Vol 18 No 12, December 2009
1674-1056/2009/18(12)/5491-05

Chinese Physics B

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Differential conductance in a normal metal/insulator/metal/d-wave superconductor junction carrying a supercurrent*

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(Received 3 March 2009; revised manuscript received 5 July 2009)

This paper applies the Bogoliubov–de Gennes equation and the Blonder–Tinkham–Klapwijk approach to study the oscillatory behaviour of differential conductance in a normal metal/insulator/metal/d-wave superconductor junction carrying a supercurrent $I_{\rm s}$. We find that (i) a three-humped structure appears at a nearly critical supercurrent $I_{\rm s}$ and $z \approx 0.5$ for the normal metal/insulator/metal/$\mathrm{d}_{x^2+y^2}$-wave superconductor junction; (ii) the zero-bias conductance peak splits into two peaks with sufficiently large applied current for the normal metal/insulator/metal/d_{xy}-wave superconductor junction; (iii) the conductance spectrum exhibits oscillating behaviour with the bias voltage and the peaks of the resonances are suppressed by increasing supercurrent $I_{\rm s}$.

Keywords: d-wave superconductor, conductance, supercurrent
PACC: 7450, 7465

Differential conductance characteristics in normal metal/insulator/superconductor junctions have been studied both theoretically and experimentally.[1−9] Tunnelling spectra depend strongly on the symmetry of the pair potential in the superconductor and therefore tunnelling spectroscopy is a very sensitive tool for superconductive study. For *s*-wave superconductors, these properties are well understood. On the other hand, theories of the tunnelling conductance of high-$T_{\rm c}$ superconductor junctions have been developed which take into account the anisotropy of the d-wave pair potential. In d-wave superconductors, the change in sign of the pair potential induces remarkable effects, i.e., a zero-bias conductance peak (ZBCP) in tunnelling experiments of high-$T_{\rm c}$ superconductors, due to the formation of zero-energy states.[3,4]

Very recently, the differential conductance in a normal metal (ferromagnet)/insulator/superconductor junction carrying a supercurrent $I_{\rm s}$ was measured and compared with theory.[10−14] It was found that the pair correlations are weakened by the supercurrent, leading to a modification of the density of states corresponding to the tunnelling spectra and leading to a reduction of gap. The coherence peaks were suppressed and broadened with increasing supercurrent $I_{\rm s}$, and the effect was the same as that caused by a magnetic field. Zhang *et al* studied theoretically the conductance characteristics of a clean normal metal/superconductor (N/S) junction with an $I_{\rm s}$ in the superconductor parallel to the interface.[10] Near critical $I_{\rm s}$ and $z \approx 0.5$, a three-humped structure appears, signalling the onset of current-induced gaplessness on the Fermi surface where the gap originally existed.

When a superconductor carries a supercurrent $I_{\rm s}$, the phase of $\Delta(\boldsymbol{k})$ has a spatial variation as $\mathrm{e}^{2\,\mathrm{i}\,q_{\rm s}r}$, where $\boldsymbol{r}$ is the centre-of-mass position of a Cooper pair, $q_{\rm s} = (m^* v_{\rm s}/2\hbar)$, with $v_{\rm s}$ the supercurrent velocity, and m^* the mass of the Cooper pair. In a two-dimensional d-wave S, the Δ_q versus q relation also depends on the directions of the supercurrent (Δ_q denotes the maximum gap in the presence of $I_{\rm s}$). Superconductivity disappears as $I_{\rm s}$ reaches the critical current. At zero temperature ($T = 0$ K), when $q < 0.3\Delta^0 (q = q_{\rm s}/k_{\rm F}$, $\Delta^0 = \Delta_0/E_{\rm F}$, here Δ_0 is the superconducting gap when $T = 0$ K, $I_{\rm s} = 0$; $k_{\rm F}$ and $E_{\rm F}$ are the Fermi momentum and energy, respectively), the changes of the order parameter of the two-dimensional d-wave S in both the antinodal ($\alpha = 0$) and nodal ($\alpha = \pi/4$) directions are almost the same.

