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The entanglement between two isolated atoms in the double mode–mode competition model*

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Extending the double Jaynes–Cummings model to a more complicated case where the mode–mode competition is considered, we investigate the entanglement character of two isolated atoms by means of concurrence, and discuss the dependence of atom–atom entanglement on the different initial state and the relative coupling strength between the atom and the corresponding cavity field. The results show that the amplitude and the period of the atom–atom entanglement evolution can be controlled by the choice of initial state and relative coupling strength, respectively. We find that the phenomenon of entanglement sudden death (ESD) is sensitive to the initial conditions. The length of the time interval for zero entanglement depends not only on the initial degree of entanglement between two atoms but also on the relative coupling strength of atom–field interaction. The ESD effect can be weakened by enhancing the mode–mode competition between the three- and single-photon processes.

Keywords: double mode–mode competition model, concurrence, entanglement, sudden death

PACC: 4250, 0365

1. Introduction

Entanglement plays a key role in quantum computation and quantum information processing. It has been extensively studied theoretically in a range of systems, including trapped ions,[1] atomic ensembles[2] and photon pairs.[3] A fundamental feature of entanglement is that it is easily degraded when the entangled system interacts with another. Therefore, it is important to study dynamical creation of entanglement and its disentangled properties. Recently, Yönac *et al*[4,5] have proposed a double Jaynes–Cummings (J–C) model, and investigated the pairwise entanglement dynamics. They have observed that a two-qubit affected by uncorrelated reservoirs can experience disentanglement in a finite time. This drastic non-asymptotic feature of entanglement, termed entanglement sudden death (ESD), has deserved great attention in recent years. Lately, it has been found that ESD is present in different scenarios for certain classes of states.[6−21] Quite recently, this phenomenon was demonstrated in experiments.[22,23] However, in their studies, a given mode is assumed to act only on one optical transition, i.e., the two modes do not act simultaneously on one transition.

From the semiclassical point of view, if an atom interacts with different fields, several nonlinear processes can take place at the same time, leading to many new dynamical aspects as a result of mode–mode competition,[24−26] such as Stokes and anti-Stokes processes.[27] In Ref.[28], Iyotsna and Agarwal found that the quantum effects were rather dominant in the mode–mode competition model. The authors have studied the atomic dynamical properties, the statistic properties of the photon and the entanglement dynamics in this model.[29−34] In this paper, we shall propose a double mode–mode competition model by extending the work in Refs.[4] and [5] to a more complicated case where the mode–mode competition is involved, and study the entanglement dynamics of two isolated atoms, each in its own mode–mode competition system. It shows that the atom–atom entanglement is highly dependent on the initial state and the relative coupling strength between the atom and the corresponding cavity field. Moreover, the ESD can be weakened by enhancing the mode–mode competi-

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tion between the three- and single-photon processes.

2. The double mode–mode competition model

We consider a system consisting of two two-level atoms labeled A and B, each interacting with a two-mode cavity field (denoted by a and b) via two different competing transitions, as shown in Fig.1. We assume that one of the modes interacts with the atom through a three-photon process and the other through a single-photon process, resonantly. Following Refs.[4] and [5], we also assume that each atom–cavity subsystem is isolated. The interaction Hamiltonian of the considered system in the rotating-wave approximation and in the interaction picture can be written as (assuming $\hbar = 1$)

$$H_{\rm I} = H_{\rm Aa} + H_{\rm Bb} \tag{1}$$

with[28]

$$\begin{aligned} H_{\rm Aa} &= k_1(\sigma_{\rm A}^+ a_1^3 + \sigma_{\rm A}^- a_1^{\dagger 3}) + g_1(\sigma_{\rm A}^+ a_2 + \sigma_{\rm A}^- a_2^\dagger), \\ H_{\rm Bb} &= k_2(\sigma_{\rm B}^+ b_1^3 + \sigma_{\rm B}^- b_1^{\dagger 3}) + g_2(\sigma_{\rm B}^+ b_2 + \sigma_{\rm B}^- b_2^\dagger), \end{aligned} \tag{2}$$

where $\sigma_\pm^i$ (i = A, B) are the raising and lowering operators of the i-th two-level atom, a_j ($a_j^\dagger$) and b_j ($b_j^\dagger$) ($j = 1, 2$) are the annihilation (creation) operators of the j-th mode of fields a and b. k_j and g_j ($j = 1, 2$) are the coupling constants for the three-photon process and the single-photon process in the j-th subsystem, respectively.

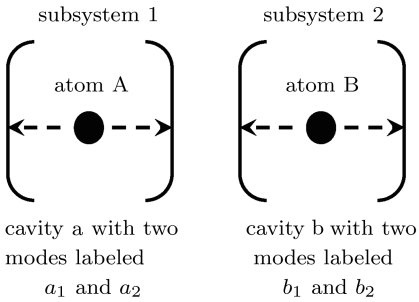


Fig.1. Schematic diagram of the system considered, two distant and non-interacting subsystems 1 and 2 each with a two-mode cavity containing a two-level atom.

