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Comparison between effects of Doppler broadening on pure and non-pure inversionless gain with frequency up-conversion*

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A study is made of the effects of Doppler broadening on pure gain without inversion, which means that neither one-photon nor two-photon inversions are allowed, and non-pure gain without inversion, which means that one-photon inversion does not occur but two-photon inversion is present, in a closed Λ-type three-level system with incoherent pumping. It is shown that when the driving field is resonant but the probe field is not, in a certain range of Doppler width, for the case of the lower degree of frequency up-conversion, generally, pure gain without inversion increases monotonically and non-pure gain without inversion does not monotonically increase or decrease with increasing Doppler width; for the case of the higher degree of frequency up-conversion, pure gain without inversion decreases monotonically but non-pure gain without inversion cannot be produced. In the case of two-photon resonance, in some range of Doppler width, pure gain without inversion does not monotonically increase or decrease while non-pure gain without inversion decreases monotonically with Doppler width increasing. Finally, an experimental scheme for examining our theoretical result is given.

Keywords: Λ-type three-level system, Doppler broadening, gain without inversion, frequency up-conversion

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1. Introduction

Since the seminal work of Kocharovskaya, Harris and Scully *et al*, lasing without inversion (LWI) has attracted tremendous attention (for example, see recent review paper[1] and papers[2−14] and references therein). Production of LWI is a result of atom coherence and quantum interference. The LWI could achieve laser action in short wavelength (e.g. the x-ray, even γ-ray, range) regions where lasing is impractical with conventional pumping schemes. Since the gain without inversion (GWI) that one can realize is usually quite small, it is important to not only identify the optimal atom and field parameters, but also explore any other issues that may affect the maximal gain that one can achieve. On the other hand, the experiment on GWI is usually performed by using atomic vapour, which is typically accompanied by obvious Doppler broadening, so in attempting to find the conditions under which the largest GWI is produced, some attention has been paid to the effect of Doppler broadening on GWI.[15−26] However, almost all these studies are devoted to the analysis of the effect of Doppler broadening on non-pure GWI, i.e. gain without one-photon inversion but with two-photon inversion. Pure GWI should be the gain that neither one-photon nor two-photon inversion is allowed to have. Only pure GWI can lead to pure LWI, i.e. LWI in the exact sense. In addition, in the studies of the effect of Doppler broadening on GWI the influence of the degree of frequency up-conversion is usually not taken into account. In the present paper we investigate the effects of Doppler broadening on pure and non-pure GWI for different degrees of frequency up-conversion in a closed Λ-type system.

2. Model and equations of motion

Consider a closed Λ-type three-level atomic system with an upper state $|1\rangle$ and two lower states $|2\rangle$ and $|3\rangle$, as illustrated in Fig.1. Transition between lev-

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els $|1\rangle$ and $|2\rangle$ is driven by a strong coherent driving field with the amplitude $\boldsymbol{E}_{\rm d}$, frequency $\omega_{\rm d}$ and Rabi frequency Ω. Incoherent pumping with a pumping rate 2Λ and a weak coherent probe field with the amplitude $\boldsymbol{E}_{\rm p}$, frequency $\omega_{\rm p}$ and Rabi frequency α are applied between levels $|1\rangle$ and $|3\rangle$. $2\gamma_1$ and $2\gamma_2$ denote the spontaneous emission rates from level $|1\rangle$ to $|3\rangle$ and $|2\rangle$, respectively.

In the interaction picture, using the rotating-wave approximation, the semiclassical interaction Hamiltonian for the system can be written as follows:

$$H_{\rm I} = \Delta_{\rm p}|1\rangle\langle 1| + (\Delta_{\rm p}-\Delta_{\rm d})|2\rangle\langle 2| - [\Omega|1\rangle\langle 2| + \alpha|1\rangle\langle 3| + \text{c.c.}], \quad (1)$$

where $\Delta_{\rm p}$ $(=\omega_{13}-\omega_{\rm p})$ and $\Delta_{\rm d}$ $(=\omega_{12}-\omega_{\rm d})$ are the detunings of the probe and driving fields from their relevant atomic transitions, respectively.

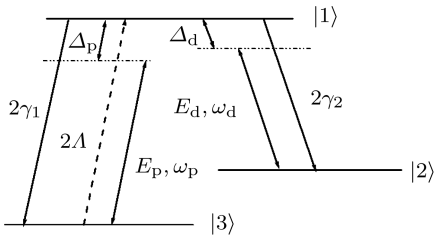


Fig.1. A closed Λ-type three level system.

