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Spin-dependent electron transport of a waveguide with Rashba spin–orbit coupling in an electromagnetic field*

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We investigate theoretically the spin-dependent electron transport in a straight waveguide with Rashba spin–orbit coupling (SOC) under the irradiation of a transversely polarized electromagnetic (EM) field. Spin-dependent electron conductance and spin polarization are calculated as functions of the emitting energy of electrons or the strength of the EM field by adopting the mode matching approach. It is shown that the spin polarization can be manipulated by external parameters when the strength of Rashba SOC is strong. Furthermore, a sharp step structure is found to exist in the total electron conductance. These results can be understood by the nontrivial Rashba subbands intermixing and the electron intersubband transition when a finite-range transversely polarized EM field irradiates a straight waveguide.

Keywords: waveguide, spin–orbit coupling, spin polarization
PACC: 7320D, 7170C, 7225

1. Introduction

Since the pioneering work of Datta and Das[1] and the experimental realization of the gate-voltage control on the strength of Rashba spin–orbit coupling (SOC) in a semiconductor heterostructure two-dimensional electron gas (2DEG),[2] the spin-related electron transport in mesoscopic systems in the presence of Rashba SOC[3−13] has been studied extensively because of their potential application in future spintronic devices. In these devices the key issue is how to inject and manipulate electron spin as the carrier of information.

A polarized EM field can be used to generate spin current. When a mesoscopic system is illuminated by a coherent external EM field, many new features will arise due to the inelastic scattering of electrons by photons.[14−24] The spin photocurrent was detected in (001)-grown n-type GaAs multiple quantum well structures under circularly polarized infrared radiation.[25] In the same structures, pure spin current was found by the irradiation of orthogonally polarized two optical harmonic pulses.[26] These results may arise from resonant intersubband transitions.

The question of what will happen when the EM field and Rashba SOC are present simultaneously in a mesoscopic system may be interesting. Recently, the spin transport properties of a 2DEG in the presence of both Rashba SOC and a longitudinally polarized terahertz laser field have been investigated theoretically.[27,28] It is found that the electron density of states, the Fermi energy and spin polarization are sensitive to the longitudinally polarized terahertz laser field. However, the spin transport properties of a low-dimensional system in the presence of both Rashba SOC and a transversely polarized EM field have been studied rarely.[29] The influence of a transversely polarized EM field on the electron transport is different from that of a longitudinally polarized EM field because the former results in intersubband transitions[20,23,24] while the latter leads to intrasubband transmissions.[19,22] Consequently, some new effects may emerge when a Rashba low-dimensional system is irradiated under a transversely polarized EM field. Furthermore, electron spin in the system may be manipulated by adjusting the parameters of the external EM field.

In this paper, we theoretically calculate the spin-dependent electron conductance and spin polarization of a straight Rashba waveguide submitted to a transversely polarized EM field at low temperatures. The effects of different EM field parameters and Rashba SOC strengths on the electron transport properties are studied by the mode matching approach. The results

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are twofold. First, the spin-dependent electron conductance is modulated by the Rashba SOC. The properties of the spin polarization are quite different when the Rashba subbands intermixing, due to the interaction between the subbands and Rashba SOC, is weak or strong. Second, the effect of the finite-range EM field on the waveguide is equivalent to that of a barrier which yields an electron reflection from lateral collisions of electrons with photons. A new feature is that a sharp step structure appears in the total electron conductance as the emitting energy or the strength of the EM field increases to a threshold value due to the effect of electronic mode split induced by the EM field. The paper is organized as follows. In Section 2 we present the model Hamiltonian and the formulation for solving the time-dependent Schrödinger equation of the system. In Section 3 we analytically calculate the electron conductance of the system. The numerical results and discussion are shown in Section 4. Finally, Section 5 concludes the paper.

