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The phenomenon of even bulk mode variance in a ferromagnetic A–A bilayer system*

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The eigenproblems of spin waves in a symmetrical ferromagnetic bilayered system with periodic boundary conditions are solved using the interface-rescaling approach (IRA). The results show that interface coupling between two sublayers would not change the excitation energy of odd bulk modes, but change excitation energy of even bulk modes. We call this peculiar phenomenon the phenomenon of even bulk mode variance (PEBMV). There are two kinds of mechanisms which cause PEBMV: phase reversal and phase translation of the magnon at the interface, corresponding, respectively, to the antiferromagnetic and ferromagnetic interface coupling cases. PEBMV embodies the selective effect of the interface on different bulk magnons.

Keywords: spin waves, magnetic multilayer, magnon
PACC: 7510D, 7540F, 7530D

1. Introduction

The spin wave, as a kind of elementary excitation, has been intensively studied in magnetic multilayers.[1−9] Many authors have studied spin-wave energy excitations by using a variety of useful theoretical methods, such as the Green function method,[10] the transfer-matrix method[11] and the Monte Carlo method.[12] Puszkarski *et al*[13] introduced an interface rescaling approach (IRA) about 20 years ago. This method can exactly solve the eigenproblems of layered magnetic materials by decomposing complex systems into independent subsystems. With this method, many authors have investigated the different magnetic properties of layered magnetic materials, such as conditions for interface spin waves,[14] the dispersion relations of spin waves and ferromagnetic resonance spectra in a system of ferromagnetic bilayers.[15,16]

Recently we have studied the eigenexcitation of the spin waves in a heterogeneous ferromagnetic bilayered system by using IRA. The results show that transverse spin waves could affect the number of the bulk modes (BM), the perfect confined modes (PCM) or the interface modes (IM) although not the total number of eigenmodes.[17] We first gave the fine structure of the energy band over the whole two-dimensional Brillouin zone in the system, discussed conditions for the existence of the interface mode and analysed the oscillating behaviour of the resonant-confined spin waves in bulk modes.[18] We also found the critical mode (CM) at the critical point between BM and PCM or PCM and IM in the system, which was composed of spin waves modulated by the interface.[19] However, the eigenexcitation of the spin waves in a symmetrical ferromagnetic bilayered system has seldom been studied. In particular, the effect of interface coupling on the bulk modes has not been reported.

In this paper, we find that interface exchange coupling can affect the magnons of even bulk modes in a symmetrical ferromagnetic bilayered system (A–A configuration), i.e. PEBMV appears in the system.

2. Theoretical model

Consider a simple cubic ferromagnetic bilayer slab consisting of two symmetrical sublayers A and B with bulk-exchange interactions $J_{\rm A} = J_{\rm B}$, interface exchange coupling $J_{\rm AB}$, spins $S_{\rm A} = S_{\rm B}$, and numbers of lattice planes $N_{\rm A} = N_{\rm B}$. For the sake of simplicity, we

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assume that the two ferromagnets still have their periodicity in the lattice planes parallel to the interface. Here, we neglect the anisotropy in the bulk, on the interface and on the surfaces, so that the inhomogeneity of the system is assumed to come from the difference between the interface couplings $J_{\rm AB}$. Under a periodic boundary condition we assume that the Heisenberg Hamiltonian consists of two terms accounting for the isotropic nearest-neighbour (NN) exchange interaction and the Zeeman energy of the spins. Hence, the Hamiltonian of such a system is written as:

$$\hat{H} = -\sum_{n,m}\sum_{i,j} J(n,i;m,j)\hat{\boldsymbol{S}}(n,i)\cdot\hat{\boldsymbol{S}}(m,j) - \mu_0\sum_{n,i} g(n,i)\boldsymbol{H}\cdot\hat{\boldsymbol{S}}(n,i), \tag{1}$$

where$N_{\rm A}+N_{\rm B}=N$, and n, m are the indices of the lattice planes and i, j are the sites in the atomic planes n and m; $g(n,i)$ and $\boldsymbol{H}$ are the Landé factor and the effective field, respectively. The interaction constant $J(n,i;m,j)$ takes, respectively, $J_{\rm A}$ for both sites in sublayer A, $J_{\rm B}$ for both sites in sublayer B, $J_{\rm AB}$ for one site in sublayer A and the other in sublayer B. The spin operators satisfy the following equation:

$$\hat{S}(n,i)\cdot\hat{S}(n,i) = S(n)[S(n)+1] = \begin{cases} S_{\rm A}(S_{\rm A}+1) \text{ for sites in A,} \\ S_{\rm B}(S_{\rm B}+1) \text{ for sites in B.} \end{cases} \tag{2}$$