*Project supported by the Natural Science Foundation of Jiangsu Provincial Education Commission of China (Grant No 06KJB140009).
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It is derived in Ref.[7] that the thermodynamic critical currents $I_{\rm sc}(0) = 0.238 env_{\rm F}\Delta_0$ at $q = q_{\rm c}(0) = 0.35\Delta_0$ and $I_{\rm sc}(\pi/4) = 0.225 env_{\rm F}\Delta_0$ at $q = q_{\rm c}(\pi/4) = 0.39\Delta_0$ for supercurrent $I_{\rm s}$ along the antinodal and nodal directions, respectively.

Another typical structure in the study of the transport properties of quasiparticles is a clean normal metal/insulator/metal/superconductor (N/I/N/S) junction with the insulator modelled as a delta function. The Andreev reflection[15] at the NS interface and the normal reflection at the delta function can construct a round-trip path, which results in a series of sub-gap differential conductance peaks.[16] In this paper, we consider a clean normal metal/insulator/metal/d-wave superconductor (N/I/N/dS) junction with a supercurrent $I_{\rm s}$ in the superconductor parallel to the interface as shown in Fig.1. There is a very thin insulator layer at $x = -L$, which is described by the delta function $V(x) = V_0\delta(x + L)$, where V_0 depends on the product of the barrier height. The d-wave pair potential has the form $\Delta(x) = \Delta_\pm = \Delta_0\cos(2\theta \mp 2\alpha)$ for $x > 0$ when $T = 0$ K, where Δ_+ (Δ_-) stands for the pair potential for electron-like (hole-like) quasiparticles, Δ_0 is constant, θ is the angle between the quasiparticle wave-vector and the interface normal, and α is the angle between the α-axis of the crystal and the interface normal.

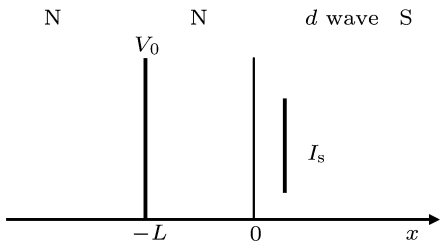


Fig.1. Schematic illustration of our model, with a supercurrent in the superconductor side parallel to the normal metal/insulator/metal/superconductor interface.

We adopt the Bogoliubov–de Gennes (BdG) approach to study the N/I/N/dS junction. The BdG equation has the form[17]

$$\begin{pmatrix} H_0 & \Delta(\boldsymbol{r}) \\ \Delta^*(\boldsymbol{r}) & -H_0^* \end{pmatrix}\begin{pmatrix} u(\boldsymbol{r}) \\ v(\boldsymbol{r}) \end{pmatrix} = E\begin{pmatrix} u(\boldsymbol{r}) \\ v(\boldsymbol{r}) \end{pmatrix}, \qquad (1)$$

where $H_0 = p^2/2m + V(x) - E_{\rm F}$ with $V(x)$ the usual static potential, E is the quasiparticle energy relative to the Fermi energy $E_{\rm F}$.

In the WKBJ approximation, equation (1) has special solutions of the form

$$\begin{pmatrix} u(\boldsymbol{r}) \\ v(\boldsymbol{r}) \end{pmatrix} = {\rm e}^{{\rm i}\,\boldsymbol{k}\cdot\boldsymbol{r}}\begin{pmatrix} u^0(\boldsymbol{r}){\rm e}^{{\rm i}\,\boldsymbol{q}_{\rm s}\cdot\boldsymbol{r}} \\ v^0(\boldsymbol{r}){\rm e}^{-{\rm i}\,\boldsymbol{q}_{\rm s}\cdot\boldsymbol{r}} \end{pmatrix}, \qquad (2)$$

where $u^0(\boldsymbol{r})$and $v^0(\boldsymbol{r})$ obey the generalized Andreev equations,[17]