In the rotating-wave and the electric dipole approximations, the semiclassical density matrix equations of motion for the closed Λ-type atomic system can be written as follows:

$$\dot\rho_{11} = -2(\gamma_1+\gamma_2)\rho_{11} + \mathrm{i}\alpha\rho_{31} - \mathrm{i}\alpha\rho_{13} + 2\Lambda\rho_{33} + \mathrm{i}\Omega\rho_{21} - \mathrm{i}\Omega\rho_{12}, \quad (2a)$$

$$\dot\rho_{22} = 2\gamma_2\rho_{11} + \mathrm{i}\Omega\rho_{12} - \mathrm{i}\Omega\rho_{21}, \quad (2b)$$

$$\dot\rho_{12} = -(\gamma_1+\gamma_2+\mathrm{i}\Delta_{\rm d})\rho_{12} + \mathrm{i}\alpha\rho_{32} - \mathrm{i}\Omega(\rho_{11}-\rho_{22}), \quad (2c)$$

$$\dot\rho_{13} = -(\gamma_1+\gamma_2+\Lambda+\mathrm{i}\Delta_{\rm p})\rho_{13} + \mathrm{i}\Omega\rho_{23} - \mathrm{i}\alpha(\rho_{11}-\rho_{33}), \quad (2d)$$

$$\dot\rho_{23} = -(\Lambda+\mathrm{i}\Delta_{\rm p}-\mathrm{i}\Delta_{\rm d})\rho_{23} + \mathrm{i}\Omega\rho_{13} - \mathrm{i}\alpha\rho_{21}, \quad (2e)$$

along with the equations of their complex conjugates, $\rho_{ij}=\rho_{ji}^*$ $(i\neq j, i, j = 1,\ 2,\ 3)$ and the closure relation $\rho_{11}+\rho_{22}+\rho_{33}=1$.

The probe gain corresponds to the imaginary part of ρ_{31}. If $\mathrm{Im}\,\rho_{31}>0$, the system exhibits gain for the probe field; if $\mathrm{Im}\,\rho_{31}<0$, the probe field is attenuated. For convenience of calculation, in the following discussion we choose α and Ω to be real, and this does not affect the generality of the discussion.

For the steady state case, $\mathrm{d}\rho_{ij}/\mathrm{d}t=0$, thus equations involved in Eq.(2) reduce into a set of coupled algebraic equations. We can obtain exact nonlinear analytical expressions for $\mathrm{Im}\,\rho_{31}, \rho_{11}, \rho_{22}, \rho_{33}$ as follows:

$$\mathrm{Im}\,\rho_{31} = \frac{b_3\alpha^3+b_1\alpha}{2(h_6\alpha^6+h_4\alpha^4+h_2\alpha^2+h_0)}, \quad (3a)$$

$$\rho_{11} = \frac{c_4\alpha^4+c_2\alpha^2+c_0}{h_6\alpha^6+h_4\alpha^4+h_2\alpha^2+h_0}, \quad (3b)$$

$$\rho_{22} = \frac{n_6\alpha^6+n_4\alpha^4+n_2\alpha^2+n_0}{2(h_6\alpha^6+h_4\alpha^4+h_2\alpha^2+h_0)}, \quad (3c)$$

$$\rho_{33} = 1-\rho_{11}-\rho_{22}, \quad (3d)$$

where b_i $(i=1,\ 3)$, c_i $(i=0,\ 2,\ 4)$ and n_i, h_i $(i=0,\ 2,\ 4,\ 6)$ are functions of Ω, α, $\Delta_{\rm p}$, $\Delta_{\rm d}$ and other parameters of the system. The concrete expressions of $\mathrm{Im}\,\rho_{31}$, ρ_{11}, ρ_{22}, ρ_{33} are very complex, so we do not list them in this paper.