We assume that the effective field is a constant H along the z axis. Using the interface-rescaling approach,[13] the Hamilitonian can be easily diagonalized and the eigenequations of the spin waves of the system can be obtained as follows:

$$\begin{aligned} E_{pk}f(n,p) = \{&[J(n,n)S(n)+J(n,n+1)S(n+1)+J(n,n-1)S(n-1)]f(n,k) \\ &- J(n,n+1)\sqrt{S(n)S(n+1)}f(n+1,k) - J(n,n-1)\sqrt{S(n)S(n-1)}f(n-1,k)\} \\ &+ \mu_0 g(n)Hf(n,k), \end{aligned} \tag{3}$$

where

$$\begin{aligned} &J(n,n) = 2(2-\cos k_x - \cos k_y)J(n,i;n,i\pm1) \equiv 2\gamma J_n, \\ &J(n,n\pm1) = J(n,i;n\pm1,i) = J(n\pm1,n). \end{aligned} \tag{4}$$

Here $E_{pk}(>0)$ is the excitation energy of the spin waves, which consists successively of the reduced excitation energy and the Zeeman excitation energy; $f(n,p)$ is an orthonormalized wave function, which satisfies the Born–Karman boundary condition. The index p in the wave functions is the wave vector of the spin waves parallel to the interface, and the parameter $\gamma=\gamma(\boldsymbol{k}_{||})=\gamma(k_x,k_y)$ defined by Eq.(4) varies from 0 to 4. We have chosen the lattice constants in both sublayers as unit length.

Considering the periodic boundary condition, we assume that the interfaces are formed between the planes n= 1, N and $n=N_{\rm A}$, $N_{\rm A}+1$ for the present slabs. We will neglect the indices p, k of E_{pk} and p of $f(n,p)$ for simplicity, and set

$$f(n) = \begin{cases} f_{\rm A}(n) \text{ for } & 1\le n\le N_{\rm A}, \\ f_{\rm B}(n) \text{ for } & N_{\rm A}+1\le n\le N. \end{cases} \tag{5}$$

Due to the periodic boundary condition and the inversion symmetry in this system, the wave functions $f(n)$ in the two sublayers may be written, respectively, as

$$f_{\rm A}(n) = \begin{cases} D_{\rm A}^{\rm C}\cos\left\{k_{\rm A}[n-\frac{1}{2}(N_{\rm A}+1)]\right\}, & P=0, \\ D_{\rm A}^{\rm S}\sin\left\{k_{\rm A}[n-\frac{1}{2}(N_{\rm A}+1)]\right\}, & P=1, \end{cases} \tag{6a}$$

$$f_{\rm B}(n) = \begin{cases} D_{\rm B}^{\rm C}\cos\left\{k_{\rm B}[n-N_{\rm A}-\frac{1}{2}(N_{\rm B}+1)]\right\}, & P=0, \\ D_{\rm B}^{\rm S}\sin\left\{k_{\rm B}[n-N_{\rm A}-\frac{1}{2}(N_{\rm B}+1)]\right\}, & P=1, \end{cases} \tag{6b}$$

where D_i^j (i=A, B; j=C, S) is a normalization constant, P represents even parity ($P = 0$) or odd parity ($P = 1$), and $k_{\rm A}$ and $k_{\rm B}$ are, respectively, the wave-vector components perpendicular to the interfaces between sublayers A and B.

