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Polarized spin transport in mesoscopic quantum rings with electron–phonon and Rashba spin–orbit coupling*

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The influence of electron–phonon (EP) scattering on spin polarization of current output from a mesoscopic ring with Rashba spin–orbit (SO) interaction is numerically investigated. There are three leads connecting to the ring at different positions; unpolarized current is injected to one of them, and the other two are output channels with different bias voltages. The spin polarization of current in the outgoing leads shows oscillations as a function of EP coupling strength owing to the quantum interference of EP states in the ring region. As temperature increases, the oscillations are evidently suppressed, implying decoherence of the EP states. The simulation shows that the magnitude of polarized current is sensitive to the location of the lead. The polarized current depends on the connecting position of the lead in a complicated way due to the spin-sensitive quantum interference effects caused by different phases accumulated by transmitting electrons with opposite spin states along different paths.

Keywords: electron–phonon scattering, Rashba spin–orbit coupling, mesoscopic ring, spin polarized transport

PACC: 6320K, 7170C, 6322, 7225

1. Introduction

Quantum phenomena, such as the Aharonov–Bohm (AB) effect, universal conductance fluctuations, and weak localization, which occur in various nanostructures or mesoscopic systems, may provide us with useful information for a further understanding of the coherence properties of electrons when they travel in different systems. The spin Hall effect, as another interesting phenomenon in mesoscopic systems, originates from spin–orbit (SO) coupling and has attracted great interest recently due to the potential applications in spintronics. The investigations focus on the properties related to the spin filter and spin polarization of current in the transport of electrons through nanostructures with various geometries and sizes.[1−10] For AB mesoscopic rings, both the Rashba and Dresselhaus types of SO interaction have been introduced in addition to the external magnetic field to control the spin precession during the travelling of electrons. The effective magnetic field created by the SO coupling will induce phase shift during the spin precession as the electrons propagate in clockwise and anticlockwise directions, which results in the interference of spin components in the wave function and then causes changes in the charge and spin conductance of the ring.[11−16] Recently several schemes for adopting the coherent effect of spin components in AB rings with SO interaction have been proposed.

On the other hand, the study of the effects of different scattering mechanisms on electron transport is one of the central issues in condensed matter physics.[16,17] It has been found that electron transport in mesoscopic structures is very sensitive to the disorder created by nonmagnetic and magnetic impurities. Besides the static impurities or disorder, electron–phonon (EP) interaction, as another important scattering mechanism, has also been studied extensively, which induces inelastic scattering during the transport of electrons and can lead to the loss of coherence or dephasing.[18−20]

In the present paper we investigate the effect of EP interaction on spin-polarized transport through a ring-shaped mesoscopic system with Rashba SO coupling. The ring is connected to one input lead and two output leads at different positions. By solving the Schrödinger equation we calculate the spin polarization of currents in the output leads when unpolarized

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electrons are injected into the input lead. The results show the interesting quantum interference of the EP states in the ring and the sensitivities of charge and spin currents to the EP interaction strength and the location of output lead.

2. Model and method

Figure 1 shows the simplified structure of the mesoscopic ring-shape conductor used to study the spin and charge transport properties in the presence of the EP interaction. Symbols L, U, and D denote three leads connecting to the ring at points 1, $N+1-d$,

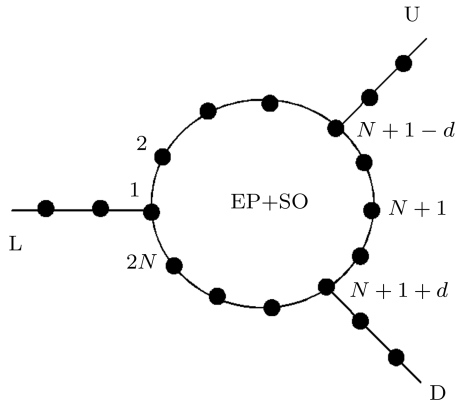


Fig.1. Mesoscopic ring-shape lattice connecting to three leads L, U and D. In the ring electrons are subject to EP interaction and SO coupling.

