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Orbital magnetization in semiconductors*

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This paper theoretically investigates the orbital magnetization of electron-doped (n-type) semiconductor heterostructures and of hole-doped (p-type) bulk semiconductors, which are respectively described by a two-dimensional electron/hole Hamiltonian with both the included Rashba spin–orbit coupling and Zeeman splitting terms. It is the Zeeman splitting, rather than the Rashba spin–orbit coupling, that destroys the time-reversal symmetry of the semiconductor systems and results in nontrivial orbital magnetization. The results show that the magnitude of the orbital magnetization per hole and the Hall conductance in the p-type bulk semiconductors are about 10^{-2}–10^{-1} effective Bohr magneton and 10^{-1}–1 e^2/h, respectively. However, the orbital magnetization per electron and the Hall conductance in the n-type semiconductor heterostructures are too small to be easily observed in experiment.

Keywords: orbital magnetization, Zeeman splitting, Berry phase effect, semiconductor

PACC: 7110C, 7170E, 7550P

1. Introduction

The orbital magnetization (OM) of Bloch electrons has been an outstanding problem in solid state physics and attracted renewed interest due to the new recognition.[1–3] In particular, in recent years, Xiao *et al*[3] proposed a semiclassical formula for calculating the equilibrium OM density at a weak external magnetic field $\boldsymbol{B}$, in which an important Berry phase correction term is added to the conventional OM. This powerful semiclassical formula is:

$$\begin{aligned}\mathcal{M}(\boldsymbol{r}) &= \sum_n \int \mathrm{d}^3\boldsymbol{k}\boldsymbol{m}_n(\boldsymbol{k}) f_n(\boldsymbol{r},\boldsymbol{k}) \\ &\quad + \frac{1}{\beta}\sum_n \int \mathrm{d}^3\boldsymbol{k}\frac{e}{\hbar}\boldsymbol{\Omega}_n \ln\left[1+\mathrm{e}^{\beta\left(\mu-\varepsilon_n^M\right)}\right] \\ &\equiv \boldsymbol{M}_c + \boldsymbol{M}_{\boldsymbol{\Omega}}, \end{aligned} \tag{1}$$

in which $\boldsymbol{M}_{\mathrm{c}}$ is the conventional term and $\boldsymbol{M}_{\boldsymbol{\Omega}}$ the Berry phase correction term. $\boldsymbol{\Omega}_n(\boldsymbol{k})$ is the Berry curvature (gauge flux) of electronic Bloch states defined by

$$\boldsymbol{\Omega}_n(\boldsymbol{k}) = \mathrm{i}\langle\nabla_{\boldsymbol{k}}u_n(\boldsymbol{k})| \times |\nabla_{\boldsymbol{k}}u_n(\boldsymbol{k})\rangle \tag{2}$$

with $|u_n(\boldsymbol{k})\rangle$ being the periodic part of the Bloch wave for the n-th band. $\varepsilon_n^M = \varepsilon_{n\boldsymbol{k}} - \boldsymbol{m}_n(\boldsymbol{k})\cdot\boldsymbol{B}$ is the electron band energy in the presence of the external magnetic field, $\boldsymbol{m}_n(\boldsymbol{k})$ is the crystal orbital magnetic moment and is conventionally defined by[4,5]

$$\begin{aligned}&\boldsymbol{m}_n(\boldsymbol{k}) \\ &= -\mathrm{i}\frac{e}{2\hbar}\langle\nabla_{\boldsymbol{k}}u_n(\boldsymbol{k})| \times \hat{H}(\boldsymbol{k}) - \varepsilon_n(\boldsymbol{k})]|\nabla_{\boldsymbol{k}}u_n(\boldsymbol{k})\rangle, \end{aligned} \tag{3}$$