$$\frac{\hbar^2}{2m}(\boldsymbol{k}+\boldsymbol{q_s})^2u^0(\boldsymbol{r}) - \frac{{\rm i}\,\hbar^2(\boldsymbol{k}+\boldsymbol{q_s})}{m}\cdot\nabla u^0(\boldsymbol{r}) + \Delta(\boldsymbol{r})\Theta(x)v^0(\boldsymbol{r}) = Eu^0(\boldsymbol{r}), \qquad (3a)$$

$$-\frac{\hbar^2}{2m}(\boldsymbol{k}-\boldsymbol{q_s})^2v^0(\boldsymbol{r}) + \frac{{\rm i}\,\hbar^2(\boldsymbol{k}-\boldsymbol{q_s})}{m}\cdot\nabla v^0(\boldsymbol{r}) + \Delta^*(\boldsymbol{r})\Theta(x)u^0(\boldsymbol{r}) = Ev^0(\boldsymbol{r}). \qquad (3b)$$

Obviously, the eigenenergy E is symmetric about $\hbar^2\boldsymbol{q_s}\cdot\boldsymbol{k}/m$ rather than zero. This leads to different energy gaps for different electron directions. For simplicity, uniform supercurrent $I_{\rm s}$ in S is assumed to be parallel to the interface in this paper.

The solutions to BdG equations in the N and the S are

$$\Psi_{\rm N1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}{\rm e}^{{\rm i}\,q_{\rm e}x\cos\theta} + a\begin{pmatrix} 0 \\ 1 \end{pmatrix}{\rm e}^{{\rm i}\,q_{\rm h}x\cos\theta} + b\begin{pmatrix} 1 \\ 0 \end{pmatrix}{\rm e}^{-{\rm i}\,q_{\rm e}x\cos\theta} \qquad (4)$$

for $x < -L$,

$$\Psi_{\rm N2} = e\begin{pmatrix} 1 \\ 0 \end{pmatrix}{\rm e}^{{\rm i}\,q_{\rm e}x\cos\theta} + f\begin{pmatrix} 0 \\ 1 \end{pmatrix}{\rm e}^{{\rm i}\,q_{\rm h}x\cos\theta} + g\begin{pmatrix} 0 \\ 1 \end{pmatrix}{\rm e}^{-{\rm i}\,q_{\rm h}x\cos\theta} + h\begin{pmatrix} 1 \\ 0 \end{pmatrix}{\rm e}^{-{\rm i}\,q_{\rm e}x\cos\theta} \qquad (5)$$

for $-L < x < 0$,

$$\Psi_{\rm S} = c\begin{pmatrix} u_+{\rm e}^{{\rm i}\,\phi_+} \\ v_+ \end{pmatrix}{\rm e}^{{\rm i}\,k_{\rm e}x\cos\theta} + d\begin{pmatrix} v_-{\rm e}^{{\rm i}\,\phi_-} \\ u_- \end{pmatrix}{\rm e}^{-{\rm i}\,k_{\rm h}x\cos\theta} \qquad (6)$$

for $x > 0$, where a, b, c and d correspond to the coefficients for the Andreev reflection,[15] normal reflection, transmission to the S as electronlike quasiparticles and transmission as holelike quasiparticles respectively. The $\phi_\pm$ stand for the phases of the S, where ${\rm e}^{{\rm i}\,\phi_\pm} = \frac{\Delta_\pm}{|\Delta_\pm|}$. In Eqs.(4)–(6).