Inserting the interface-rescaling coefficient R[15] into Eq.(3) and using the periodic boundary condition, one can obtain

$$\begin{aligned} E &= 2J_{\rm A}S_{\rm A}(1+\gamma-\cos k_{\rm A})+\mu_0 g_{\rm A}H \\ &= 2J_{\rm B}S_{\rm B}(1+\gamma-\cos k_{\rm B})+\mu_0 g_{\rm B}H, \end{aligned} \tag{7}$$

and

$$\cot\left[\frac{1}{2}k_i(N_i+1)\right]=\frac{-\sin k_i}{\cos k_i-\mu_i^{-1}},\ P=0, \tag{8a}$$

$$\tan\left[\frac{1}{2}k_i(N_i+1)\right]=\frac{\sin k_i}{\cos k_i-\mu_i^{-1}},\ P=1, \tag{8b}$$

where the Landé factor $g_{\rm A}$ in sublayer A is equal to $g_{\rm B}$ in sublayer B, the index i represents A or B, and the parameters $\mu_{\rm A}$ and $\mu_{\rm B}$ are given in Ref.[17]. Solving Eqs.(7) and (8), one can obtain $k_{\rm A}$ and $k_{\rm B}$. When $k_{\rm A}$ and $k_{\rm B}$ take different values, we can obtain the body mode, the interface mode, the perfect confined mode, and the critical mode.[19]

The Zeeman excitation energy is eliminated from Eq.(7). Thus, in the following we only consider the reduced excitation energy.

3. Results and discussion

Choose the value of the parameter $J_{\rm A}$, $J_{\rm B}$, $S_{\rm A}$, $S_{\rm B}$, $N_{\rm A}$ and $N_{\rm B}$ as, respectively, 1.0, 1.0, 1/2, 1/2, 11 and 11. When $0 < J_{\rm AB} \le J_{\rm A}$, the excitation energy of bulk modes at the Γ point of the two-dimensional (2D) Brillouin zone is given in Fig.1, where (a) and (b), respectively, denote odd parity and even parity, $J_{\rm AB} = J_{\rm A} = 1.0$ represents the case without the interface between two sublayers. In this system, the total number of spin-wave modes is 22 ($= N_{\rm A} + N_{\rm B}$), in which there are 10 odd parity modes and 12 even parity modes.

Table 1. The bulk excitation energy E of odd parity for different $J_{\rm AB}$.

$J_{\rm AB}$	E									
	m=1	2	3	4	5	6	7	8	9	10
1.0	0.040	0.159	0.345	0.584	0.858	1.142	1.415	1.655	1.841	1.959
0.8	0.040	0.152	0.345	0.559	0.858	1.094	1.415	1.590	1.841	1.905
0.5	0.040	0.136	0.345	0.505	0.858	1.007	1.415	1.505	1.841	1.868
1.5	0.040	0.169	0.345	0.621	0.858	1.212	1.415	1.748	1.841	—
2.5	0.040	0.177	0.345	0.651	0.858	1.260	1.415	1.786	1.841	—
–0.5	0.040	0.252	0.345	0.788	0.858	1.379	1.415	1.831	1.841	—
–1.5	0.040	0.214	0.345	0.739	0.858	1.349	1.415	1.822	1.841	—

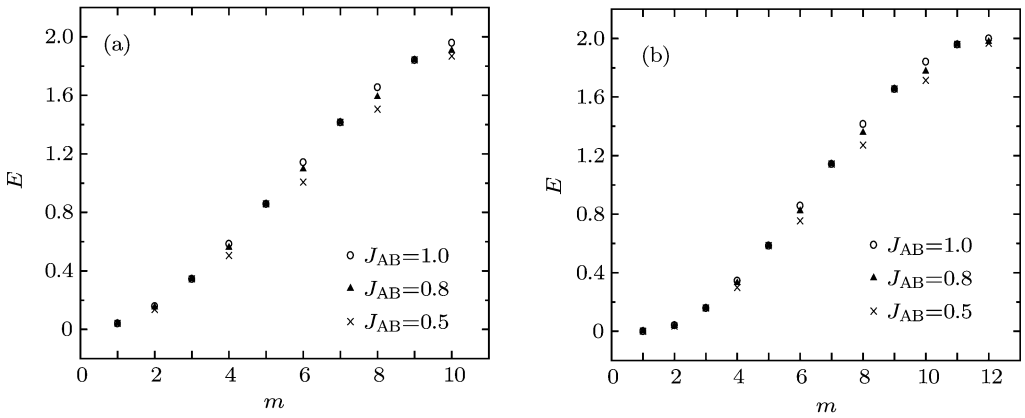


Fig.1. The excitation energy of bulk modes in the A–A system for $0 < J_{\rm AB} \le J_{\rm A}$ at the Γ point of the 2D Brillouin zone, (a) and (b) corresponding to the odd parity case and the even parity case, respectively. Here, m denotes the number of the bulk mode.