2. Model and formalism

The system considered is sketched in Fig.1, where a straight waveguide of width d is smoothly connected with two metal electrodes at the ends. The x-axis is longitudinally along the wire, while the y-axis describes the transversal direction, and part of the waveguide ($0 < x < L$) is irradiated under a transversely polarized EM field in an unspecified way, so that the EM field transports along the z-axis. The Rashba SOC is assumed to be only present in the irradiated area. Because the size of the waveguide in the z-axis is much smaller than the wavelength of the illuminating EM field, the vector potential can be written as $\boldsymbol{A}(t) = -(\varepsilon/\omega)\cos(\omega t)\hat{y}$, where ω and ε are the frequency and amplitude of the EM field respectively, and $\hat{y}$ is the unit vector of the y coordinate. The z-axis is chosen as the spin-quantized axis.

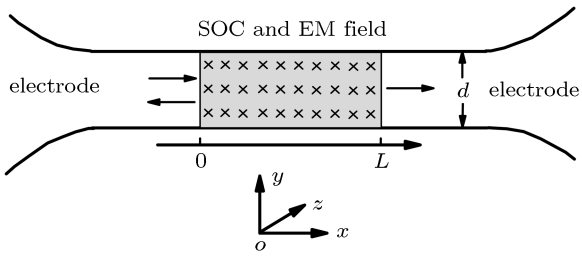


Fig.1. Schematic view of the system considered here. The width of the waveguide is d. The grey area represents the Rashba SOC and the EM field region with its length L. The arrows denote the propagation directions of electrons.

At low temperatures, we can use the free electron model to describe the electronic dynamics. The appropriate time-dependent Schrödinger equation for an electron, therefore, is

$$\mathrm{i}\,\hbar\frac{\partial}{\partial t}\Psi(x,y,t) = \left\{\frac{(\hat{p}-\frac{e}{c}\boldsymbol{A})^2}{2m^*} + U(y) + \frac{1}{2\hbar}[\alpha(x)(\hat{\sigma}\times\hat{p})_z + (\hat{\sigma}\times\hat{p})_z\alpha(x)]\right\}\Psi(x,y,t), \tag{1}$$

where $\hat{p} = -\mathrm{i}\,\hbar(\partial/\partial x + \partial/\partial y)$ is the momentum, m^* and e are the electron's effective mass and charge, $\hat{\sigma}$ is a Pauli matrix. $\alpha(x)$ are the effective parameters describing the strength of the Rashba effect: $\alpha(x) = \alpha$ for $0 < x < L$, and $\alpha(x) = 0$ elsewhere.

The form of the electronic wavefunction in the irradiated area can be expressed as

$$\Psi(x,y,t) = \sum_k \mathrm{e}^{\mathrm{i}\,kx}\sum_{n,s}\phi_n(y)c_n^s(t)|\sigma\rangle\mathrm{e}^{-\mathrm{i}(E/\hbar)t}, \tag{2}$$

where k is the longitudinal wave vector of the electron, $c_n^s(t)$ is the time-dependent amplitude of different subbands and spin states, $|\,s\rangle = |\uparrow\rangle$ for spin-up and $|\,s\rangle = |\downarrow\rangle$ for spin-down, $\phi_n(y)$ is the eigenfunction of the steady-state Schrödinger equation

$$\left[\frac{\hat{p}_y^2}{2m^*} + U(y)\right]\phi_n(y) = \epsilon_n\phi_n(y), \tag{3}$$

in which ϵ_n is the transversal subband energy level.

For the sake of simplicity, only the lowest two subbands in the irradiated area are assumed to be involved in the transport. Inserting Eqs.(2) and (3) into Eq.(1) and neglecting the $\sim A^2$ term (this term only gives an extra phase that is common to all modes) we obtain the matrix form of the Schrödinger equation

$$\mathrm{i}\hbar\frac{\partial}{\partial t}V=\begin{bmatrix} E_1 & \alpha k & \mathrm{i}\gamma\cos(\omega t) & -\beta \\ \alpha k & E_1 & \beta & \mathrm{i}\gamma\cos(\omega t) \\ -\mathrm{i}\gamma\cos(\omega t) & \beta & E_2 & \alpha k \\ -\beta & -\mathrm{i}\gamma\cos(\omega t) & \alpha k & E_2 \end{bmatrix} V, \tag{4}$$