3. Effect of Doppler broadening on GWI

As already stated above, $\mathrm{Im}\,\rho_{31}$, ρ_{11}, ρ_{22}, ρ_{33} given by equations (3) are only valid for static atoms. Now we consider atomic motion and the resulting GWI when the Doppler effect is included. For a single atom, moving at a speed v along the z-axis, the probe frequency $\omega_{\rm p}(v)$, as seen by the atom, is given by

$$\omega_{\rm p}(v) = \omega_{\rm p}(1\pm v/c), \quad (4a)$$

where the negative (positive) sign corresponds to co-propagating (counter-propagating) atom and probe. Similarly, the frequency of the driving field $\omega_{\rm d}(v)$, as seen by the atom is

$$\omega_{\rm d}(v) = \omega_{\rm d}(1\pm v/c). \quad (4b)$$

We denote by $\delta_{\rm p}(v)$ $(\delta_{\rm p}(v)=\omega_{13}-\omega_{\rm p}(v))$ and $\delta_{\rm d}(v)$ $(\delta_{\rm d}(v)=\omega_{12}-\omega_{\rm d}(v))$ the detunings of the probe and driving fields from their respective transitions, as

seen by the moving atom. In terms of $\delta_{\rm p}(v)$ and stationary parameters, $\delta_{\rm d}(v)$ can be written as

$$\delta_{\rm d}\left(v\right)=\Delta_{\rm d}\pm\frac{1}{R}\left[\delta_{\rm p}\left(v\right)-\Delta_{\rm p}\right], \tag{5a}$$

$$\delta_{\rm p}\left(v\right)=\Delta_{\rm p}\mp\frac{\omega_{\rm p}v}{c}\ , \tag{5b}$$

where $R=\omega_{\rm p}/\omega_{\rm d}$ denotes the degree of the frequency up-conversion. Assuming that the atomic velocities have a Maxwell–Boltzmann distribution, we denote a probability distribution function for $\delta_{\rm p}(v)$, which is given by

$$\rho(\delta_{\rm p})=\exp[-(\delta_{\rm p}-\Delta_{\rm p})^2/2D^2]/\sqrt{2\pi D^2}. \tag{6}$$

The gain averaged over the Doppler distribution can be represented as

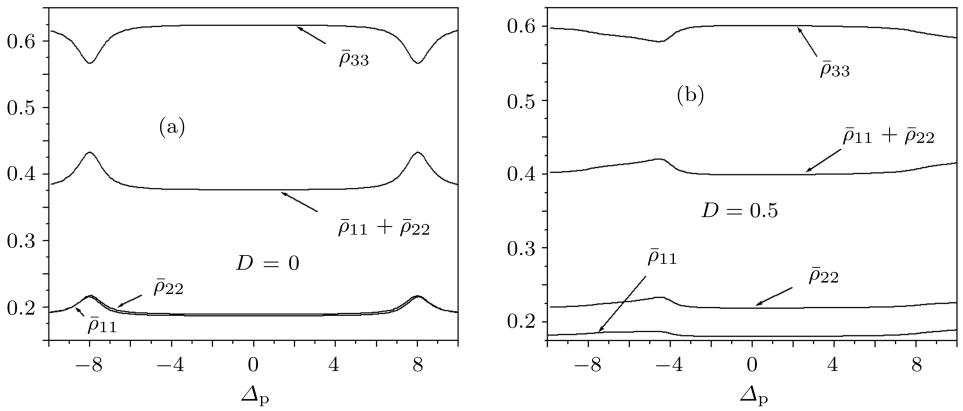
$$G=\frac{Nd_{13}^2}{\hbar\alpha}{\rm Im}\int_{-\infty}^{\infty}\rho_{31}(\delta_{\rm p})\rho(\delta_{\rm p})\,{\rm d}\delta_{\rm p}, \tag{7}$$

where N is the medium density, d_{13} is the electric dipole matrix element of the optical transition $|1\rangle\rightarrow|3\rangle$. Similarly, the population ρ_{11}, ρ_{22} and ρ_{33} averaged over the Doppler distribution can be represented as

$$\bar{\rho}_{ii}=\int_{-\infty}^{\infty}\rho_{ii}\left(\delta_{\rm p}\right)\rho\left(\delta_{\rm p}\right)\,{\rm d}\delta_{\rm p}\,, \tag{8}$$