and $N+1+d$, respectively. In the tight-binding representation the total number of sites in the ring is $2N$, and the number of included phonon modes is M. The SO and EP couplings only exist in the ring region. With this model, the Hamiltonian of the whole system can be written as follows:

$$\begin{aligned}\widehat{H} &= \widehat{H}_{\rm ring} + \widehat{H}_{\rm L} + \widehat{H}_{\rm U} + \widehat{H}_{\rm D} + \widehat{H}_{\rm L,ring} \\ &\quad + \widehat{H}_{\rm U,ring} + \widehat{H}_{\rm D,ring}, \end{aligned} \tag{1}$$

where $\widehat{H}_{\rm ring}$ describes the motion of electrons in the ring subject to both Rashba SO coupling and EP interaction and is given by

$$\widehat{H}_{\rm ring} = \widehat{H}_{\rm el} + \widehat{H}_{\rm ph} + \widehat{H}_{\rm el-ph},$$

where the fist term represents the sub-Hamiltonians of electrons in an isolated ring subjected to Rashba SO interaction as

$$H_{\rm el} = \frac{\widehat{\boldsymbol{p}}^2}{2m^*} + \frac{\alpha}{\hbar}(\widehat{\boldsymbol{\sigma}} \times \widehat{\boldsymbol{p}})_z,$$

where m^* is the electron effective mass, $\widehat{\boldsymbol{p}}$ is the momentum operator, $\widehat{\boldsymbol{\sigma}}$ is the vector of the Pauli spin operators, and α is the strength of the Rashba SO coupling. In cylindrical coordinates it is

$$\begin{aligned}\widehat{H}_{\rm el} &= -\frac{\hbar^2}{2m^*R^2}\frac{\partial^2}{\partial\varphi^2} - \frac{{\rm i}\alpha}{R}(\cos\varphi\widehat{\sigma}_x + \sin\varphi\widehat{\sigma}_y)\frac{\partial}{\partial\varphi} \\ &\quad - \frac{{\rm i}\alpha}{2R}(\cos\varphi\widehat{\sigma}_y - \sin\varphi\widehat{\sigma}_x), \end{aligned} \tag{2}$$

where R is the radius of a one-dimensional (1D) ring ($R = Na/2\pi$ with α being the lattice spacing along the radial direction), and φ is the angular coordinate. In the second quantized representation, it is given by[1,9]

$$\begin{aligned}&\widehat{H}_{\rm el} \\ &= \sum_{i,s}\varepsilon_1 c^\dagger_{R,i,s}c_{R,i,s} + \sum_{i,s,s'}[t_0\widehat{I}_s - {\rm i}t_{\rm so}(\cos\varphi_{i,i+1}\widehat{\sigma}_x \\ &\quad + \sin\varphi_{i,i+1}\widehat{\sigma}_y)]c^\dagger_{R,i,s}c_{R,i+1,s'} + {\rm H.c.}, \end{aligned} \tag{3}$$

where $c^\dagger_{R,i,s}(c_{R,i,s})$ creates (annihilates) an electron with spin $s(=\uparrow,\downarrow)$ at site i of the ring, which is the lattice site index along the radial direction; ε_1 is the on-site energy; $\varphi_{i,i+1} = (\varphi_i + \varphi_{i+1})/2$; $\varphi_i = 2\pi(i-1)/N$; $t_0 \equiv \hbar^2/2m^*a^2$; $t_{\rm so} \equiv \alpha/2a$ is the SO hopping parameter; $\widehat{I}_s$ is the 2×2 identity matrix. The second part is sub-Hamiltonians of phonons

$$\widehat{H}_{\rm ph} = \sum_q \hbar\omega_q a^\dagger_q a_q.$$

The interaction between them is described by the following Hamiltonian:

$$\widehat{H}_{\rm el-ph} = \sum_{i,s}\sum_q (\lambda_{q,i}c^\dagger_{R,i,s}c_{R,i,s}a_q + {\rm H.c.}), \tag{4}$$