where $V=[c_1^{\uparrow}(t)\ \ c_1^{\downarrow}(t)\ \ c_2^{\uparrow}(t)\ \ c_2^{\downarrow}(t)]^{\mathrm{T}}$ is a time-dependent row vector, here T means transposition, $E_{1(2)}=\hbar^2k^2/2m^*+\epsilon_{1(2)}$ is the total energy of electrons on the lower (upper) subband, $\beta=\alpha\int_{-d/2}^{d/2}\phi_1^*(y)\frac{\mathrm{d}}{\mathrm{d}y}\phi_2(y)$ and $\gamma=\frac{2e\varepsilon}{c\omega}\int_{-(d/2)}^{(d/2)}\phi_1^*(y)\frac{\partial}{\partial y}\phi_2(y)\mathrm{d}y=\frac{2e\varepsilon}{c\omega}\int_{-(d/2)}^{(d/2)}\phi_2^*(y)\frac{\partial}{\partial y}\phi_1(y)\mathrm{d}y$ denote the Rashba SOC and the EM field interaction with the lowest two subbands, respectively.

Applying a time-dependent unitary transformation and rotating wave approximation, the time-independent Schrödinger equation is obtained

$$\begin{bmatrix} E_1+\frac{\hbar\omega}{2} & \alpha k & \frac{\mathrm{i}\gamma}{2} & -\beta \\ \alpha k & E_1+\frac{\hbar\omega}{2} & \beta & \frac{\mathrm{i}\gamma}{2} \\ -\frac{\mathrm{i}\gamma}{2} & \beta & E_2-\frac{\hbar\omega}{2} & \alpha k \\ -\beta & -\frac{\mathrm{i}\gamma}{2} & \alpha k & E_2-\frac{\hbar\omega}{2} \end{bmatrix} V'=\lambda V', \tag{5}$$

in which $V'=[c_1^{\uparrow\prime}\ \ c_1^{\downarrow\prime}\ \ c_2^{\uparrow\prime}\ \ c_2^{\downarrow\prime}]^{\mathrm{T}}$ is a new time-independent eigenvector. Solving this secular function, the four eigenvalues $E_1^{\pm}$ and $E_2^{\pm}$ and the corresponding four time-independent eigenvectors V_j' $(j=1,2,3,4)$ are obtained.

Then through an inverse transformation, the time-dependent row vectors of the electron in the irradiated area V_j $(j=1,2,3,4)$ are finally achieved, so that the electronic wavefunction in the EM field irradiated area is

$$\begin{aligned}\Psi(x,y,t)=&\, m_1\mathrm{e}^{\mathrm{i}K_1x}\mathrm{e}^{-\mathrm{i}(E_1^+/\hbar)t}V_1\\ &+m_2\mathrm{e}^{\mathrm{i}K_2x}\mathrm{e}^{-\mathrm{i}(E_1^-/\hbar)t}V_2\\ &+m_3\mathrm{e}^{\mathrm{i}K_3x}\mathrm{e}^{-\mathrm{i}(E_2^+/\hbar)t}V_3\\ &+m_4\mathrm{e}^{\mathrm{i}K_4x}\mathrm{e}^{-\mathrm{i}(E_2^-/\hbar)t}V_4,\end{aligned} \tag{6}$$

where m_1, m_2, m_3 and m_4 are arbitrary constants; K_1, K_2, K_3 and K_4 are the wave vectors in the irradiated area that can be determined by the four eigenvalues.