where $i=1,\ 2,\ 3$. $\bar{\rho}_{11}+\bar{\rho}_{22}-\bar{\rho}_{33}<0$ means that neither one-photon nor two-photon inversion occurs; $\bar{\rho}_{11}-\bar{\rho}_{33}<0$ but $\bar{\rho}_{11}+\bar{\rho}_{22}-\bar{\rho}_{33}>0$ corresponds to the absence of one-photon inversion but the presence of two-photon inversion. When $G>0$ and $\bar{\rho}_{11}+\bar{\rho}_{22}-\bar{\rho}_{33}<0$ we obtain pure GWI, which means that neither one-photon nor two-photon inversion is allowed; when $G>0$ and $\bar{\rho}_{11}-\bar{\rho}_{33}<0$ but $\bar{\rho}_{11}+\bar{\rho}_{22}-\bar{\rho}_{33}>0$, we acquire gain without one-photon inversion but with two-photon inversion (non-pure GWI). In the following we will discuss the effect of Doppler broadening on GWI by using the numerical results. The numerical algorithm involves calculating ${\rm Im}\,\rho_{31}$ and $\rho_{11},\rho_{22},\rho_{33},\rho_{11}+\rho_{22}$ in the presence of Doppler broadening, i.e., we replace $\Delta_{\rm p}$ and $\Delta_{\rm d}$ in Eqs.(3) with $\delta_{\rm p}(v)$ and $\delta_{\rm d}(v)$. For given D and $\Delta_{\rm p}$, we generate a probability distribution of $\delta_{\rm p}(v)$ via the use of Eq.(6). The resulting values of $\delta_{\rm p}$ are centred at the selected value of $\Delta_{\rm p}$. By choosing optimal intervals in the Doppler line-width D we assure that sufficient atoms participate in producing consistent and accurate results. In the following, we will study the case where the driving field and the probe field co-propagate.

Figure 2 shows population $\bar{\rho}_{11}$, $\bar{\rho}_{22}$, $\bar{\rho}_{33}$, $\bar{\rho}_{11}+\bar{\rho}_{22}$ as functions of the probe detuning for different Doppler widths when the probe field and the driving field are co-propagating, the driving field is resonant ($\Delta_{\rm d}=0$) and the degree of the frequency up-conversion $R=\omega_{\rm p}/\omega_{\rm d}=1.2$. The numerical result shows that for the parameter values given in the caption of Fig.2, with the Doppler width increasing, $\bar{\rho}_{11}$ and $\bar{\rho}_{33}$ decrease but $\bar{\rho}_{22}$ increases, when $D\leq1.4$, $\bar{\rho}_{11}+\bar{\rho}_{22}$ is always smaller than $\bar{\rho}_{33}$ (shown in Figs.3(a)–3(c)) in the probe detuning region corresponding to the gain (see Fig.3), i.e. neither one-photon inversion nor two-photon inversion occurs; when $D>1.4$, $\bar{\rho}_{11}-\bar{\rho}_{33}<0$ but $\bar{\rho}_{11}+\bar{\rho}_{22}-\bar{\rho}_{33}>0$ (shown in Fig.3(d)), i.e., one-photon inversion is absent but two-photon inversion is present.



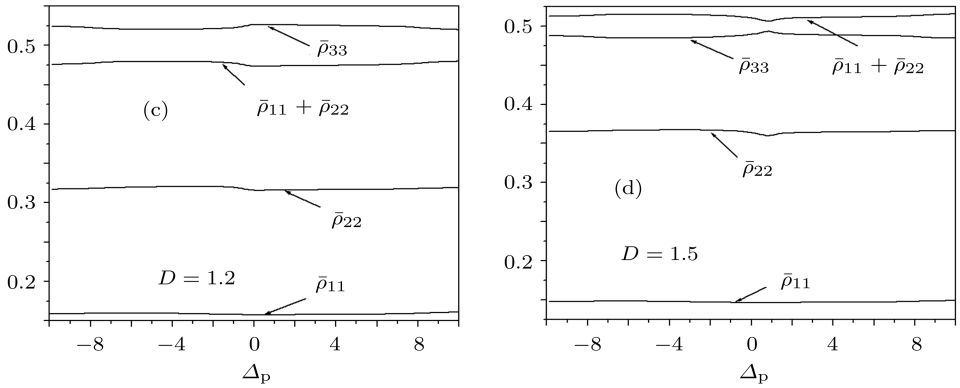


Fig.2. Populations $\bar{\rho}_{11}, \bar{\rho}_{22}, \bar{\rho}_{33}, \bar{\rho}_{11}+\bar{\rho}_{22}$ versus probe detuning for different Doppler widths when PDF are co-propagating. The parameter values are $\Delta_d = 0$, $R = \omega_p/\omega_d = 1.2$, $\Omega = 8.0$, $\alpha = 0.3$, $2\gamma_2 = 1.0$ and $2\Lambda = 0.3$ and Ω, α, $2\gamma_2, 2\Lambda$, Δ_d, Δ_p and D are scaled by $2\gamma_1$. For the ^{87}Rb atomic system described in this paper, $2\gamma_1 = 38.11$ MHZ. Panel (a) is for $D = 0$, panel (b) for $D = 0.5$, panel (c) for $D = 1.2$, and panel (d) for $D = 1.5$.

