focal length can be used to obtain high resolution tomographic images at terahertz frequencies.[7] However, very few results which treat the diffraction of terahertz waves after passing through a Fresnel lens are found in the literature. In the present paper, we generalize the discussion to the case of terahertz pulses that have passed through a Fresnel lens, and put special emphasis on the spatiotemporal and spectral properties of the terahertz pulses propagating in free space and undergoing focusing. The results show that the Fresnel lens may be a useful alternative approach to being a terahertz filter.

2. Discussion and results

A Fresnel lens is a Fresnel zone plate with phase modulated patterns, formed by a series of concentric ring structures. The focal length is defined as[5]

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$$f_\nu = \frac{r_\mathrm{p}^2}{2\lambda} = \frac{r_\mathrm{p}^2}{2c}\nu \propto \nu, \tag{1}$$

where r_p^2 is the Fresnel zone period with a dimension of area, λ is the wavelength, c is the speed of light, and ν is the light frequency. The focal length f_ν is linearly proportional to frequency ν. In the case of single-frequency illumination in the paraxial ray approximation, a Fresnel lens focusing system acts as a conventional lens. For a non-monochromatic wave, the focus shift for a diffractive Fresnel lens with main focal length f_0 designed at frequency ν_0 is given by

$$f_\nu = \frac{f_0}{\nu_0}\nu. \tag{2}$$

In the present paper, a diffractive phase Fresnel lens array with a 20.35-mm diameter, consisting of a total of 23 zones, designed for $\nu_0 = 1$ THz with focal length $f_0 = 30$ mm, is considered in our simulation.

Figure 1 schematically shows the terahertz Fresnel lens focusing system. To demonstrate this non-monochromatic focusing concept, a broadband terahertz wave pulse is used in our simulation; its frequency distribution is shown in the left upper corner of Fig.1, and the inset shows its time domain waveform. The useful amplitude spectrum extends from 0.1 THz to 3 THz. The broad frequency range of the pulse makes it an ideal candidate to test Fresnel lens focusing.

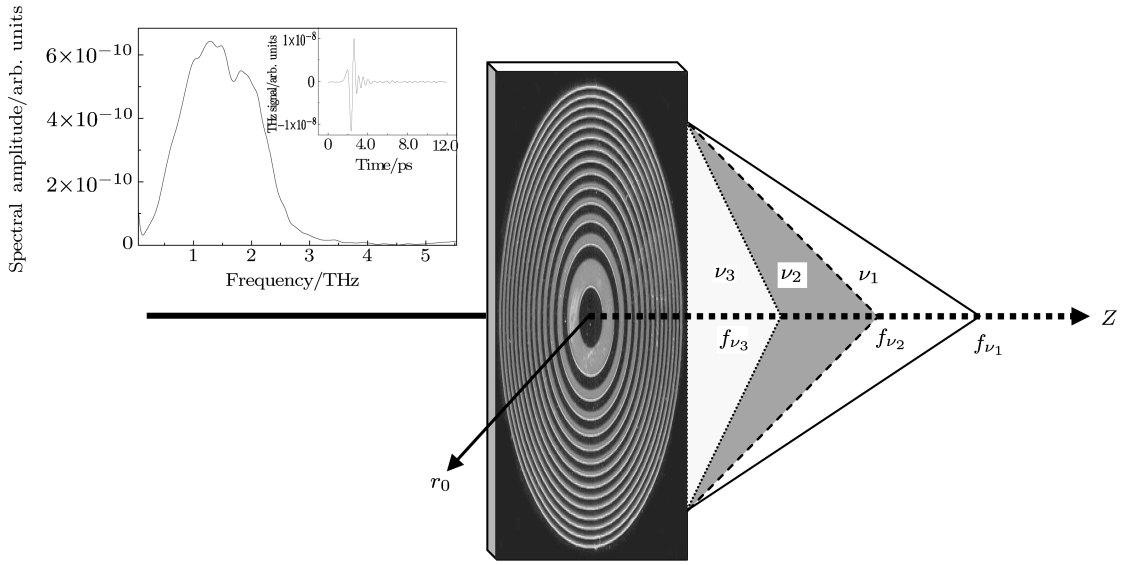


Fig.1. Schematic diagram of the terahertz focusing system with a Fresnel lens. The left upper panel shows the Fourier spectra of the incident terahertz pulse waveform, and the inset displays its time domain waveform.