Figure 1 shows that the excitation energy of even bulk modes (EEEBM) reduces as $J_{\rm AB}$ decreases, but the excitation energy of odd bulk modes (EEOBM) in no way changes. The phenomenon is called the phenomenon

of even bulk mode variance (PEBMV). Because of the analogy between the odd parity case and the even parity case, we only give the bulk excitation energy of the odd parity in Table 1. When $J_{AB} > J_A$, EEEBM rises with J_{AB} increasing while EEOBM does not change. In the same way, EEEBM reduces as $|J_{AB}|$ increases while EEOBM shows no variation for the case of $J_{AB} < 0$. When $J_{AB} > J_A$ or $J_{AB} < 0$, there are interface modes in the system,[14,19] so the number of bulk modes of the odd parity is nine (see Table 1).

The cause of PEBMV is related to the selective effect of the interface on different bulk magnons. If spin waves of bulk magnons are symmetrical with respect to the interface between two sublayers, the interface will not affect these magnons; otherwise, the interface will affect the bulk magnons. Because spin waves of odd bulk magnons are symmetrical with respect to the interface while those of even bulk magnons are unsymmetrical with respect to the interface, odd bulk magnons will not be affected (see Fig.2) by the interface but even bulk magnons will be (see Figs.3 and 4). There are two kinds of mechanisms which cause PEBMV. One is phase translation and the other is phase reversal. It is phase translation for $J_{AB} > 0$, namely the excitation energy of even bulk modes reduces or rises because their wavelength is lengthened (see Fig.3) or shortened (see Fig.4(a)) due to the phase translation in the interface. It is phase reversal for $J_{AB} < 0$, namely the excitation energy of these modes rises because their wavelength is shortened (see Fig.4(b)) due to the phase reversal in the interface. Consequently, either the ferromagnetic or antiferromagnetic interface has an influence on the propagation of even bulk magnons.

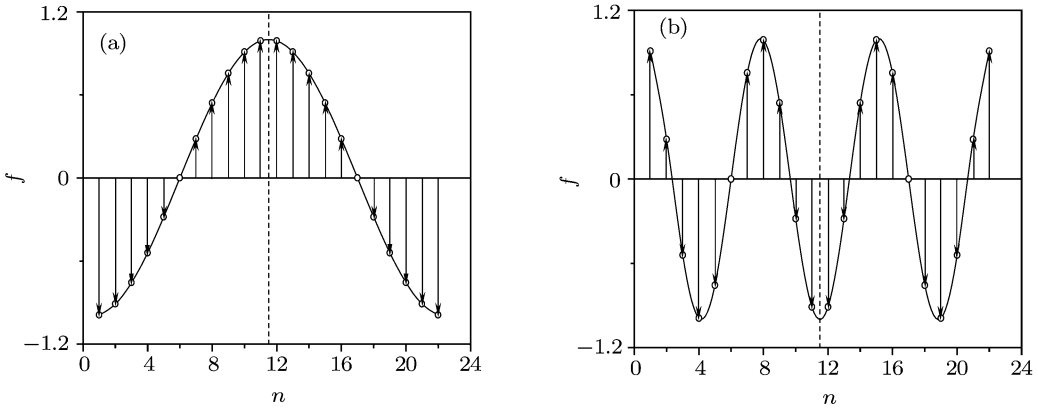


Fig.2. (a) and (b) represent, respectively, the spin wave of the 1st bulk mode and that of the 3rd bulk mode in the odd parity for an arbitrary J_{AB}. Here, n denotes the number of the lattice along the z axis. The vertical dashed line represents the interface between two sublayers. The open circles and the arrows denote, respectively, the amplitude and the vibrating direction of the magnon at different lattices, where the up-arrow represents the positive phase and the down-arrow represents the negative phase.