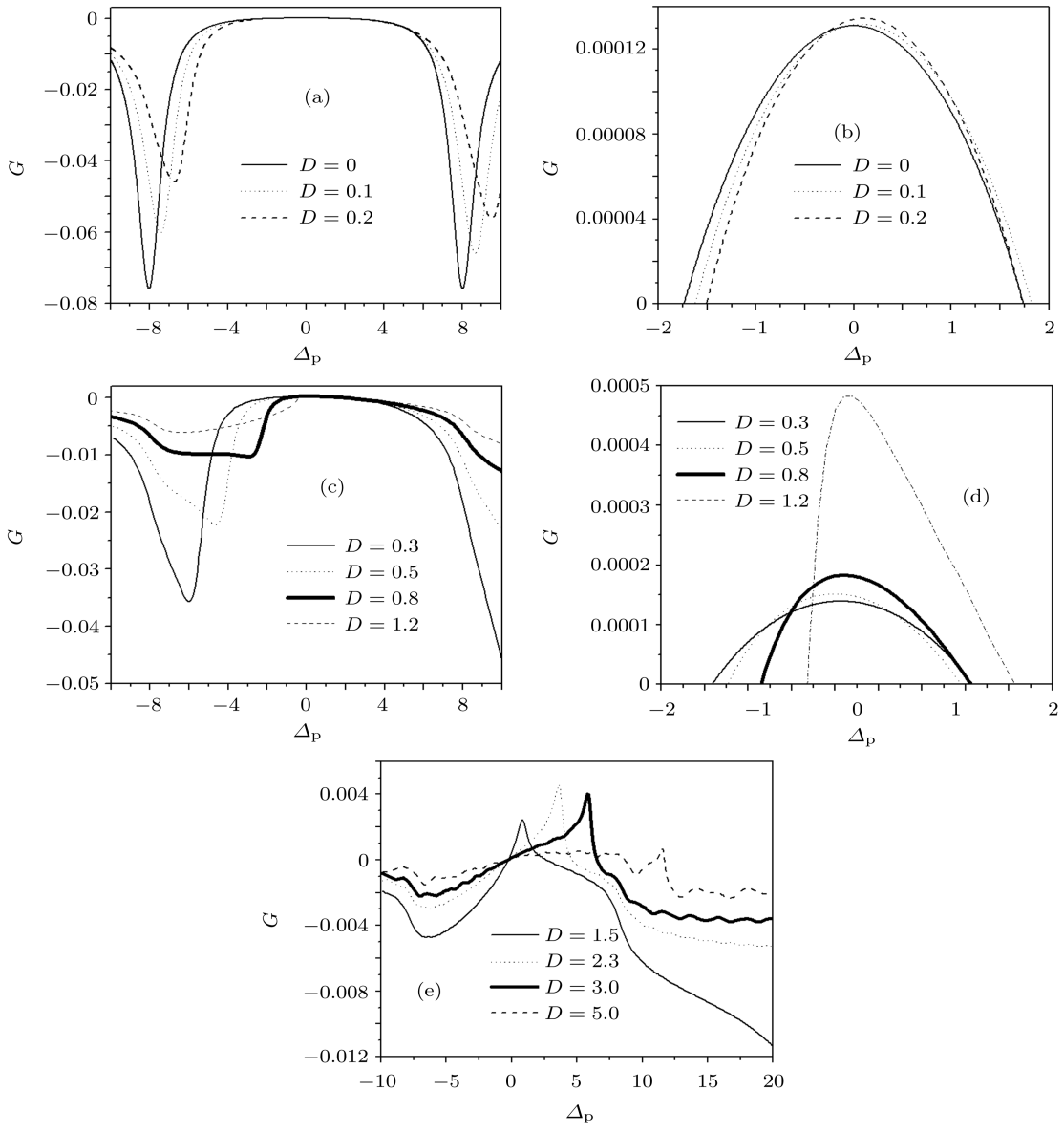


Fig.3. Curves for gain versus probe detuning for different Doppler widths when the probe field and the driving field are co-propagating with the same parameter values as those in Fig.2.