where $\hat{H}(\boldsymbol{k})$ is the crystal Hamiltonian acting on $|u_n(\boldsymbol{k})\rangle$. In Eq.(1) $f_n(\boldsymbol{r},\boldsymbol{k})$ is the local equilibrium Fermi–Dirac distribution function for the n-th band, μ is the electron chemical potential, $(-e)$ is the electronic charge and $\beta = 1/k_{\mathrm{B}}T$. The Berry phase correction term is of topological nature, arising from a bulk consideration. At zero temperature and magnetic field the general expression (1) reduces to[2,3]

$$\begin{aligned}&\mathcal{M} \\ &= \sum_n \int^{\mu_0} \mathrm{d}^3\boldsymbol{k}\left(\boldsymbol{m}_n(\boldsymbol{k}) + \frac{e}{\hbar}\boldsymbol{\Omega}_n(\boldsymbol{k})\left[\mu_0 - \varepsilon_{n\boldsymbol{k}}\right]\right), \end{aligned} \tag{4}$$

where the upper limit means that the integral is over states with energies below the Fermi energy μ_0.

The Berry phase effect on the OM was until now partially presented in very few models,[3,6] which include the two-dimensional (2D) Haldane model,[7] the 2D ferromagnetic Kagomé lattice with spin chirality,[8] and the three-dimensional (3D) antiferromagetic distorted face-centred-cubic (fcc) lattice.[9] These models are rare examples to show the zero-field quantum Hall

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effect. Due to its basic importance in understanding the magnetism and transport features of the materials, obviously, more work is needed on exploiting the properties of the OM in various kinds of realistic physical systems.

On the other hand, electron-doped (n-type) semiconductor heterostructures (SHs) and hole-doped (p-type) bulk semiconductors (BSs) are extensively studied theoretically and experimentally in condensed matter physics for their fundamental quantum mechanics and technical applications.[10−15] However, the orbital magnetism is still confused in these two popular types of semiconductors. Although the time-inversion symmetry contained in these semiconductor systems results in trivial OM, the Zeeman splitting due to the external weak magnetic field or the internal exchange field will break this time-inversion symmetry. So, the main goal of the present paper is focused on exploiting the properties of the OM in these semiconductors in the presence of a weak Zeeman splitting field.

Before investigating the OM in these semiconductors in detail, we present the main results obtained in this paper. Firstly, with the help of the semiclassical formula (1), we derived analytical expressions of the OM in the n-type SHs and the p-type BSs at zero temperature. Then, when the manipulated parameters in the analytical expressions are chosen as those which can be realized in experiment, as we will show in the following, the OM in the p-type BSs is about 0.1 effective Bohr magnetons while the OM remains trivial in the n-type SHs. Finally, we also calculate the Hall conductance (HC) σ_{xy} in these two types of semiconductors. We find that the magnitude of the HC is about e^2/h in the p-type BSs, which is about 10^3 times larger than that in n-type SHs. Moreover, the OM and the HC satisfy the thermodynamical relation $\partial\mathcal{M}/\partial\mu = (-1/e)\sigma_{xy}$. This is a remarkable relation between the OM and the HC, because it definitely tells one that the topological ingredient in the OM may be faithfully mapped out through the HC.

2. The n-type semiconductor heterostructures

Now let us consider the n-type SHs, which can be described by a 2D electron gas (2DEG) Hamiltonian. This Hamiltonian includes the kinetic energy term and the Rashba spin–orbit (SO) interaction arising from the quantum well asymmetry in the growth (z) direction, and is given by

$$H_0^{\mathrm{SH}} = \gamma k^2 + \alpha\left(\sigma_x k_y - \sigma_y k_x\right), \tag{5}$$

where $\gamma = \hbar^2/2m^*$ with m^* being the band effective mass, $\sigma_i\,(i = x, y, z)$ denote the Pauli matrices, and α is the Rashba SO coupling coefficient. Note that although the Rashba SO coupling behaves like an effective magnetic field in k-space, it does not destroy time-reversal symmetry. However, Zeeman splitting due to an external weak magnetic field or the internal exchange field will destroy this time-reversal symmetry and results in a nontrivial OM. When this Zeeman splitting term is included, the Hamiltonian is given by