According to the optical scalar diffraction theory, the propagation of a monochromatic wave can be well described; a mathematical solution was given by Kirhhoff *et al* [8] and Sommerfeld *et al* [9] as

$$U(P_0) = \frac{1}{\mathrm{i}\,\lambda}\iint_{\Sigma} U(P_1)\frac{\exp(\mathrm{i}\,k|r_{01}|)}{|r_{01}|}\cos(\boldsymbol{n},\boldsymbol{r}_{01})\mathrm{d}s, \tag{3}$$

where P_1 is a point in the plane Σ, $\boldsymbol{n}$ is the normal to the plane at P_1, $\boldsymbol{r}_{01}$ is the displacement vector from P_1 to P_0, and $k = 2\pi/\lambda$ is the wave vector. This solution is basically a superposition of spherical waves, which successfully describes the propagation of a monochromatic wave in most practical circumstances, as long as the condition $\lambda \ll |r_{01}|$ is satisfied. To generalize this solution to the case of a non-monochromatic optical pulse $u\,(P_1,\ t)$, $U\,(P_1)$ in Eq.(3) can be regarded as one of the Fourier components of the optical pulse.[10] Applying Eq.(3) to each Fourier component of $u\,(P_1,\ t)$, the electrical field at P_0 is calculated as

$$u(P_0,t) = \iint_{\Sigma}\frac{\cos(\boldsymbol{n},\boldsymbol{r}_{01})}{2\pi c|r_{01}|}\frac{\partial}{\partial t}u\left(P_1, t-\frac{|r_{01}|}{c}\right)\mathrm{d}s. \tag{4}$$

Equation (4) indicates that the electrical field, at position P_0 and time t, depends on the time deriva-

tive of the field in the plane Σ at a retarded time $t - |r_{01}|/c$; the temporal waveform of the pulsed radiation changes during the propagation. For most of the optical pulses, the temporal waveform can be described as a monochromatic wave modulated by a time envelope, and the change of the waveform during propagation in free space is rarely noticed or observed in experiments. However, if an ultrashort pulse, such as a terahertz pulse or a laser pulse with time duration of only a few femtoseconds, is considered, the time envelope cannot be well determined and the change of temporal waveform during propagation can no longer be ignored according to Eq.(4).

For terahertz waves passing through a Fresnel lens, correspondingly, a phase delay factor $\Delta\phi$ should be added into Eq.(4), leading to

$$u(P_0,t) = \iint\limits_{\Sigma} \frac{\cos(\boldsymbol{n},\boldsymbol{r}_{01})}{2\pi c|r_{01}|} \times \frac{\partial}{\partial t} u\left(P_1, t - \frac{|r_{01}|}{c} - \Delta\phi\right) \mathrm{d}s. \tag{5}$$

For a conventional lens, all rays focused at the focal point have the same phases. However, for a Fresnel lens, rays that come from different zones have different phases. The phase difference is an integer multiple of 2π.