Taking the same parameter values as those in Fig.2, figure 3 shows the gain G as a function of the probe detuning for different Doppler widths. In Fig.3 and the following figures, the values of G are marked in units of $Nd_{13}^2/(\varepsilon_0\hbar)$. Both Fig.3(b) and Fig.3(d) are the enlarged parts corresponding to Fig.3(a) and Fig.3(c), respectively. The following conclusions can be drawn from Figs.2 and 3. 1) When $D = 0$, i.e., Doppler broadening is absent, we obtain pure GWI, which means that neither one-photon nor two-photon inversion is allowed, and we can obtain the maximum value at $\Delta_{\rm p} = \Delta_{\rm d} = 0$ (two-photon resonance). 2) When $0 < D \leq 1.4$, we can obtain pure GWI, the gain maximum value increases monotonically with increasing Doppler width; the value of the probe detuning corresponding to the maximum value increases but the region of the pure GWI spectrum decreases as Doppler width increases. Corresponding to some values of D, for example, $D = 1.2$ and 1.4, pure GWI does not monotonically decrease or increase as the probe detuning increases. 3) When $D > 1.4$ (Fig.3(e)), we obtain non-pure GWI; the gain maximum value does not monotonically decrease or increase with increasing Doppler width; corresponding to a fixed value of D, non-pure GWI does not monotonically decrease or increase as the probe detuning increases.

Figure 4 presents the values of gain maximum $G_{\max}$ as a function of Doppler width D for different values of the degree of the frequency up-conversion, $R(= \omega_{\rm p}/\omega_{\rm d})$. Figure 4(b) shows an enlarged part of Fig.4(a). The solid dots, which are marked by the values of D, denote the critical points between the region of neither one-photon nor two-photon inversion and the region without one-photon but with two-photon inversion for different values of R. Figure 4 indicates the following points. (i) When Doppler broadening is absent, we can obtain pure GWI and its maximum values for different values of R are the same. (ii) When Doppler broadening is present, generally, for the same value of D, a smaller value of R corresponds to a larger maximum value of pure GWI; with increasing Doppler width the variation rules of the maximum values of pure GWI corresponding to different values of R become very different. When $R = 1$ and 1.2, the maximum values of pure GWI increase monotonically with the increase of Doppler width. When $R = 1.3$, pure GWI, absorption and pure GWI arise successively with the increase of Doppler width; A comparison between the cases of $R = 1$, 1.2, 3 shows that we can obtain larger maximum values of pure GWI. When $R = 3$, the maximum value of pure GWI decreases rapidly and monotonically with the increase of Doppler width. (iii) For $R = 1$, 1.2, 1.3, the case of the lower degree of frequency up-conversion, the maximum value of non-pure GWI does not monotonically increase or decrease as Doppler width increases, we can obtain the largest gain maximum value at a suitable Doppler width; for the same value of D, a smaller value of R corresponds to a larger maximum value of non-pure GWI. For $R = 3$, the case of the higher degree of frequency up-conversion, non-pure GWI does not occur.

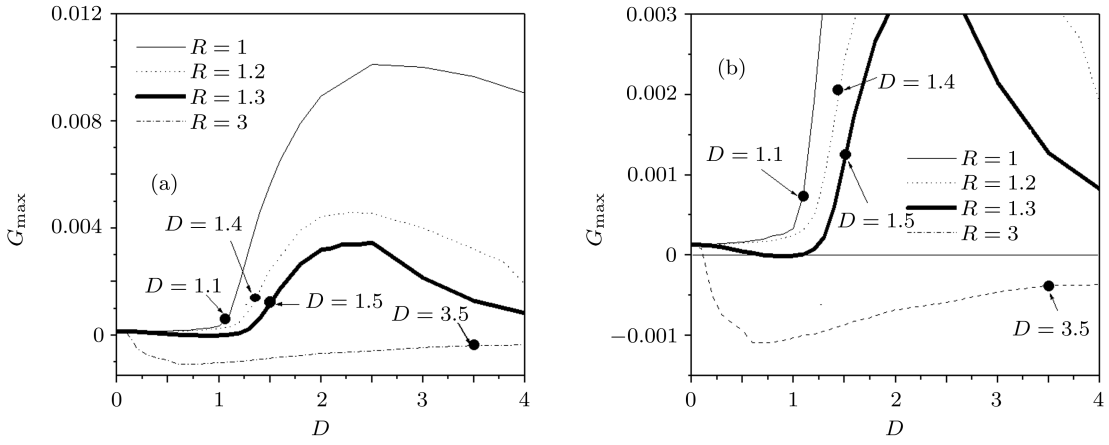


Fig.4. Curves for gain maximum value versus Doppler width for different values of R with the same parameter values as those in Fig.2.