where $a^\dagger_q$ is the creation operator for a phonon in the q-th mode with energy $\hbar\omega_q$, and $\lambda_{q,i}$ is the EP coupling strength between the q-th mode and the electron at site i. In the investigation we take into account only a finite number of acoustic modes with uniform frequency distribution $\omega_i = {\rm i}\omega_0$ with ω_0 being the frequency spacing. If the wavelengths of the acoustic waves are larger than the size of the ring, the position dependence of $\lambda_{q,i}$ can be neglected. The strength of EP interaction is dependent on the phonon energy and can be written as[18,21] $\lambda_q = c_0/\hbar\omega_q$ with coefficient c_0 depending on density, dielectric constant, system size, and average strain.

$\widehat{H}_{\rm L}$, $\widehat{H}_{\rm U}$, and $\widehat{H}_{\rm D}$ are sub-Hamiltonians of electrons in leads L, U, and D, respectively, and can be written as

$$\widehat{H}_Q = t_0 \sum_{\langle i,i'\rangle,s} c^\dagger_{Q,i,s} c_{Q,i',s} + \sum_{i,s} \varepsilon_0 c^\dagger_{Q,i,s} c_{Q,i,s}, \quad (5)$$

where Q = L, U, D, $\widehat{c}_{Q,i,s}$ creates an electron with spin s at site i of the Q-th lead, and ε_0 is the site energy in the leads.

The sub-Hamiltonian of the coupling between the ring and the Q-th lead is

$$\widehat{H}_{Q,\rm ring} = t_0 \sum_s (c^\dagger_{Q,1,s} c_{R,l_Q,s} + \text{H.c.}), \quad (6)$$

where the connecting point between the ring and the Q-th lead is indexed as l_q in the ring and as 1 in the lead. So, $l_{\rm L} = 1$, $l_{\rm U} = N+1-d$, and $l_{\rm D} = N+1+d$ are defined as the connection positions in the ring of the corresponding leads, while for the leads the connecting sites are the starting points to index. For simplicity and to reduce the number of parameters, we assume that the hopping strength between the leads and the ring is t_0 and the hopping integrals in the leads and the ring are the same.

In this model the full Hilbert space is spanned by many-body quantum states each of which can be expressed as the direct product of an electron state and a multi-phonon state. A wave function of the system, denoted as $|\psi\rangle$, can be written as a linear combination of these basis functions[9]

$$|\psi\rangle = \sum_{P,i,s} \sum_{\{n_q\}} b_{P,i,s,\{n_q\}} c^\dagger_{P,i,s} \prod_q \frac{(a^\dagger_q)^{n_q}}{\sqrt{n_q!}} |0\rangle , \quad (7)$$

where the sum for P runs over the leads (L, U, and D) and the ring (R); $|0\rangle$ denotes the vacuum; n_q is the number of phonons in the q-th mode; $b_{P,i,s,\{n_q\}}$ is the corresponding coefficient in the linear combination; $\{n_q\}$ denotes a set of phonon numbers in the modes $n_1, n_2, \ldots, n_q, \ldots$, and can be used to specify the phonon state in the ring.

The system is open and the same values of bias voltage $V_{\rm b}$ are applied between leads U and L and between leads D and L separately. We suppose that initially an incident electron wave with unity amplitude, spin s, and energy ε_μ is injected into lead L, and the phonon state in the ring is $\{n_q\}_\mu$. Because the total energy E of wave function $|\psi\rangle$, including the energies of electron and phonons, should be a good quantum number under the Hamiltonian, for this injected wave we have $E = \varepsilon_\mu + \sum_q n^\mu_q \hbar\omega_q$ with $n_q \in \{n_q\}_\mu$. This electron wave can be transmitted to lead U or D, or be reflected back to lead L, with the phonon state in the ring changed to $\{n_q\}_{\mu'}$. In this case the coefficients $b_{P,i,s',\{n_q\}}$ of the wave function outside the ring can be written in the form of plane waves[9]