$$H^{\mathrm{SH}} = H_0^{\mathrm{SH}} + h_0\sigma_z, \tag{6}$$

where $h_0 = g^* B\mu_{\mathrm{B}}/2$ denotes Zeeman splitting with g^* being the effective g-factor and μ_{B} being Bohr magnetons. The eigenvalues of the Hamiltonian H^{SH} can be easily obtained as $\varepsilon_{1,2}(\boldsymbol{k}) = \gamma k^2 \mp \Delta_{\boldsymbol{k}}$ with $\Delta_{\boldsymbol{k}} = \left(h_0^2 + \alpha^2 k^2\right)^{1/2}$. The corresponding Bloch eigenstates are given by

$$\Phi_1(\boldsymbol{r}) = \begin{pmatrix} \sin\dfrac{\theta_{\boldsymbol{k}}}{2}\mathrm{e}^{\mathrm{i}\chi_{\boldsymbol{k}}} \\ -\cos\dfrac{\theta_{\boldsymbol{k}}}{2} \end{pmatrix} \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}} \equiv u_{1\boldsymbol{k}}\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}, \tag{7}$$

$$\Phi_2(\boldsymbol{r}) = \begin{pmatrix} \cos\dfrac{\theta_{\boldsymbol{k}}}{2}\mathrm{e}^{\mathrm{i}\chi_{\boldsymbol{k}}} \\ \sin\dfrac{\theta_{\boldsymbol{k}}}{2} \end{pmatrix} \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}} \equiv u_{2\boldsymbol{k}}\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}, \tag{8}$$

where $u_{n\boldsymbol{k}}\,(n = 1, 2)$ are introduced to denote the momentum-dependent spinors. The factors $\theta_{\boldsymbol{k}}$ and $\chi_{\boldsymbol{k}}$ in Eqs.(7) and (8) are defined by $\cos\theta_{\boldsymbol{k}} = \alpha k/\Delta_{\boldsymbol{k}}$ and $\tan\chi_{\boldsymbol{k}} = k_x/k_y$.

After a straightforward calculation with Eqs.(2) and (3), one can get the Berry curvatures $\boldsymbol{\Omega}_n(\boldsymbol{k})$ and the crystal orbital magnetic moment $\boldsymbol{m}_n(\boldsymbol{k})$ along the z direction with the final expressions as

$$\Omega_1^z(\boldsymbol{k}) = -\Omega_2^z(\boldsymbol{k}) = \frac{1}{2}\frac{h_0\alpha^2}{\Delta_{\boldsymbol{k}}^3}, \tag{9}$$

and

$$m_1^z(\boldsymbol{k}) = m_z^z(\boldsymbol{k}) = -\frac{e}{\hbar}\frac{h_0\alpha^2}{2\Delta_{\boldsymbol{k}}^2}, \tag{10}$$

respectively. When $h_0 = 0$, the time-reversal symmetry is recovered and the Berry curvature and the conventional magnetic moment for both bands vanish. Substituting Eqs.(9) and (10) into Eq.(4), one obtains the analytical expression of the OM as

$$\mathcal{M} = \frac{e}{2h}h_0\left(\mu_0 + \frac{\gamma h_0^2}{\alpha^2}\right)\left(\frac{1}{F_2} - \frac{1}{F_1}\right) - \frac{e}{2h}\frac{h_0\gamma}{\alpha^2}\left(F_1 - F_2\right), \tag{11}$$

where $F_n = (h_0^2 + \alpha^2 k_{f_n}^2)^{1/2}$ with k_{f_n} being the Fermi vector of the energy band n. The corresponding HC is also easily obtained, and is given by

$$\sigma_{xy} = -\frac{e^2}{\hbar}\sum_{n=1}^{2}\int \frac{\mathrm{d}^2k}{(2\pi)^2}\Omega_n^z(\boldsymbol{k}) = \frac{e^2}{2h}h_0\left(\frac{1}{F_1}-\frac{1}{F_2}\right). \quad (12)$$