Using the same values of system parameters as those in Fig.2 except $\Delta_p = 0$, we obtain the values of gain G as a function of Doppler width for the cases of two-photon resonance, i.e., $\Delta_p = \Delta_d$,[27] and $\Delta_p = \Delta_d = 0$, as shown in Fig.5. The calculation results corresponding to Fig.5 indicate that when $D \leq 1.4$, neither one-photon inversion nor two-photon inversion occurs; when $D > 1.4$ one-photon inversion is still absent but two-photon inversion is present. Considering the numerical result, from Fig.5 we can see the following points. When $D \leq 1.4$, the gain is pure GWI, the gain does not monotonically increase or decrease with increasing Doppler width and is always larger than that without Doppler broadening; when $D = 1.3$, the system obtains the largest pure GWI. The variation of the gain with increasing Doppler width is similar to that when $\Delta_d = 0$ and $\Delta_p \neq 0$ (see Fig.4(b)). When $D > 1.4$ the gain is non-pure GWI and decreases monotonically with increasing Doppler width. Comparing Fig.5 with the curve corresponding to $R = 1.2$ in Fig.4, we find that with increasing Doppler width, the variation rule of pure GWI (non-pure GWI) for the case of $\Delta_p = \Delta_d = 0$ is very different from that of the maximum value of pure GWI (non-pure GWI) for the case where $\Delta_d = 0$ and $\Delta_p \neq 0$.

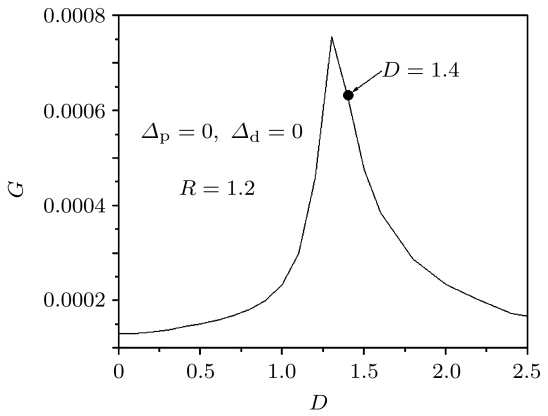


Fig.5. Curves for gain versus Doppler width in the case of two-photon resonance with the same parameter values as those in Fig.2 but $\Delta_p = 0$.

It should be pointed out that in the above discussion on the effect of Doppler broadening on pure and non-pure GWI, we have taken 0.3 as the value of the incoherent pumping rate (2Λ), but this value has no special meaning. Our numerical calculation result shows that varying the value of the incoherent pumping rate, with the values of the other parameters fixed, will produce obvious effects on the size of the population and gain and even the properties of the gain. Generally speaking, with the incoherent pumping rate increasing, gain increases, the population of the upper level $|1\rangle$ increases but the populations of two lower levels $|2\rangle$ and $|3\rangle$ decrease. Now we take an example to show the above conclusion. Taking the same parameters values as those in Fig.2 and Fig.3 except for the incoherent pumping rate, we can obtain the following conclusions. i) When $0.16 > 2\Lambda$, $G < 0$ always holds when $\bar{\rho}_{11} + \bar{\rho}_{22} - \bar{\rho}_{33} < 0$ which means that neither one-photon nor two-photon inversion occurs but no gain exists for any Doppler width; ii) when $0.16 < 2\Lambda < 0.5$, then $G > 0$ when $\bar{\rho}_{11} + \bar{\rho}_{22} - \bar{\rho}_{33} < 0$ or $\bar{\rho}_{11} - \bar{\rho}_{33} < 0$ but $\bar{\rho}_{11} + \bar{\rho}_{22} - \bar{\rho}_{33} > 0$, i.e. pure GWI or non-pure GWI can be realized, and this depends on the value of the Doppler width; iii) when $0.5 < 2\Lambda < 1.3$, then only $\bar{\rho}_{11} - \bar{\rho}_{33} < 0$ but $\bar{\rho}_{11} + \bar{\rho}_{22} - \bar{\rho}_{33} > 0$, hence only non-pure GWI is possible; iv) when $2\Lambda > 1.3$, then $\bar{\rho}_{11} - \bar{\rho}_{33} > 0$ i.e. both one- and two-photon inversions exist simultaneously, only gain with inversion can be obtained.

Now there arises the question of why GWI can be obtained under the effect of Doppler broadening only in some cases but not in any other cases. The answer is briefly given below.