$$b_{Q,i,s',\{n_q\}\mu'} = \begin{cases} \mathrm{e}^{\mathrm{i}k_\mu i}\delta_{\mu,\mu'}\delta_{s,s'} + r_{s\mu,s'\mu'}\,\mathrm{e}^{-\mathrm{i}k_{\mu'}i}, & \text{for} \quad Q = \text{L}, \\ t_{Q,s\mu,s'\mu'}\,\mathrm{e}^{\mathrm{i}k_{\mu'}i}, & \text{for} \quad Q = \text{U} \ \text{ or } \ \text{D}, \end{cases} \quad (8)$$

where $r_{s\mu,s'\mu'}$ ($t_{Q,s\mu,s'\mu'}$) is the amplitude of reflection (transmission) to lead Q with spin flipped from s to s' and the phonon state in the ring changed from $\{n_q\}_\mu$ to $\{n_q\}_{\mu'}$, and k_μ is the wave vector of the plane wave satisfying $2t_0 \cos k_\mu + \varepsilon_0 = \varepsilon_\mu$. The reflection amplitude and the transmission amplitude, together with other coefficients $b_{R,i,s,\{n_q\}}$ within the ring, are unknown variables which can be solved by a set of linear equations obtained from the Schrödinger equation $H|\psi\rangle = E|\psi\rangle$. Obviously the calculated transmission amplitudes are functions of E.

With the solved coefficients, under a small bias voltage $V_{\rm b}$ the current to an output lead Q (U or D) polarized in spin s' can be calculated as[9]

$$\begin{aligned} I^{s'}_Q(V_{\rm b},T) = \frac{e}{h}\int \mathrm{d}E \sum_{\mu,\mu',s} \frac{|t_{Q,s\mu,s'\mu'}(E)|^2\,|\sin k_{\mu'}|\,F_\mu(T)}{|\sin k_\mu|} \Bigg\{ f\left(E - \sum_q n^\mu_q \hbar\omega_q, \frac{eV_{\rm b}}{2}, T\right) \\ \times \left[1 - f\left(E - \sum_q n^{\mu'}_q \hbar\omega_q, -\frac{eV_{\rm b}}{2}, T\right)\right]\Bigg\} - f\left(E - \sum_q n^\mu_q \hbar\omega_q, -\frac{eV_{\rm b}}{2}, T\right) \\ \times \left[1 - f\left(E - \sum_q n^{\mu'}_q \hbar\omega_q, \frac{eV_{\rm b}}{2}, T\right)\right], \end{aligned} \quad (9)$$

where

$$f(\varepsilon, V, T) = \frac{1}{1 + \exp\left(\dfrac{\varepsilon - E_F - V}{k_B T}\right)}$$

is the Fermi distribution of electrons in the leads with given bias and temperature T, and

$$F_\mu(T) = \frac{1}{Z} \exp\left(-\frac{\sum_q n_q^\mu \hbar\omega}{k_B T}\right)$$

with $z = \sum_\mu \exp(-\sum_q n_q^\mu \hbar\omega/(k_B T))$ being the statistical probability of the phonon state $\{n_q\}_\mu$ in the ring. By taking the limitation $V_\mathrm{b} \to 0$, we can obtain the charge transport conductance from the lead L to both leads U and D as follows:

$$G_\mathrm{c}(T) = \lim_{V_\mathrm{b}\to 0} \sum_{Q=\mathrm{U,D}} \sum_s \frac{\partial I_Q^s(V_\mathrm{b}, T)}{\partial V_\mathrm{b}}. \qquad (10)$$

The difference $I_Q^{s=\uparrow} - I_Q^{s=\downarrow}$ gives the spin polarization of the output current in lead Q when unpolarized electrons are injected into the input lead L.