Comparing Eq.(11) with Eq.(12), one can find

$$\frac{\partial \mathcal{M}}{\partial \mu_0} = -\frac{1}{e}\sigma_{xy}. \quad (13)$$

This is an important relation between the OM and HC, which can be straightforwardly derived by the thermodynamical relation $\partial\mathcal{M}/\partial\mu_0 = \partial N/\partial B$, where the quasiparticle density has the form in the semiclassical picture $N = \sum_n \int \mathrm{d}^2k/[1/(2\pi)^2][1 + (e/\hbar)B\Omega_n]f_n$ with f_n being the Fermi distribution. This thermodynamical relation between the OM and HC in Eq.(13) definitely tells one that the topological ingredient in the OM may be faithfully mapped out through the HC. Note that this thermodynamical relation in Eq.(13) obtained here can be reduced to a linear relation $\mathcal{M} = (-\mu_0/e)\sigma_{xy}$ in some special physical systems described by an effective two-level massless Dirac Hamiltonian.[16]

At zero temperature, the Fermi vectors are determined by the following equations:

$$\mu_0 = \varepsilon_1(\boldsymbol{k})|_{|\boldsymbol{k}|=k_{f_1}} = \varepsilon_2(\boldsymbol{k})|_{|\boldsymbol{k}|=k_{f_2}}, \quad (14)$$

and

$$N_\mathrm{e} = \sum_{n=1}^{2}\int \frac{\mathrm{d}^2k}{(2\pi)^2}\theta\left[\mu_0 - \varepsilon_n(\boldsymbol{k})\right] = \frac{1}{4\pi}\left(k_{f_1}^2 + k_{f_2}^2\right), \quad (15)$$

where N_e is the electron number density. Solving the simultaneous equations (11), (14) and (15), one can get the OM in this 2DEG model in Eq.(6) describing the n-type SHs.

Now let us consider a realistic GaAs 2DEG system with parameters $\gamma = \hbar^2/2m^* = 568\ \mathrm{meV\cdot nm^2}$ with $m^* = 0.067m_\mathrm{e}$ and $N_\mathrm{e} = 0.05\ \mathrm{nm^{-2}}$. The Rashba SO coupling strength α can be varied over a wide range by engineering the external potential which is applied perpendicularly to the plane. The weak Zeeman splitting is set here as $h_0 = 0.1$ meV. Figure 1 plots the energy spectrum of this 2DEG system with α chosen as 18 meV·nm. Under these parameters, numerical calculations give the Fermi vectors as 0.578 $\mathrm{nm^{-1}}$ and 0.543 $\mathrm{nm^{-1}}$, respectively. Simultaneously, the Fermi energy μ_0 is obtained as 178.1 meV (see the dotted line in Fig.1).

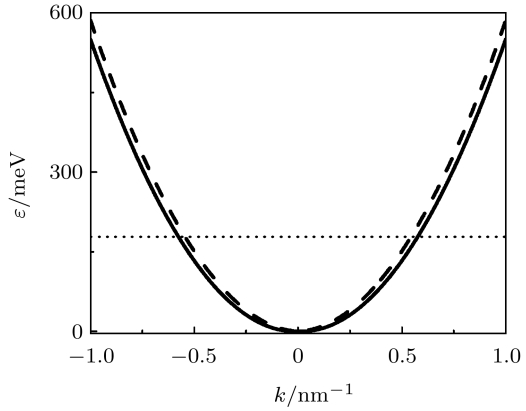


Fig.1. Energy spetrum of the n-type SHs described by the Hamiltonian (6) with $\gamma = 568\ \mathrm{meV\cdot nm^2}$, $\alpha = 18\ \mathrm{meV\cdot nm}$ and $h_0 = 0.1$ meV. The dotted line denotes the Fermi energy $\mu_0 = 178.1$ meV.