We denote the excited and ground states of the atoms by $|e\rangle$ and $|g\rangle$ respectively. We take the two atoms initially in Bell-like pure states, and the fields initially in vacuum states. Thus the initial state for the double mode–mode competition system is given by

$$\begin{aligned} |\Psi(0)\rangle &= \left\{\cos\left(\frac{\theta}{2}\right)|eg\rangle + \sin\left(\frac{\theta}{2}\right)|ge\rangle\right\} \\ &\quad \otimes |00\rangle \otimes |00\rangle, \end{aligned} \tag{3}$$

and

$$\begin{aligned} |\Phi(0)\rangle &= \left\{\cos\left(\frac{\theta}{2}\right)|ee\rangle + \sin\left(\frac{\theta}{2}\right)|gg\rangle\right\} \\ &\quad \otimes |00\rangle \otimes |00\rangle, \end{aligned} \tag{4}$$

where $0 \le \theta \le \pi/2$.

The state vector at any time t can be expressed in terms of a complete set of possible states in the interaction picture as

$$\begin{aligned} |\Psi(t)\rangle &= x_1(t)|eg0000\rangle + x_2(t)|ge0000\rangle \\ &\quad + x_3(t)|gg3000\rangle + x_4(t)|gg0100\rangle \\ &\quad + x_5(t)|gg0030\rangle + x_6(t)|gg0001\rangle, \end{aligned} \tag{5}$$

where the probability coefficients are given by

$$\begin{aligned} x_1(t) &= \sin\left(\frac{\theta}{2}\right)\cos(\Omega_1 t), \\ x_2(t) &= \cos\left(\frac{\theta}{2}\right)\cos(\Omega_2 t), \\ x_3(t) &= \frac{-{\rm i}\sqrt{6}k_1}{\Omega_1}\sin\left(\frac{\theta}{2}\right)\sin(\Omega_1 t), \\ x_4(t) &= \frac{-{\rm i}g_1}{\Omega_1}\sin\left(\frac{\theta}{2}\right)\sin(\Omega_1 t), \\ x_5(t) &= \frac{-{\rm i}\sqrt{6}k_2}{\Omega_2}\cos\left(\frac{\theta}{2}\right)\sin(\Omega_2 t), \\ x_6(t) &= \frac{-{\rm i}g_2}{\Omega_2}\cos\left(\frac{\theta}{2}\right)\sin(\Omega_2 t) \end{aligned} \tag{6}$$

with

$$\Omega_1 = \sqrt{g_1^2 + 6k_1^2}, \quad \Omega_2 = \sqrt{g_2^2 + 6k_2^2}, \tag{7}$$

and

$$\begin{aligned} |\Phi(t)\rangle &= y_1(t)|ee0000\rangle + y_2(t)|gg0000\rangle \\ &\quad + y_3(t)|ge3000\rangle + y_4(t)|ge0100\rangle \\ &\quad + y_5(t)|eg0030\rangle + y_6(t)|eg0001\rangle \\ &\quad + y_7(t)|gg3030\rangle + y_8(t)|gg3001\rangle \\ &\quad + y_9(t)|gg0130\rangle + y_{10}(t)|gg0101\rangle, \end{aligned} \tag{8}$$

where the complex amplitudes are derived from the Schrödinger equation in the interaction picture. It is difficult to obtain the analytical solution for the coefficients in expression (8), so we use the fourth-order Runge–Kutta method to obtain their numerical solution. Here, $|egij\xi\eta\rangle$ represents the atom A in the excited state $|e\rangle$, the atom B in the groud state $|g\rangle$, i, j and ξ, η photons in the field modes a_1, a_2 and b_1, b_2, respectively.

3. Entanglement between two isolated atoms

Several different measures have been proposed to identify entanglement between two qubits, and we choose the Wootters entanglement measure,[35] the concurrence C, defined as

$$C = \max\left(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\right), \quad (9)$$

where $\lambda_1, \ldots, \lambda_4$ are the eigenvalues of the matrix $\tilde{\rho} = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$. ρ is the density matrix representing the quantum state, and the matrix elements are taken with repect to the basis of $|ee\rangle$, $|eg\rangle$, $|ge\rangle$, $|gg\rangle$. The range of the concurrence is from 0 to 1. For unentangled qubits $C = 0$ whereas $C = 1$ for maximally entangled qubits.