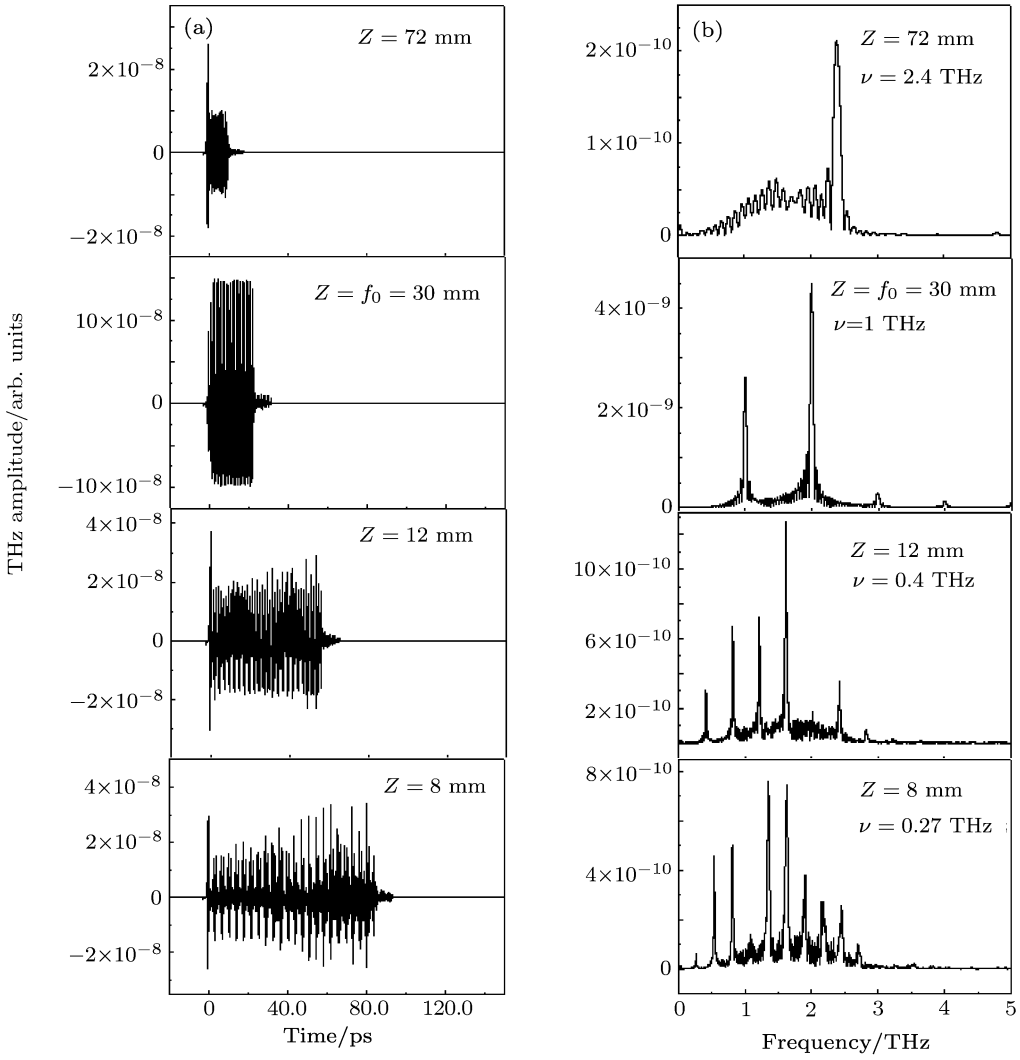


Fig.2. Focused terahertz (a) waveforms and (b) Fourier spectra at on-axis points of the Fresnel lens with different propagation distances (Z = 8, 12, 30, 72 mm) and corresponding different focusing frequencies (ν = 0.27, 0.4, 1.7, 2.4 THz), respectively, from bottom to top.

To indicate how a spectrally broadband pulse changes its shape and spectrum during diffraction, we calculate the evolution of the temporal terahertz waveforms along the axis, propagating in free space, by using Eq.(5). Figures 2(a) and 2(b) depict the waveforms and the spectra of the terahertz waves focused at different focal points by a Fresnel lens, respectively. The multiple oscillation waveforms of the focused terahertz waves result from the superposition of the terahertz waves coming from different zones. Compared with the terahertz fields at the on-axis points of the Fresnel lens, the focused terahertz fields compress dramatically as propagation distance increases. This corresponds to the enhanced terahertz wave superposition efficiency, which is because the difference in phase among the pulses that come from different points of the Fresnel lens decreases as propagation distance increases.