3. Numerical results

In the whole numerical calculations we consider only the case where the phonon number in every mode does not exceed 1. We first study how the spin polarizations of the outgoing current, $I_\mathrm{U}^\uparrow - I_\mathrm{U}^\downarrow$ and $I_\mathrm{D}^\uparrow - I_\mathrm{D}^\downarrow$, vary with the temperature and strength of EP coupling. Figures 2 and 3 present the simulation results of the polarized current for a fixed value of the phonon frequency. The oscillation behaviour of the polarized current can be observed in the up and down outgoing leads within a range of small EP coupling strengths. By increasing the temperature these oscillations are obviously suppressed as can be seen from Fig.3 where the amplitude and the frequency of the oscillations decrease rapidly although some oscillation behaviours still exist.

Excluding the phonons from our system for a while, when electrons travel along a ring, the phase difference acquired by opposite spin states plays an important role for the modulation of the polarized current, which can be achieved by solving the eigenstates

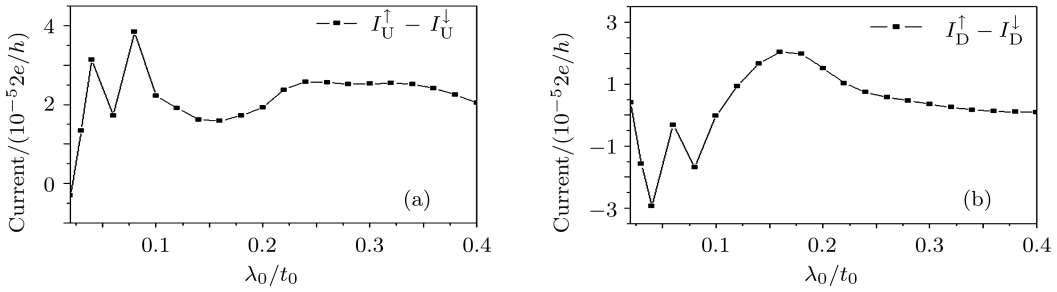


Fig.2. Spin current $I^\uparrow - I^\downarrow$ in two outgoing leads as a function of EP coupling strength λ_0 at temperature $k_\mathrm{B}T = 0.002t_0$ (a) in the outgoing lead L and (b) in the outgoing lead D. The number of phonon modes and the size of the ring are set to be $M = 10$ and $N = 20$ respectively. The positions of connecting points of the leads are specified by $d = 5$ as illustrated in Fig.1. The Rashba SO interaction strength is $V_\mathrm{so} = 0.6t_0$. The phonon energy is $\hbar\omega_0 = 0.0008t_0$.

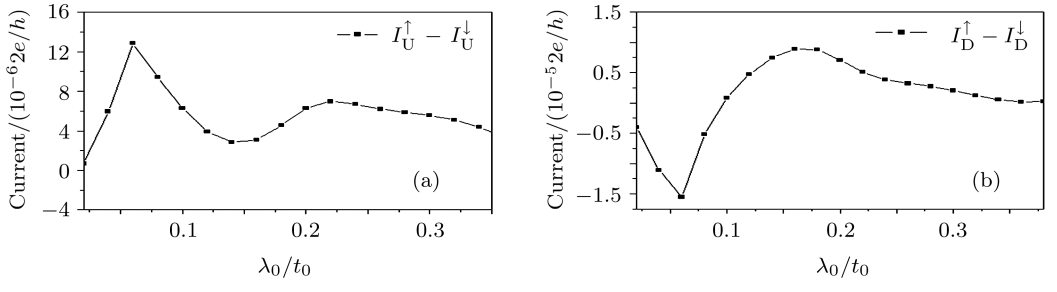


Fig.3. Values of spin current $I^\uparrow - I^\downarrow$ in two outgoing leads as a function of EP coupling strength λ_0 at temperature $k_\mathrm{B}T = 0.003t_0$ (a) in the outgoing lead L and (b) in the outgoing lead D. Other parameters are the same as those in Fig.2.