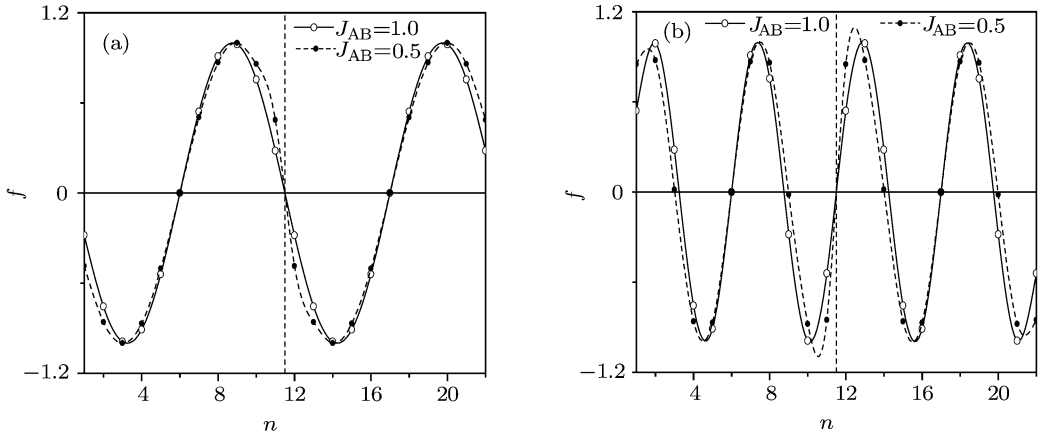


Fig.3. (a) and (b) represent, respectively, the spin wave of the 2nd bulk mode and that of the 4th bulk mode in the odd parity for $J_{AB} = 0.5$.

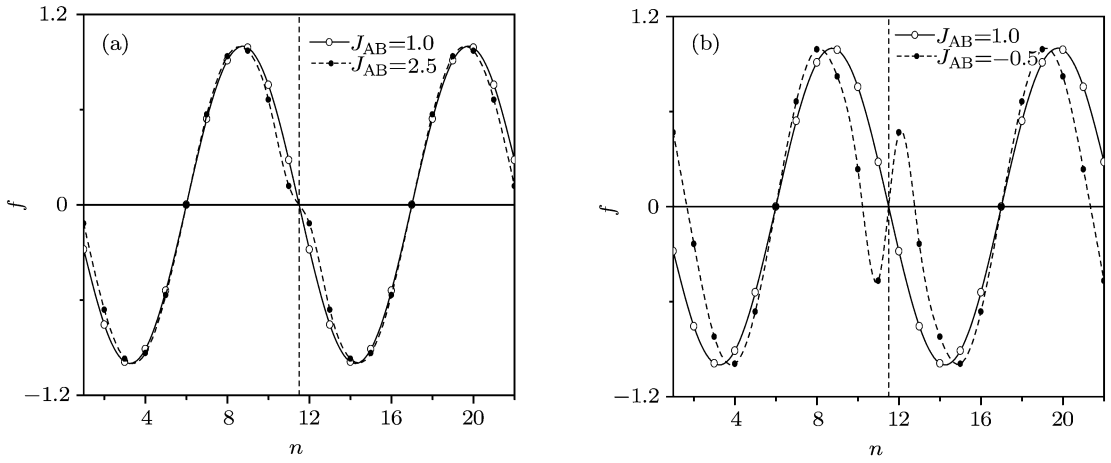


Fig.4. The spin wave of the 2nd bulk mode in the odd parity, (a) and (b) corresponding to $J_{AB} = 2.5$ and $J_{AB} = -0.5$, respectively.

This phenomenon is similar to Compton scattering. Because magnons of even bulk modes are scattered at the interface, changes in their spin wave vectors take place and thereby cause changes in their excitation energy.

4. Conclusions

In summary, the phenomenon of even bulk mode variance (PEBMV) is found in a symmetrical ferromagnetic bilayered system. When the interface is ferromagnetic coupling, namely, $J_{AB} > 0$, PEBMV results from the phase translation of spin waves; when the interface is antiferromagnetic coupling, namely, $J_{AB} < 0$, PEBMV results from the phase reversal of spin waves. Thus, the interface coupling J_{AB} not only affects the interface mode,[13] but also selectively affects the bulk mode.

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