Due to the frequency-dependent focal lengths, a Fresnel lens focuses different frequencies onto different focal points. For each propagation distance Z, the corresponding focal frequency ν can be derived from Eq.(2), and the calculated values of focal frequency ν are 0.27, 0.4, 1, 2.4 THz for $Z = 8$, 12, 30, 72 mm, respectively. The Fourier transformation of the measured terahertz pulses is performed to allow the frequency-dependent response of the Fresnel lens to be studied as shown in Fig.2(b). It is interesting to see that the terahertz spectra show that the peaks appear not only at the expected focal frequencies, but also at integer multiples of these frequencies. For a Fresnel lens, rays that come from different zones have the phase difference

$$\Delta\phi = k(l_i - l_j) = k\Delta l = \frac{2\pi}{\lambda}\Delta l = \frac{2\pi}{c}\Delta l \cdot \nu,$$

where l_i and l_j are the optical lengths from the i, j-level zones to a certain focal point f_ν, respectively. For frequency ν corresponding to the focal length f_ν, $\Delta\phi$ should be an integer multiple of 2π in order to focus at f_ν. In this way, for frequency $\nu' = m\nu$, where m is an integer, the phase difference

$$\Delta\phi' = \frac{2\pi}{c}\Delta l \cdot \nu' = m \cdot \Delta\phi$$

is also an integer multiple of 2π for the focal point f_ν, and thus the Fresnel lens is able to focus at $\nu' = m\nu$ frequencies for focal point f_ν. The frequency response of a Fresnel lens consists of a series of regularly-spaced spikes, giving the appearance of a comb; therefore, the Fresnel lens can be used as a comb filter at terahertz frequencies.

Several factors may limit the resolution of the Fresnel lens focusing along the z-axis, which is defined as the focusing resolution. The first one is the terahertz frequency measurement resolution, which can be estimated by differentiating Eq.(1), given as

$$\Delta f_\nu = \frac{r_{\mathrm{p}}^2}{2c}\Delta\nu. \tag{6}$$

For a broadband terahertz pulse with frequency resolution $\Delta\nu = 190$ GHz, from Eq.(6), the estimated focusing resolution $\Delta f_\nu = 5.7$ mm. The second factor limiting the focusing resolution is the spatial distribution of terahertz pulses, which is discussed below.

Figure 3 shows the Fourier spectra of terahertz waveforms at different distances from the axis in the planes (a) $Z = f_0 = 30$ mm, corresponding to the focusing frequency $\nu = 1.0$ THz, and (b) $Z = 45$ mm, corresponding to $\nu = 1.5$ THz. The values of distance from the axis, R, are 0, 0.5, 10 mm, i.e. $R = 0$, 0.5, 10 mm, respectively, from bottom to top. It can be seen that, at the on-axis points, the terahertz spectra show peaks at expected frequencies; as the distance from the axis increases, sharp dips appear at the expected frequencies; in the region far from the axis, the dips are broadened considerably and become obvious. This can be explained as that the focusing frequency components are efficiently gathered at focal points, so that at off-axis positions, there is a shortage of these frequency components but dips appear there. It is concluded that the diffraction not only induces focusing on-axis but also inhibit focusing at off-axis points. In this way, only points on the axis of Fresnel points can be utilized to achieve the comb filtering function.

The distributions of terahertz fields in the focal plane ($Z = 45$ mm) of the Fresnel lens at different delay times ($\Delta t = 0$, 0.15, 0.45, 0.6, 3.9, 7.5 ps) are shown in Fig.4; the time zero ($\Delta t = 0$) corresponds to the peak position of the focused terahertz waveform at the focal point ($Z = 45$ mm, $R = 0$ mm). In this way, the propagation properties of the focused terahertz fields can be observed on the sub-picosecond time scale. It can be seen that the terahertz field is approximately in a single-peak form at time zero; as the delay time increases, the peak oscillates up and down, and then expands to distant spatial positions. It is very like the propagation behaviour of water waves. To our imagination, both the terahertz fields and the spectra should focus on the axis of the Fresnel lens. However, the simulation shows that as delay time increases, the terahertz fields are not restricted to the on-axis position but propagate to distant positions.