We plot the OM per electron (in units of effective Bohr magneton) and its two components in Fig.2(a) as a function of the weak Zeeman splitting h_0 with other parameters being set the same as those in Fig.1. Clearly, one can see that the total OM is very tiny (about zero) over the whole varying regions of h_0. However, the magnitudes of its two components are about 10^{-4}–10^{-3} effective Bohr magneton. They increase with increasing h_0 and overall M_c and M_Ω have opposite contributions to $\mathcal{M}$, which implies that these two parts carry oppositely-circulating currents. We also draw the HC in Fig.2(b) as a function of Zeeman splitting h_0. The results show that the HC in the n-type SHs is about 10^{-4}–$10^{-3}e^2/h$, which is also a very faint signal detected in experiment.

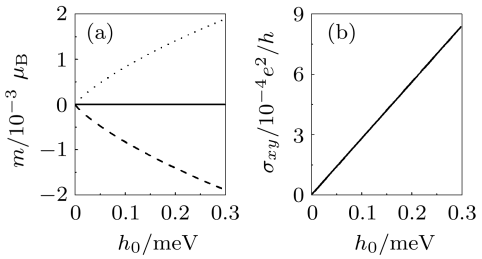


Fig.2. (a) The OM (solid line) and its two components (dashed line for M_c and dottted line for M_Ω) and (b) the HC σ_{xy} in the n-type SHs versus Zeeman splitting h_0. In both figures, $N_\mathrm{e} = 0.05\ \mathrm{nm^{-2}}$ and the other parameters are the same as those in Fig.1.

3. The p-type bulk semiconductors

Now we turn to consider the properties of the OM in the p-type BSs, which can be described by a 2D hole gas (2DHG) system. The spin–orbit interaction in this system is modelled by combined Luttinger and spin-3/2 space-inversion asymmetric Rashba spin–orbit coupling terms. The resultant $\boldsymbol{k}\cdot\boldsymbol{p}$ Hamiltonian reads[17,18]

$$H_{\boldsymbol{k}\cdot\boldsymbol{p}} = \left(\gamma_1 + \frac{5}{2}\gamma_2\right)\frac{k^2}{2m} - \frac{\gamma_2}{m}\left(\boldsymbol{k}\cdot\boldsymbol{S}\right)^2 + \alpha(\boldsymbol{S}\times\boldsymbol{k})\cdot\hat{z}. \tag{16}$$

For a typical band structure for GaAs, $\gamma_1 = 6.92$ and $\gamma_2 = 2.1$. Since the light hole bands are usually unoccupied by the holes in the experiment, one can approximate the four-band Hamiltonian (16) by an effective two-band one.[18] The effective two-band Hamiltonian for heavy holes has a usual form given by

$$H_0^{\rm BS} = \gamma k^2 + \mathrm{i}\frac{\alpha}{2}(k_-^3\sigma_+ - k_+^3\sigma_-). \tag{17}$$

The above equation uses the notations $k_\pm = k_x \pm \mathrm{i}k_y$, $\sigma_\pm = \sigma_x \pm \mathrm{i}\sigma_y$. These Pauli matrices operate on the total angular momentum states with spin projection $\pm 3/2$ along the growth direction; in this sense they represent a pseudospin degree of freedom rather than a genuine spin 1/2.[19] $\gamma = \hbar^2/2m^*$ with m^* being the band effective mass. α is the Rashba spin–orbit coupling coefficient due to structure inversion asymmetry across the quantum well grown along the [001]-direction chosen as the z-axis. For a symmetrically grown quantum well, the coefficient α is essentially proportional to an electric field applied across the well and therefore experimentally tunable.[19–22] When weak Zeeman splitting is introduced to the Hamiltonian (17), one can get the final total Hamiltonian as