3. Electron conductance of the system

When a spin-up electron with energy E emits from the left metal electrode to the interface at $x=0$, transmission and reflection will take place simultaneously. Because the electron has a certain probability of absorbing a photon after penetrating the interface, a transition from lower mode to upper mode happens, hence there are two energy components E and $E+\hbar\omega$ in the reflection wave in the region of $x<0$. The electronic wavefunction in this area is

$$\begin{aligned}\Psi(x<0,y,t)=&\,\mathrm{e}^{\mathrm{i}k_1x-\mathrm{i}(E/\hbar)t}\phi_1(y)\,|\uparrow\rangle+r_1\mathrm{e}^{-\mathrm{i}k_1x-\mathrm{i}(E/\hbar)t}\phi_1(y)\,|\uparrow\rangle+r_2\mathrm{e}^{-\mathrm{i}k_1x-\mathrm{i}(E/\hbar)t}\phi_1(y)\,|\downarrow\rangle\\ &+r_3\mathrm{e}^{-\mathrm{i}k_2x-\mathrm{i}[(E+\hbar\omega)/\hbar]t}\phi_2(y)\,|\uparrow\rangle+r_4\mathrm{e}^{-\mathrm{i}k_2x-\mathrm{i}[(E+\hbar\omega)/\hbar]t}\phi_2(y)\,|\downarrow\rangle,\end{aligned} \tag{7}$$

where r_1, r_2, r_3 and r_4 are the reflection coefficients, $k_1=\frac{\sqrt{2m^*(E-\epsilon_1)}}{\hbar}$, $k_2=\frac{\sqrt{2m^*(E+\hbar\omega-\epsilon_2)}}{\hbar}$.

According to Eq.(6), the electronic wavefunction in the irradiated area can be written as

$$\begin{aligned}\Psi(0<x<L,y,t)=\sum_{\pm}\{&m_{1\pm}\mathrm{e}^{\mathrm{i}K_{1\pm}x}\mathrm{e}^{-\mathrm{i}(E/\hbar)t}V_{1\pm}+m_{2\pm}\mathrm{e}^{\mathrm{i}K_{2\pm}x}\mathrm{e}^{-\mathrm{i}(E/\hbar)t}V_{2\pm}\\ &+m_{3\pm}\mathrm{e}^{\mathrm{i}K_{3\pm}x}\mathrm{e}^{-\mathrm{i}(E/\hbar)t}V_{3\pm}+m_{4\pm}\mathrm{e}^{\mathrm{i}K_{4\pm}x}\mathrm{e}^{-\mathrm{i}(E/\hbar)t}V_{4\pm}\},\end{aligned} \tag{8}$$

where $m_{j\pm}$ are arbitrary constants, $K_{j\pm}$ and $V_{j\pm}$ $(j=1,2,3,4)$ are, respectively, the wave vectors and eigenvectors, which can be determined by Eq.(5).

The electronic wavefunction in the region of $x > L$ is given by

$$\Psi(x > L, y, t) = t_1 \mathrm{e}^{\mathrm{i}\, k_1 x - \mathrm{i}(E/\hbar)t}\phi_1(y)\,|\uparrow\rangle + t_2 \mathrm{e}^{\mathrm{i}\, k_1 x - \mathrm{i}(E/\hbar)t}\phi_1(y)\,|\downarrow\rangle + t_3 \mathrm{e}^{\mathrm{i}\, k_2 x - \mathrm{i}[(E+\hbar\omega)/\hbar]t}\phi_2(y)\,|\uparrow\rangle + t_4 \mathrm{e}^{\mathrm{i}\, k_2 x - \mathrm{i}[(E+\hbar\omega)/\hbar]t}\phi_2(y)\,|\downarrow\rangle, \quad (9)$$

where t_1 and t_3 are the transmission coefficients for the spin-conserved electron, and t_2 and t_4 are the transmission coefficients for the spin-flip electron.