3.1. Atom–atom entanglement for the initial atomic state $|\Psi(0)\rangle$

We are only interested in the entanglement dynamics of the two isolated atoms. In order to obtain the atom–atom density operator $\rho_{\rm I}^{\rm AB}(t)$, we trace over the field variables from Eq.(5). For the initial atomic state $|\Psi(0)\rangle$, the atom–atom density matrix $\rho_{\rm I}^{\rm AB}(t)$ written in the basis of $|ee\rangle$, $|eg\rangle$, $|ge\rangle$, $|gg\rangle$ is given by

$$\rho_{\rm I}^{\rm AB}(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & |x_1(t)|^2 & x_1(t)x_2^*(t) & 0 \\ 0 & x_1^*(t)x_2(t) & |x_2(t)|^2 & 0 \\ 0 & 0 & 0 & |x_3(t)|^2 + |x_4(t)|^2 + |x_5(t)|^2 + |x_6(t)|^2 \end{pmatrix}. \quad (10)$$

The concurrence of the density matrix (10) is expressed by

$$C^{\rm AB}(t) = 2|x_1(t)||x_2(t)|. \quad (11)$$

Now we study the influence of the initial state on the atom–atom entanglement. For simplicity, we suppose that the correponding coupling strength in both subsystems is equivalent (so-called symmetric atom–field couplings), i.e., $k_1 = k_2 = k$, $g_1 = g_2 = g$. So we can find that $\Omega_1 = \Omega_2 = \Omega$. The expression (11) is easily reduced to

$$C^{\rm AB}(t) = |\sin(\theta)|\cos^2(\Omega t). \quad (12)$$

We plot the time evolution of the concurrence $C^{\rm AB}(t)$ given by Eq.(12) when $k/g = 1$ for different θ in Fig.2. In the case of $\theta = \pi/2$, that is, the two atoms are maximally entangled initially, the concurrence $C^{\rm AB}(t)$ evolves periodically between 0 and 1 (solid line), and the period is $\pi/\sqrt{7}$. When θ takes the value $\pi/4$ and $\pi/8$, we can see that the maximum of $C^{\rm AB}(t)$ is reduced to 0.7071 (dashed line) and 0.3827 (dot-dashed line), respectively. It is clear that the smaller the initial degree of entanglement between the two atoms, the smaller the maximum of $C^{\rm AB}(t)$. But the evolution period of the atom–atom entanglement keeps invariant, which is easily obtained from the expression (12).

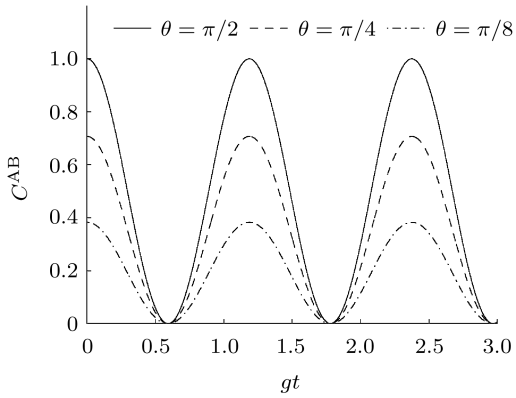


Fig.2. The time evolution of the concurrence $C^{\rm AB}$ with the initial atomic state $|\Psi(0)\rangle$ when $k/g = 1$ for different θ.

Now we shall examine the influence of k_j/g_j $(j = 1, 2)$ (the relative coupling strength between the atom and the corresponding cavity field) on the atom–atom entanglement when the two atoms are initially in the state $\sqrt{2}/2(|eg\rangle + |ge\rangle)$ $(\theta = \pi/2)$, as shown in Fig.3 and Fig.4. Figure 3 displays the case of symmetric atom–field couplings, i.e., $k_1 = k_2 = k$, $g_1 = g_2 = g$. Comparing the two curves in Fig.3 with the solid line in Fig.2, we can find: (i) the concurrence $C^{\rm AB}(t)$ exhibits periodical evolution between 0 and 1; (ii) the evolution period of concurrence $C^{\rm AB}(t)$ is $\pi/\sqrt{1 + 6k^2/g^2}$, decreasing with k/g. Figure 4 illustrates the case where the relative coupling strength

in both subsystems is not equivalent (so-called asymmetric atom–field couplings), i.e., $k_1/g_1 \neq k_2/g_2$. For simplicity, we also suppose that $g_1 = g_2 = g$. In this case, the expression (11) becomes

$$C^{\rm AB}(t) = |\sin(\theta)\cos(\Omega_1 t)\cos(\Omega_2 t)|. \qquad (13)$$

Fixing the value of $k_1/g = 0.01$, the dependence on time of the concurrence $C^{\rm AB}(t)$ changes when k_2/g is varied from 2 (Fig.4(a)) to 5 (Fig.4(b)). Drawing a comparison between Fig.4(a) and Fig.4(b), we can see that the number of the peaks in the latter is larger than that in the former. This is explained as follows. From expression (13), we can see that the atom–atom entanglement has two periods: one is the envelope oscillation period with a value of $\pi/\Omega_1 \approx \pi$, the other is the oscillation period π/Ω_2, which decreases with the increase of k_2/g. We also see that the time evolution of atom–atom entanglement in the case of symmetric atom–field couplings (marked by * in Fig.4(b)) is modulated by the asymmetric atom–field couplings.