$$q_{\rm e} = k_{\rm F}\sqrt{1 + (-\hbar^2q_{\rm s}^2/2m + E')/E_{\rm F}}, \qquad (7a)$$

$$q_{\rm h} = k_{\rm F}\sqrt{1 + (-\hbar^2q_{\rm s}^2/2m - E')/E_{\rm F}}, \qquad (7b)$$

which indicates different wave-vectors for electrons and holes in N, with $E' = E - \hbar^2 q_s k_F \sin\theta/m$. In S

$$k_e = k_F\sqrt{1+(-\hbar^2 q_s^2/2m+\Omega_+)/E_F}, \quad (8a)$$

$$k_h = k_F\sqrt{1-(-\hbar^2 q_s^2/2m+\Omega_-)/E_F}, \quad (8b)$$

and

$$u_\pm = \sqrt{[1+\Omega_\pm/(E-\hbar^2 k_F q_s\sin\theta/m)]/2}, \quad (9a)$$

$$v_\pm = \sqrt{[1-\Omega_\pm/(E-\hbar^2 k_F q_s\sin\theta/m)]/2} \quad (9b)$$

with $\Omega_\pm = \sqrt{(E-\hbar^2 k_F q_s\sin\theta/m)^2 - |\Delta_\pm|^2}$.

The wave functions must satisfy the boundary conditions[18]

$$\Psi_{N1}(x=-L) = \Psi_{N2}(x=-L), \quad (10a)$$

$$\left(\frac{d\Psi_{N2}}{dx}\right)_{x=-L} - \left(\frac{d\Psi_{N1}}{dx}\right)_{x=-L} = \frac{2m}{\hbar^2}V_0\Psi_{N1}(x=-L), \quad (10b)$$

$$\Psi_{N2}(x=0) = \Psi_S(x=0), \quad (11a)$$

$$\left(\frac{d\Psi_{N2}}{dx}\right)_{x=0} - \left(\frac{d\Psi_S}{dx}\right)_{x=0} = 0. \quad (11b)$$

From Eqs.(4)–(6), (10) and (11) we obtain

$$a = \frac{v_+ e^{-i\phi_-}}{u_+(1+z^2)\eta}, \quad (12a)$$

$$b = \frac{-iz e^{-2iq_h L}}{(1+iz)\eta}\left\{1 - \frac{v_+ v_-}{u_+ u_-}e^{i[\phi_- - \phi_+ + 2(q_e - q_h)L]}\right\}, \quad (12b)$$

$$\eta = 1 - \frac{z^2 v_+ v_-}{(1+z^2)u_+ u_-}e^{i[\phi_- - \phi_+ + 2(q_e - q_h)L]},$$

and $z = mV_0/(\hbar^2 k_F\cos\theta)$. We have used the Andreev approximation in the boundary conditions, with $k_{e(h)} = k_F$ and $q_{e(h)} = k_F \pm mE/\hbar k_F$. By using the Blonder–Tinkham–Klapwijk model,[18] we are able to calculate the differential conductance of the N/I/N/dS junction at temperature $T = 0$ K,

$$G(eV) = G_0\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta \times \int_{-\infty}^{\infty} dE\,[1+A-B]\delta(E-eV), \quad (13)$$

where $A = |a|^2$, $B = |b|^2$. The reflection coefficients are evaluated for $E = eV$, where V is the bias voltage.