of Eq.(2) and calculating the quantum probability of transmission between incoming and outgoing states based on the Landauer formula.[5,22] The spin-dependent phases are signatures of the Aharonov–Casher effect[23−25] for spins travelling in the presence of an electric field, which is the electromagnetic dual of the Aharonov–Bohm effect. When EP coupling is considered, inelastic scattering can lead to complicated phase-breaking processes, which suppress the quantum interference of electrons. However, on the other hand, the EP states themselves, as a whole, can be coherent among the tunnelling paths associated with different phonon states because of the conservation of the whole energy. Consequently, the interference effect remains which can be observed from oscillations of the curves in Fig.2 when the EP coupling is not too large. The oscillations are obviously suppressed by increasing the temperature. As the temperature increases, more phonon states are involved in the transport process, which in turn destroys the coherence of the EP states. On the other hand, both figures show that by increasing the EP coupling strength the oscillating feature of the polarized outgoing current is also suppressed, implying that weak EP scattering may induce quantum interference more easily than strong EP scattering.

To illustrate the effect of the connection position of the outgoing lead on the spin current and find an effective way to control the magnitude of the outgoing spin current the strong dependence of the polarized current on the connection location of the lead is shown in Fig.4 for different temperatures.

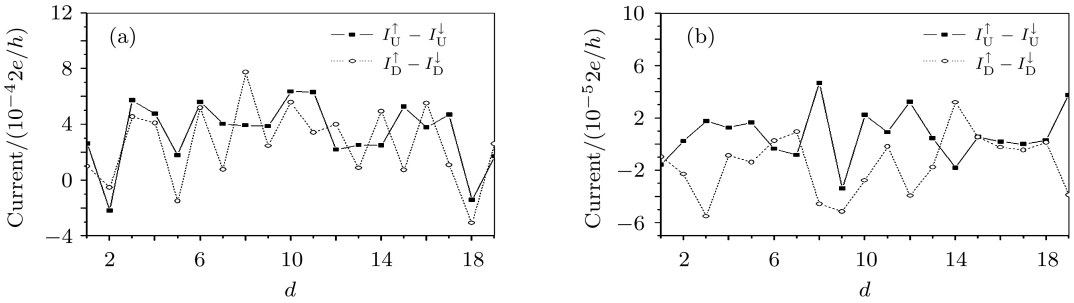


Fig.4. Values of spin current $I^\uparrow - I^\downarrow$ in three outgoing leads as a function of different connecting sites of up and down leads to the quantum ring at (a) zero temperature and (b) at finite temperature $k_BT = 0.002t_0$. The Rashba SO interaction strength is $V_{so} = 0.3t_0$, the EP coupling strength is $\lambda_0 = 0.4t_0$, and the phonon energy is $\hbar\omega_0 = 0.0008t_0$.

As the locations of the up and down leads change, oscillations in both the magnitude and the sign of the polarized current can take place. This behaviour can be attributed to the position dependence of the phase difference accumulated by opposite spin electron transmitting states between the clockwise and the counter clockwise paths within the range of elastic phonon scattering, which does not break the quantum interference of electrons within weak EP coupling strength and low environmental temperature. The amplitude of the oscillations is rapidly reduced by increasing the temperature from zero. This is due to the destruction of quantum coherence by inelastic scattering with phonons which is enhanced by hot phonons at higher temperatures. This oscillation behaviour dependence on connection locations for spin-polarized current may provide another potential application for the control of polarized current in spintronics.

4. Conclusion

In this paper we studied the spin-polarized transport influenced by inelastic scattering due to EP interaction in a ring-shape mesoscopic system with Rashba SO interaction. The ring is connected to three leads at different locations and a small bias is applied between one input lead and two output leads. The effect of the EP interaction is clearly exhibited in the polarized spin currents in the two output leads, which indicates that when the EP coupling strength is not large enough the EP states are quantum-mechanically coherent. As a result, quantum interference can be induced when electrons pass through the whole region of the ring. The dependence of the spin current on the strength of the EP interaction and ambient temperature is also calculated.

The destruction of quantum coherence at finite temperatures and large EP coupling strength caused by inelastic scattering with phonons can be seen from the decrease of the oscillation amplitude of the spin current. We also studied the influence of the position of the outgoing lead on the magnitude of the spin polarized current. The sensitive character can be found in our simulation results, which provides the possibility of control of spin-polarized current.

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