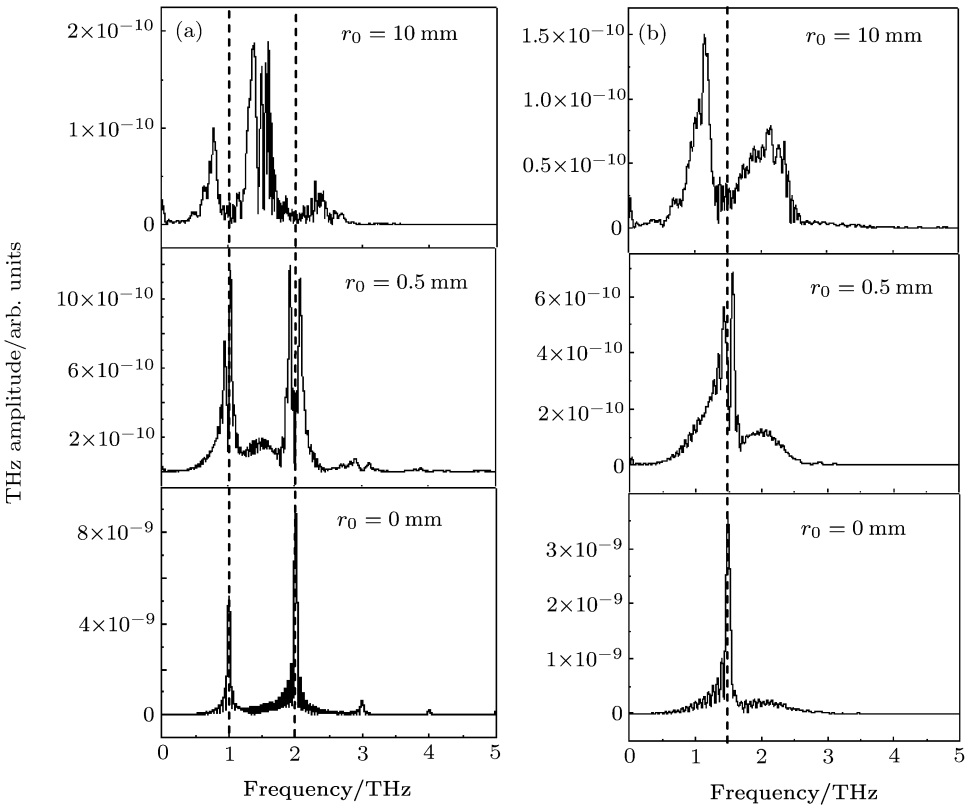


Fig.3. Fourier spectra of terahertz waveforms at different distances from the axis in the planes (a) $Z = f_0 = 30$ mm, corresponding to the focusing frequency $\nu = 1.0$ THz; (b) $Z = 45$ mm, corresponding to $\nu = 1.5$ THz. The values of distance from the axis r_0 are 0, 0.5, 10 mm, respectively, from bottom to top.