Let us first study the quantum interference effect on tunnelling conductance in the N/I/N/$d_{x^2+y^2}$($\alpha = 0$, $\Delta_\pm = \Delta_0\cos(2\theta)$) junction carrying a supercurrent. With the help of Eqs.(12) and (13), the dependence of the differential conductance on the bias voltage for various z and q is shown in Fig.2. Equation (13) shows that while ordinary reflection reduces the conductance, Andreev reflection increases it. When a uniform supercurrent I_s passes through a two-dimensional d-wave S, the phase of $\Delta(\boldsymbol{k})$ has a spatial variation as $e^{2iq_s r}$. The spatially varying phase leads to an anisotropic quasiparticle excitation spectrum in a clean S besides the intrinsic anisotropic property. For $L = 0$, the central peak due to Andreev reflection is gradually suppressed and broadened by increasing q with $Z = 0$. With increasing barrier strength Z, Andreev reflection is greatly suppressed, so the ZBCP diminishes rapidly. There are two conductance peaks at $|eV| \approx \Delta_0$, called coherence peaks. When $q \neq 0$, the eigenenergy E is symmetric about $\hbar^2\boldsymbol{q}_s\boldsymbol{k}/m$ rather than zero. The energy corresponding to the coherence peak is determined by $|E| - \hbar^2\boldsymbol{q}_s\boldsymbol{k}/m \approx \Delta_0$. As q increases, the coherence peaks are suppressed and move outward, because of the gap anisotropy caused by the supercurrent. A three-humped structure including a peak at zero bias appears at nearly critical I_s and $z = 0.5$ because the system has become gapless. With the insulator layer moving away from the NS interface, the multiple scatterings in the two regions interplay with each other. For $L/\xi_0 = 6$, conductance spectrum exhibits oscillating behaviour with the bias voltage except when $z = 0$. The numbers and the position of the resonances are related to q. The peaks of the resonances are also suppressed by increasing z or q. In the N/I/N/d junction carrying a supercurrent, the oscillations of differential conductance arise mainly from quantum interference effects within the intermediate N layer. When $L = 0$ or $Z = 0$, the oscillation behaviour of G in the N/I/N/d junctions carrying a supercurrent vanishes. This is because the N/I/N/d junctions carrying a supercurrent reduce to N/I/d junctions carrying a supercurrent.

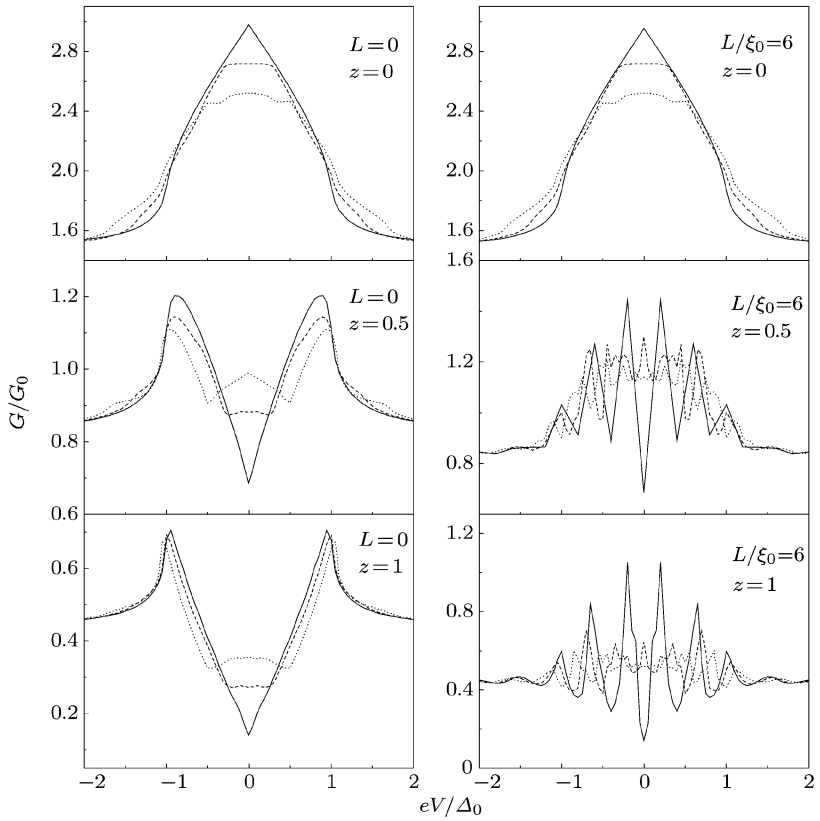


Fig.2. Differential conductance as a function of the bias voltage for the N/I/N/$d_{x^2+y^2}$ ($\alpha = 0$) junction. $q = 0$ (solid), $q = 0.2\Delta^0$ (dash) and $q = 0.35\Delta^0$ (dot).