We now match the electronic wavefunctions at the interfaces $x = 0$ and $x = L$, where the wavefunctions and their flux must be continuous. The transmission coefficients t_1, t_2, t_3 and t_4 can be determined by the boundary conditions. Based upon the Landauer–Büttiker formula, the spin-dependent electron conductance and the total electron conductance can be obtained

$$\begin{aligned} G^{\uparrow\uparrow} &= (|t_1|^2 + |t_3|^2)G_0, \\ G^{\downarrow\uparrow} &= (|t_2|^2 + |t_4|^2)G_0, \\ G^{\mathrm{total}} &= G^{\uparrow\uparrow} + G^{\downarrow\uparrow}, \end{aligned} \quad (10)$$

where $G_0 = e^2/h$ is the conductance quantum. The definition of spin polarization is

$$P = \frac{G^{\uparrow\uparrow} - G^{\downarrow\uparrow}}{G^{\mathrm{total}}}. \quad (11)$$

4. Numerical results and discussion

In our numerical example, the waveguide is taken to be that in a 2DEG of high-mobility GaAs/Al_xGa_{1-x}As with a typical electron density $n \sim 2.5 \times 10^{11}/\mathrm{cm}^2$ and $m^* = 0.067m_\mathrm{e}$, where m_e is the mass of the free electron. The structural parameters are $d = 50$ nm and $L = 400$ nm. The boundary of the waveguide is determined by the hard-well confining potential, i.e., $U(y) = 0$ when $|y| < d/2$ (else ∞), which can eliminate the possibility of Rashba SOC due to the effective electric field coming from the nonuniformity of the confining potential.[30,31]

According to Refs.[23] and [24], an electron has the most possible probability to absorb a photon in the resonance case, namely, the energy of photon equals the energy gap between the lowest two subbands. Thereby, only the resonance ($\omega = 10.2$ THz) spin-dependent electron conductance and spin polarization of the system are shown in the following calculation.

The resonance spin-dependent electron conductance as a function of the emitting energy of electrons is illustrated in Fig.2(a) with parameters of $\alpha = 1.05 \times 10^{-11}$ eV·m and $\varepsilon = 60$ V/mm. The solid line stands for the total electron conductance, while the dashed and dotted lines stand for spin-up electron conductance and spin-down electron conductance, respectively. One can see that the electron is blocked by the EM field when the emitting energy $E < 5.31$ meV. Interestingly, there is a 0.5 G_0 step arising in the total electron conductance as E increases to 5.31 meV. Further, another 0.5 G_0 step appears as E reaches 5.90 meV. The conductance profiles in Fig.2(a) can be explained by the resonance energy-subband dispersion in Fig.2(b). When an electron penetrates through the interface to the region with EM field and Rashba SOC, the transversal levels of the electron in the waveguide are dressed and one electron mode is split into two pairs of electron modes. All the electron modes are evanescent (nonpropagating) modes when $E < 5.31$ meV, which contribute nothing to the conductance so that the conductance is zero. When 5.31 meV$< E < 5.90$ meV, the lower pair of modes become propagating modes whereas the upper pair of modes still are nonpropagating modes so that the conductance is only about 0.5 G_0. Furthermore, when $E > 5.90$ meV, the upper pair of modes also become propagating, which results in another 0.5 G_0 conductance. This pure external field effect cannot be achieved for the waveguide irradiated under a longitudinally-polarized EM field. Apart from the step structures in Fig.2(a), another character, the oscillation of electron conductance, is also found in our results. This oscillation originates from the interference between different pairs of modes and the backscattering caused by the EM-field-induced barrier. In addition, the spin polarization (see the inset) remains constant, i.e., $P = 0.47$, with the variation of the emitting energy of electrons. In Fig.3 we show the resonance spin-dependent electron conductance as a function of the strength of the EM field with parameters of $\alpha = 1.05 \times 10^{-11}$ eV·m and $E = 6.12$ meV. The 0.5 G_0 step structure also can be found in the total electron conductance when the EM field strength