Quantum coherence and interference in an atomic system lead to amplification without inversion and lasing without inversion. Generally speaking, Doppler broadening will produce detuning of the laser frequencies from the atomic transitions, which may affect the coherent recycling in atoms and damp the intensity of the laser radiation, thereby leading to a gain decrease. The effects from the Doppler broadening, coherent driving and probe fields will produce quantum destructive or constructive interference which is mainly determined by the Doppler width, propagating direction, strength and detuning of the driving and probe fields and is also related to the incoherent pumping and spontaneous emission rates. In some cases, quantum destructive interference arises and leads to absorption or gain decreases; in other cases, quantum constructive interference appears and leads to gain and gain increases.

In order to validate the above theoretical result, one can perform an experiment with ^{87}Rb atomic vapour contained in a Pyrex cell. The energy-level diagram of ^{87}Rb is shown in Fig.6. The extended cavity diode lasers are tuned to the D_2 line and form a Λ-type system. The probe (with Rabi frequency α) and drive (with Rabi frequency Ω) laser fields excite

transitions $5S_{1/2}(F = 1) \rightarrow 5P_{3/2}$ $(F' = 0,\ 1,\ 2)$ and $5S_{1/2}(F = 2) \rightarrow 5P_{3/2}(F' = 1,\ 2,\ 3)$, respectively, as shown in Fig.6. The double and single lines represent the drive and probe laser fields, respectively. The drive and probe fields cannot couple the ground states via the hyperfine levels $F' = 0$ and 3 of $5P_{3/2}$ state due to the selection rules, and these states are marked by dashed lines. The spontaneous emission rates of both transitions are $2\gamma_1 = 2\gamma_2 = 38.11$ MHZ. The amplitudes of the dipole moments $\boldsymbol{d}_{12}$ and $\boldsymbol{d}_{13}$ are $4.227(5)ea_0$, where e and a_0 are elementary charge and Bohr radius, respectively. $E_{\rm p}$ and $E_{\rm d}$ are in V/cm. To be accurate, one can choose the driving frequency around $5S_{1/2}(F = 2) \rightarrow 5P_{3/2}(F' = 1)$ transition. The Rabi frequency Ω of the driving laser can be estimated from the formula $\Omega = 2\pi\Gamma\sqrt{I/8}$, where I is the drive intensity (mW/cm^2) and $\Gamma(\approx 6$ MHZ) is the natural width of

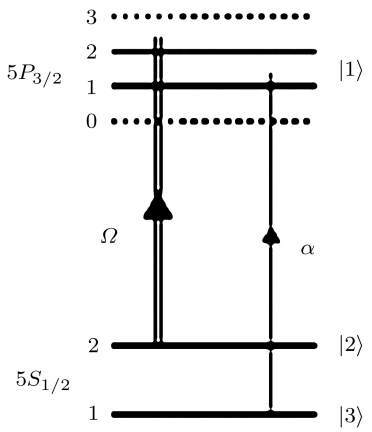


Fig.6. Energy-level diagram of atomic ^{87}Rb. Double and single lines represent the driving and probe laser fields, respectively. Dashed lines depict uncoupled states in the energy diagram.

the P-state of the Rb atom.[28] The incoherent pumping with pumping rate 2Λ, which is applied between levels $|1\rangle$ and $|3\rangle$, can be realized by an incoherent pump laser.[29] Generally speaking, in the experiment, we can obtain the required value of the incoherent pumping rate by modulating the intensity of the incoherent pumping laser field, but the concrete manipulation manner is determined by concrete experimental conditions.

4. Conclusion

In this paper, using the numerical calculation results from the steady analytical solution of the density matrix equations of motion, we studied the effect of Doppler broadening on pure and non-pure GWI in a closed Λ-type three-level system with an incoherent pump applied between the high level $|1\rangle$ and the low level $|3\rangle$. Our findings are as follows. When the driving field is in resonance but the probe field is not as Doppler width increases, generally speaking, for the case of the lower degree of frequency up-conversion, pure GWI increases monotonically, but non-pure GWI does not monotonically increase or decrease; for the case of the higher degree of frequency up-conversion, pure GWI decreases monotonically, non-pure GWI does not arise. In the case of two-photon resonance, in a certain range of the Doppler width, pure GWI does not monotonously increase or decrease while non-pure GWI decreases monotonically with increasing Doppler width. Finally, an experimental scheme, in which the ^{87}Rb atomic system is used to examine the above theoretical result, is given.

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