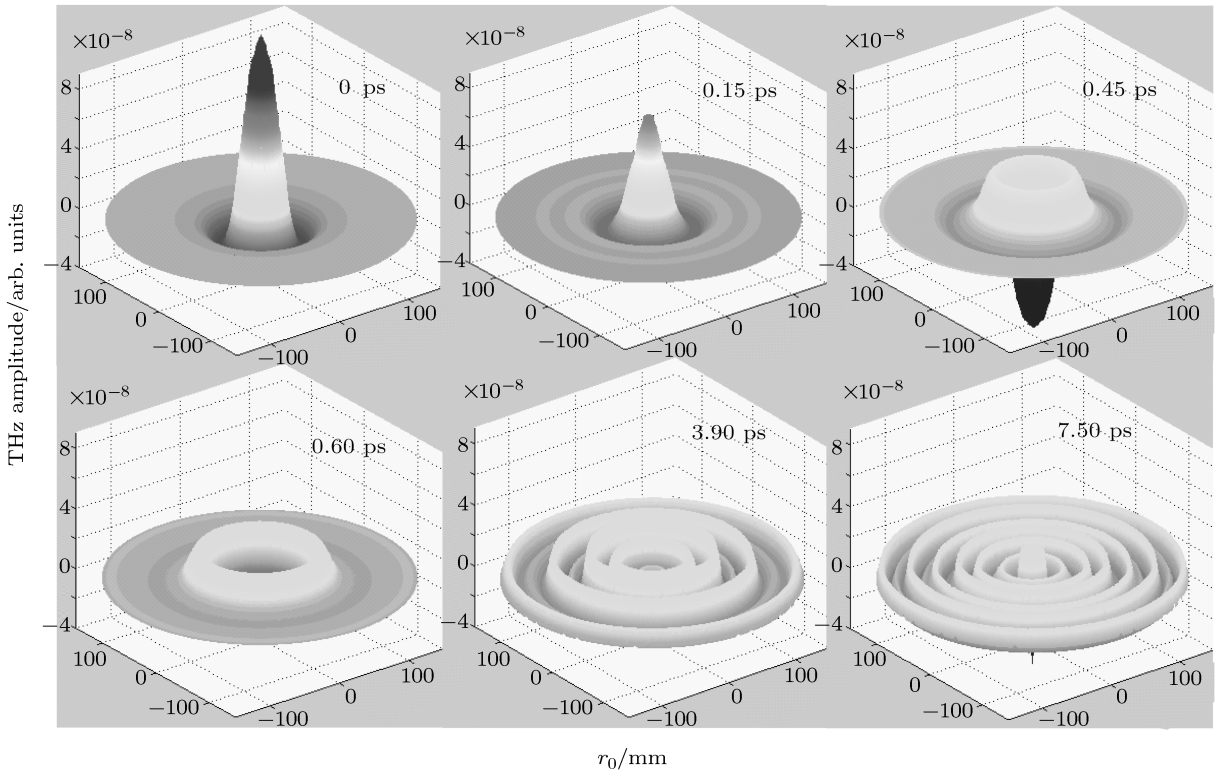


Fig.4. Distribution of terahertz fields on the focal plane ($Z = f_0 = 30$ mm) of the Fresnel lens at different delay times ($\Delta t = 0$, 0.15, 0.45, 0.6, 3.9, 7.5 ps), respectively.

Because the evolution of the shape of terahertz pulses discussed above is basically due to diffraction, any manipulation changing the spatial distribution of the incident terahertz pulses will lead to changes of the time waveforms and the spectra of the focused pulses. To discuss this effect, we consider terahertz pulses with Gaussian spatial distributions at $Z = 0$, which have a general form of $F(r) = \exp(-r^2/a_0^2)$, where a_0 is the waist radius of the Gaussian beam radius. The simulation results are indicated in Fig.5. It can be seen that the waveform decreases and concentrates dramatically as the value of a_0 increases. However, in the spectra, the peaks, which are related to the focusing frequency, are broadened considerably as the value of a_0 decreases, and as $a_0 = 2$ mm, the peaks even disappear completely. This is because the spatial distribution deviates largely from the plane wave as a_0 decreases, and simultaneously, the focusing efficiency decreases.