increases to 47 V/mm, but its direction is opposite to

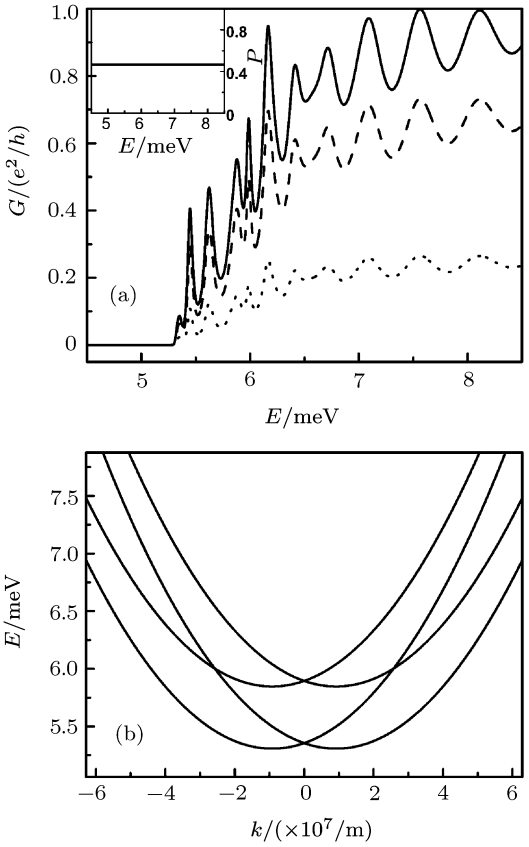


Fig.2. (a) The resonance spin-dependent electron conductance as a function of the emitting energy of electrons. The dashed line represents $G^{\uparrow\uparrow}$, the dotted line $G^{\downarrow\uparrow}$, and the solid line G^{total}. Inset: spin polarization vs. the emitting energy of electrons. (b) The resonance energy-subband dispersion of the waveguide with EM field and Rashba SOC. The parameters are $\alpha = 1.05 \times 10^{-11}$ eV·m and $\varepsilon = 60$ V/mm.

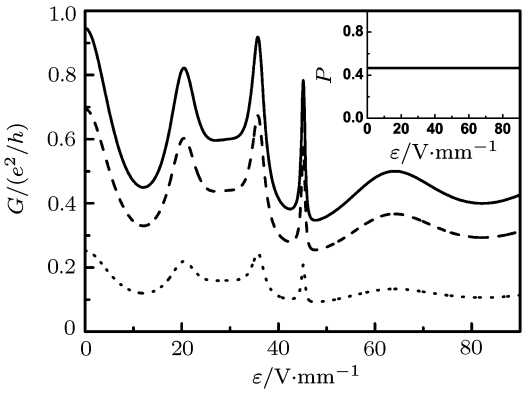


Fig.3. The resonance spin-dependent electron conductance as a function of the strength of the EM field. Inset: spin polarization vs. the strength of the EM field. The line presentation and the parameters are the same as those in Fig.2(a) except for $E = 6.12$ meV.

that in Fig.2(a) due to the upper pair modes changing from propagating modes to nonpropagating modes so that the conductance is suppressed to near 0.5 G_0. Interestingly, the spin polarization (see the inset) not only keeps constant but also has its magnitude equal to that in Fig.2(a) as the strength of the EM field varies, which can be attributed to the Rashba subbands intermixing being weak enough to be negligible.

Figure 4(a) plots the resonance spin-dependent electron conductance as a function of the emitting energy of electrons with another Rashba SOC strength $\alpha = 3.45 \times 10^{-11}$ eV·m. The other parameters are the same as those for Fig.2(a). The 0.5 G_0 step structures and oscillation behaviours also can be found to exist in the total electron conductance, which can be understood from the resonance energy-subband dispersion in Fig.4(b). The physical origin of these effects is the same as that in Fig.2(a). However, the spin polarization (see the inset) does not remain unchanged but varies as the emitting energy increases.