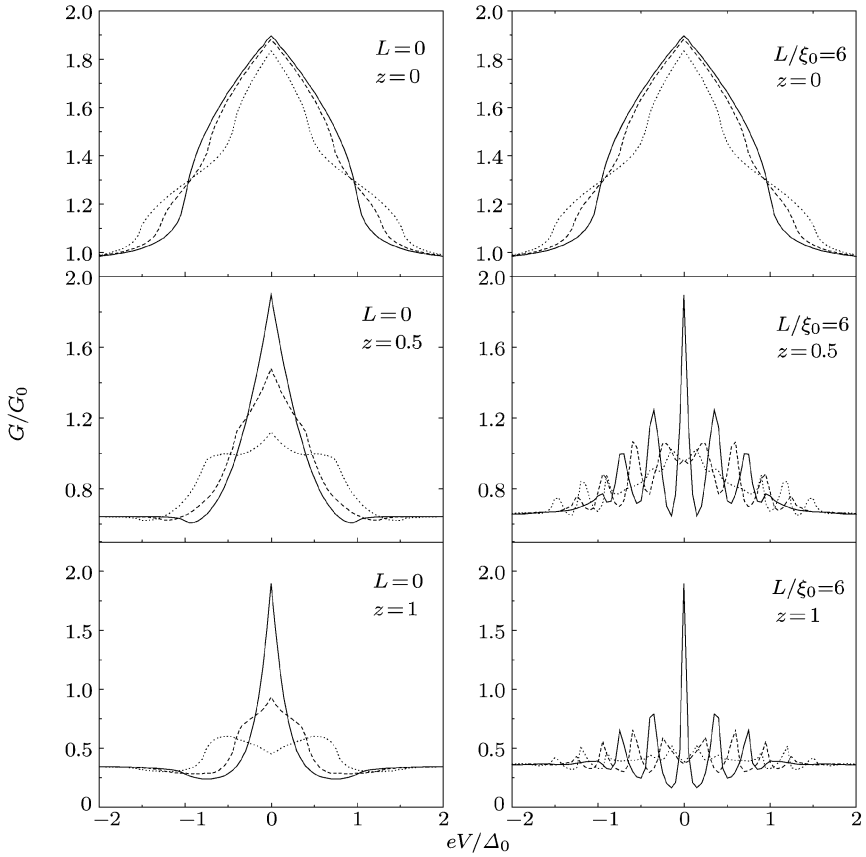


Fig.3. Differential conductance as a function of the bias voltage for the N/I/N/d_{xy} ($\alpha = \pi/4$) junction. $q = 0$ (solid), $q = 0.2\Delta^0$ (dash) and $q = 0.39\Delta^0$ (dot).

The differential conductance in the N/I/N/d$_{xy}$ ($\alpha = \pi/4$, $\Delta_{\pm} = \pm\Delta_0 \sin(2\theta)$) junction with a supercurrent in the superconductor side is shown in Fig.3. For $L = 0$, the ZBCP induced by midgap surface states is suppressed, broadened, and eventually split at sufficiently large z when q is increased. When z decreases, the conductance peak is lowered. When $z = 1$ and $q = 0.35\Delta^0$, the ZBCP disappears and the conductance spectra show a zero-bias conductance dip structure. For $L/\xi_0 = 6$, the number and position of resonances in the differential conductance for the N/I/N/d$_{xy}$ ($\alpha = \pi/4$) junction is different from that for the N/I/N/d$_{x^2+y^2}$ ($\alpha = 0$) junction.

In summary, we have studied the differential conductance in a clean N/I/N/dS junction with I_s in the superconductor parallel to the interface. For $L = 0$, the central peak is gradually suppressed and broadened by increasing q in the N/I/N/dS junction with a supercurrent in the superconductor side. There is a three-peak structure at nearly critical I_s and $z = 0.5$ in the N/I/N/d$_{x^2+y^2}$ ($\alpha = 0$) junction. With increasing L, the conductance spectrum exhibits oscillating behaviour with the bias voltage and the peaks of the resonances are suppressed by increasing supercurrent I_s.

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