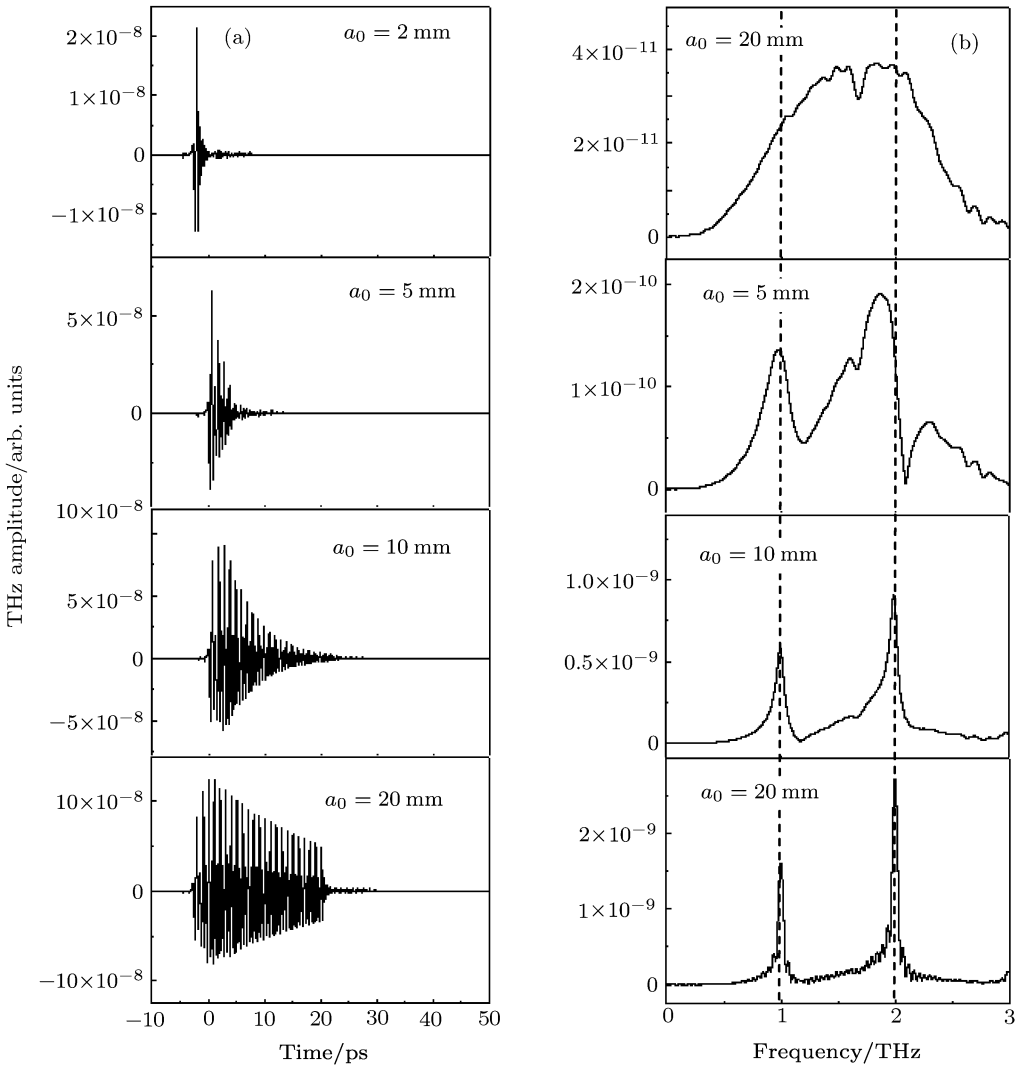


Fig.5. Focused terahertz (a) waveforms and (b) spectra on the focal point ($Z = f_0 = 30$ mm) with different Gaussian spatial distributions, which have a general form of $F(r) = \exp(-r^2/a_0^2)$, and a Gaussian beam waist radius $a_0 = 20$, 10, 5, 2 mm, respectively, from bottom to top.

3. Conclusion

Terahertz pulses with broad spectra passing through a Fresnel lens are studied by using the scalar diffraction theory. The Fresnel lens contributes a phase modulation to the temporal waveforms; the focused terahertz waveforms compress with increasing propagation distance. The Fresnel lens focuses multi-frequencies at different corresponding foci, and the incident spatial distribution results in significant effects on the temporal waveforms

and spectra of the pulses. Additionally, the distribution of terahertz fields in focal planes is also discussed, and it is found that the diffraction not only induces focusing on-axis but also inhibits focusing at off-axis positions. Fresnel lenses are much lighter and more compact than conventional terahertz optics and their excellent focusing functions lend themselves to unique beam forming and terahertz filtering, and so they may be expected to have increasing applications in the future.

References

[1] McGowan R W, Gallot G and Greschkowsky D 1999 *Opt. Lett.* **24** 20

[2] Xu J, Wang L and Yang G 2000 *Chin. Phys. Lett.* **17** 8

[3] Xu J, Wang L and Yang G 2001 *Chin. Sci. Bull.* **46** 11

[4] Fu Y, Li B, Yang Y, Zhang C, Zhang H and Zhou L 2009 *Acta Phys. Sin.* **58** 3302 (in Chinese)

[5] Jahns J and Walker S J 1990 *Appl. Opt.* **29** 931

[6] Wang S, Yuan T, Walsby E D, Blaikie R J, Durbin S M, Cumming D R S, Xu J and Zhang X C 2002 *Opt. Lett.* **27** 1183

[7] Wang S and Zhang X C 2003 *Appl. Phys. Lett.* **82** 1821

[8] Kirhhoff G 1882 *Berl. Ber.* **17** 641
Kirhhoff G 1883 *Ann. Der Physik* (Leipzig) **18** 663

[9] Sommerfeld A 1896 *Math. Ann.* **47** 317

[10] Goodman J W 1968 *Introduction to Fourier Optics* (New York: McGrawHill)