$$H^{\rm BS} = H_0^{\rm BS} + h_0\sigma_z. \tag{18}$$

Similar to the case in the n-type SHs, one can get the eigenvalues of the Hamiltonian $H^{\rm BS}$ which are $\varepsilon_{1,2}(\boldsymbol{k}) = \gamma k^2 \mp \Delta_{\boldsymbol{k}}$ with $\Delta_{\boldsymbol{k}} = \left(h_0^2 + \alpha^2 k^6\right)^{1/2}$. The corresponding Bloch eigenstates are given by

$$\Phi_1(\boldsymbol{r}) = \begin{pmatrix} \sin\dfrac{\theta_{\boldsymbol{k}}}{2} \\ -\cos\dfrac{\theta_{\boldsymbol{k}}}{2}\mathrm{e}^{\mathrm{i}\chi_{\boldsymbol{k}}} \end{pmatrix}\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}} \equiv u_{1\boldsymbol{k}}\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}, \tag{19}$$

$$\Phi_2(\boldsymbol{r}) = \begin{pmatrix} \cos\dfrac{\theta_{\boldsymbol{k}}}{2}\mathrm{e}^{\mathrm{i}\chi_{\boldsymbol{k}}} \\ \sin\dfrac{\theta_{\boldsymbol{k}}}{2} \end{pmatrix}\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}} \equiv u_{2\boldsymbol{k}}\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}, \tag{20}$$

where $u_{n\boldsymbol{k}}$ $(n = 1, 2)$ is introduced to denote momentum-dependent spinors. The factors θ_k and $\chi_{\boldsymbol{k}}$ in Eqs.(19) and (20) are defined by $\cos\theta_{\boldsymbol{k}} = h_0/\Delta_{\boldsymbol{k}}$ and $\cos\chi_{\boldsymbol{k}} = k_y(3k_x^2 - k_y^2)/k^3$. After a straightforward calculation with the definitions (2) and (3), one can get the Berry curvatures $\boldsymbol{\Omega}_n(\boldsymbol{k})$ and the crystal orbital magnetic moment $\boldsymbol{m}_n(\boldsymbol{k})$, which are given by

$$\Omega_1^z(\boldsymbol{k}) = -\Omega_2^z(\boldsymbol{k}) = \frac{9h_0\alpha^2k^4}{2\Delta_{\boldsymbol{k}}^3}, \tag{21}$$

and

$$m_1^z(\boldsymbol{k}) = m_2^z(\boldsymbol{k}) = -\frac{e}{\hbar}\frac{9h_0\alpha^2k^4}{2\Delta_{\boldsymbol{k}}^2}, \tag{22}$$

respectively. Substituting Eqs.(21) and (22) into Eq.(4), one can get the OM of this 2DHG system at zero temperature as follows:

$$\mathcal{M} = \frac{3e}{2h}h_0\mu_0\left(\frac{1}{G_2} - \frac{1}{G_1}\right) - \frac{9e}{2h}h_0\gamma\alpha^2\int_{k_{f_2}}^{k_{f_1}}\frac{k^7\mathrm{d}k}{\Delta_{\boldsymbol{k}}^3}, \tag{23}$$

where $G_n = (h_0^2 + \alpha^2 k_{f_n}^6)^{1/2}$ with k_{f_n} being the Fermi vector of the energy band n. The corresponding HC of the p-type BSs with the Hamiltonian (18) is also calculated and obtained as[23]

$$\sigma_{xy} = -\frac{e^2}{\hbar}\sum_{n=1}^{2}\int\frac{\mathrm{d}^2k}{(2\pi)^2}\Omega_n^z(\boldsymbol{k})\,\theta\left[\mu_0 - \varepsilon_n(\boldsymbol{k})\right] = \frac{3e^2}{2h}h_0\left(\frac{1}{G_1} - \frac{1}{G_2}\right). \tag{24}$$

Comparing Eq.(24) with Eq.(23), one can find that the thermodynamical relation between the OM and the HC (Eq.(13)) also holds true. At zero temperature, the chemical potential and the Fermi wave vectors can be also obtained by solving the simultaneous equations (14) and (15). Note that the electron number density $N_{\rm e}$ in Eq.(15) becomes the hole number density $N_{\rm h}$ in this case.