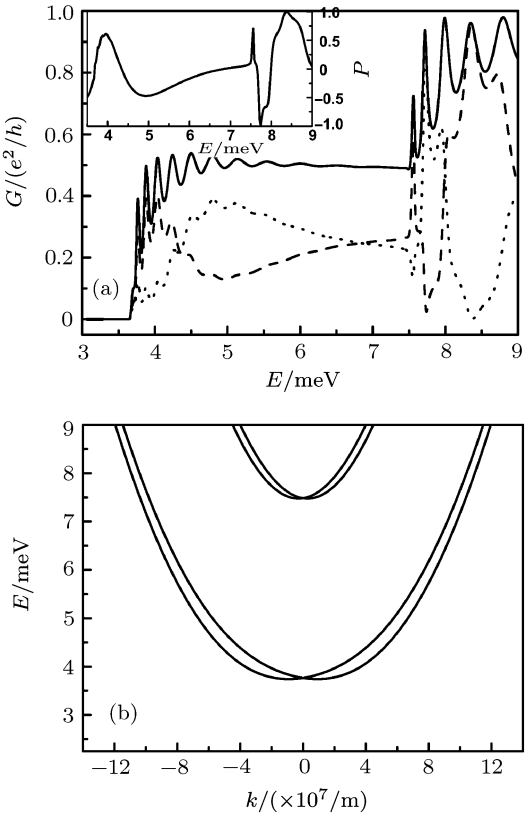


Fig.4. (a) The resonance spin-dependent electron conductance as a function of the emitting energy of electrons. The line presentation is the same as that in Fig.2(a). Inset: spin polarization vs. the emitting energy of electrons. (b) The resonance energy-subband dispersion of the waveguide with EM field and Rashba SOC. The parameters are the same as those in Fig.1 except for $\alpha = 3.45 \times 10^{-11}$ eV·m.

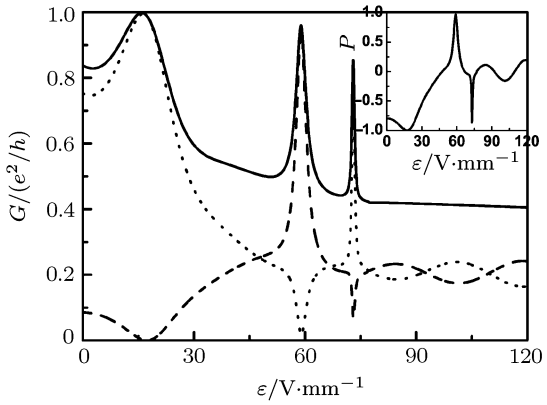


Fig.5. The resonance spin-dependent electron conductance as a function of the strength of the EM field. Inset: spin polarization vs. the strength of the EM field. The line presentation is the same as that in Fig.2(a). The parameters are $\alpha = 3.45 \times 10^{-11}$ eV·m and $E = 7.59$ meV.

The spin polarization almost equals -1 when $E = 7.7$ meV, implying full spin flip. Then it almost equals 1 when $E = 8.4$ meV, which indicates that the electron spin keeps its original state after passing the waveguide. Figure 5 shows the resonance spin-dependent electron conductance as a function of the strength of EM field with parameters of $\alpha = 3.45 \times 10^{-11}$ eV·m and $E = 7.59$ meV. Interestingly, the spin polarization (see the inset) also varies with the increase of the strength of the EM field; full spin flip and spin conservation can also be achieved when the strength of the EM field equals 27.0 V/mm and 59.7 V/mm, respectively. In this case, the spin polarization has a strong external parameter dependence because the Rashba subbands intermixing is so strong that it cannot be neglected. This effect may provide new tunable parameters, i.e., the EM field strength and the emitting energy of the electron, to manipulate electron spin in the waveguide.

5. Conclusion

In conclusion, we have clarified the spin-dependent electron transport properties of a straight Rashba waveguide, placed in a transversely polarized EM field, and found that (i) the spin polarization remains constant when the strength of Rashba SOC is weak; (ii) electron spin in the waveguide can be manipulated by the emitting energy of electrons and the external EM field when the strength of Rashba SOC is strong; (iii) a sharp step structure appears in the total electron conductance. An all-electrical nonmagnetic spintronic device may be desirable under an appropriate choice of external control parameters. An experiment is worth doing to observe these effects.

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