Now we use the above analytical expressions to study the properties of the OM and the HC in a realistic 2DHG system, which is consists of GaAs-based quantum well samples with heavy-hole densities of a few 10^{-4} nm^{-2}.[21] Given the value of the effective Zeeman factor, $g^* = 2.2$ for bulk GaAs, the Zeeman splitting h_0 can be varied over a range of 0–0.30 meV, which is relatively small in comparison with the other energy scales. However, the situation is totally different if the Zeeman splitting is tuned by

magnetic impurity doping in the paramagnetic semiconductor hosts. Recent advances in diluted magnetic semiconductors[24] have made it possible to manipulate the kinetic exchange interaction of the free carriers in a controllable way. In the mean-field approximation, the kinetic exchange interaction splitting can be expressed as $h_0 = -N_0\beta x S B_S(\xi)$. Here, $N_0\beta$ is the exchange integral, x is the magnetic ion concentration, $B_S(\xi)$ is the standard Brillouin function[25] and $\xi = g_S\mu_B SB/k_BT$ with S and g_S being the total spin quantum number and the Landau actor of the magnetic ions. For Mn doping in a GaAs host, it has been acknowledged[26] that the Mn ions substitute the cations, and are in $3d^5$ configurations, thus leading to $S = 5/2$ and $g_S = 2$. The p–d exchange integral in (Ga,Mn)As has been measured[27] to be $N_0\beta \approx -1$ eV. The exchange interaction h_0 can be changed over a wide range of 4–16 meV under a weak external magnetic field of the order 10–100 mT at a typical low temperature $T = 1$ K and Mn concentration $x = 0.05$.

Figure 3(a) plots the energy spectrum of the 2DHG system with the Rashba SO coefficient and the Zeeman splitting which are set at reasonable values of 400 meV$\cdot$nm^3 and 0.02 meV, respectively. The effective mass of the hole is chosen as $m^* = 0.5m_e$, which results in $\gamma = 76$ meV$\cdot$nm^2. Taking these parameters into Eqs.(14) and (15) and after a self-consistent calculation, one can get the Fermi wave vectors $k_{f_1} = 0.096$ nm^{-1}, $k_{f_2} = 0.059$ nm^{-1} and the chemical potential $\mu_0 = 0.34$ meV. The Fermi wave vectors are shown in Fig.3(b) as a function of the Zeeman splitting h_0 with the hole number density $N_h = 10^{-3}$ nm^{-2}, the parameters are chosen as those in Fig.3(a).

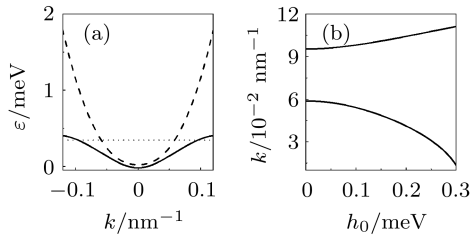


Fig.3. (a) Energy spectrum of the p-type BSs described by the Hamiltonian (18) with $\gamma = 76$ meV$\cdot$nm^2, $\alpha = 400$ meV$\cdot$nm^3, and $h_0 = 0.02$ meV. (b) The Fermi vectors as a function of the Zeeman splitting h_0. The hole number density $N_h = 10^{-3}$ nm^{-2} and the other parameters are the same as those in Fig.3(a).

The OM of the 2DHG is shown in Fig.4(a) versus the Zeeman splitting. One can clearly see from this figure that the magnitude of the OM increases with increasing h_0 and is about 0.1 effective Bohr magneton. The conventional and Berry phase correction terms are also shown in the inset in Fig.4(a). Similar to that in the n-type SHs, overall M_c and M_Ω have opposite contributions to $\mathcal{M}$. With the results obtained in the n-type SHs and other systems, one can summarize that the Berry phase correction always weakens the contribution from the conventional term. The corresponding HC as a function of h_0 has also been obtained and is shown in Fig.4(b), which is about e^2/h. Compared with Fig.2(b), one finds that the magnitude of the HC in the p-type BSs is about 10^3 times larger than that in the n-type SHs.

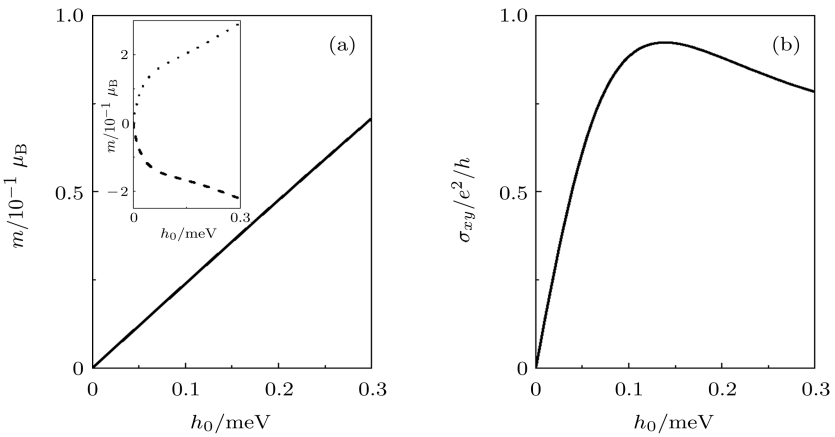


Fig.4. (a) The OM (solid line) and its two components (dashed line for M_c and dotted line for M_Ω in the inset) as a function of the Zeeman splitting h_0. (b) The corresponding HC as a function of the Zeeman splitting. The parameters in both figures are the same as those in Fig.3(b).

Before ending this section, we summarize in Fig.5 the behaviour of the OM in the parameter space (h_0, N_h) (contour plot). The Rashba coupling has been fixed to be $\alpha = 400$ meV$\cdot$nm^3. One can see that the magnitude of the OM increases as the hole density or Zeeman splitting strength increases.

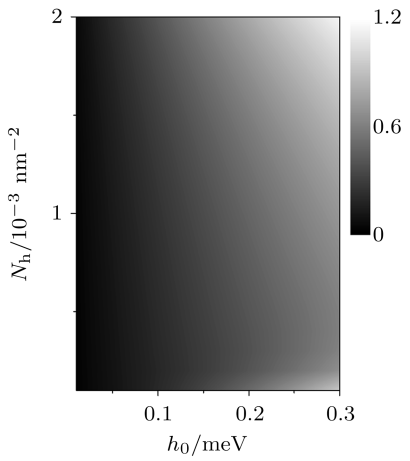


Fig.5. The contour plot of the OM (in units of 10^{-2} effective Bohr magneton) of the p-type BSs modelled by the 2DHG Hamiltonian (18) in the parameter space (h_0, N_h) by fixing the values of $\gamma = 76$ meV$\cdot$nm^2 and $\alpha = 400$ meV$\cdot$nm^3.

4. Conclusion

In conclusion, we theoretically studied the properties of the OM in n-type SHs and in p-type BSs, which are respectively described by a 2DEG and 2DHG Hamiltonian with the included Rashba SO coupling and Zeeman splitting. Based on the semiclassical formula proposed in Ref.[3], we obtained analytical expressions of the OM in both these semiconductors at zero temperature. The results show that the OM in the n-type SHs is too small to be observed. In the p-type BSs, the magnitude of the OM is about $10^{-2}-10^{-1}$ effective Bohr magneton. The corresponding HC in the semiconductors is also calculated. The magnitude of the HC in the p-type BSs is about 10^{-1}–1 e^2/h, which is about 10^3 times larger than that in the n-type SHs. In addition, we obtain a very important thermodynamics relation between the OM and the HC in Eq.(13), which tells one that the OM and the HC have the same topological nature.

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