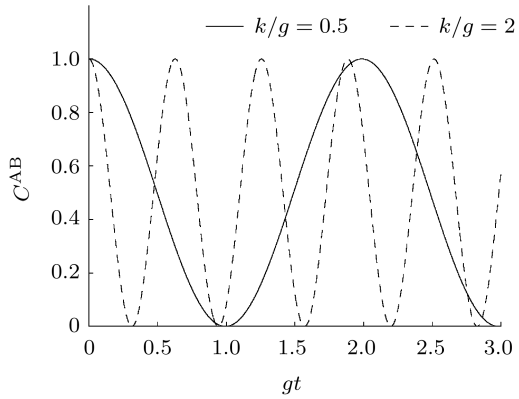


Fig.3. The time evolution of the concurrence $C^{\rm AB}$ with the initial atomic state $|\Psi(0)\rangle$ when $\theta = \pi/2$ for the case of symmetric atom–field couplings.

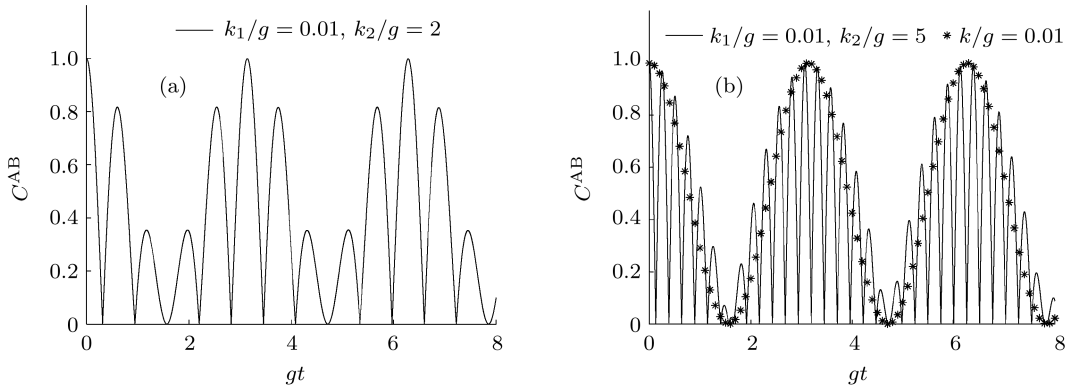


Fig.4. The time evolution of the concurrence $C^{\rm AB}$ with the initial atomic state $|\Psi(0)\rangle$ when $\theta = \pi/2$ for the case of asymmetric atom–field couplings.

3.2. Atom–atom entanglement for the initial atomic state $|\Phi(0)\rangle$

For $|\Phi(0)\rangle$, the atom–atom density matrix $\rho_{\rm I}^{\rm AB}(t)$ is obtained by tracing over the field variables from Eq.(8)

$$\rho_{\rm I}^{\rm AB}(t) = \begin{pmatrix} a(t) & 0 & 0 & w(t) \\ 0 & b(t) & 0 & 0 \\ 0 & 0 & c(t) & 0 \\ w^*(t) & 0 & 0 & d(t) \end{pmatrix}, \qquad (14)$$

where

$$\begin{aligned} a(t) &= |y_1(t)|^2, \\ b(t) &= |y_5(t)|^2 + |y_6(t)|^2, \\ c(t) &= |y_3(t)|^2 + |y_4(t)|^2, \\ d(t) &= |y_2(t)|^2 + |y_7(t)|^2 + |y_8(t)|^2 \\ &\quad + |y_9(t)|^2 + |y_{10}(t)|^2, \\ w(t) &= y_1(t)y_2^*(t). \end{aligned} \qquad (15)$$

The concurrence for this matrix is

$$\begin{aligned} &C^{\rm AB}(t) \\ &= 2\max\{0, |y_1(t)y_2^*(t)| \\ &\quad - \sqrt{(|y_3(t)|^2 + |y_4(t)|^2)(|y_5(t)|^2 + |y_6(t)|^2)}\}. \end{aligned} \qquad (16)$$

The results for the concurrence $C^{\rm AB}(t)$ with the initial atomic $|\Phi(0)\rangle$ are illustrated in Figs.5, 6 and 7, which differ from those with the initial atomic state $|\Psi(0)\rangle$. This is due to a "dark" state $|gg0000\rangle$ in $|\Phi(0)\rangle$, i.e., the state $|gg0000\rangle$ is unaffected by coupling to the fields. Unlike the previous case, figure

5 shows that the atom–atom entanglement can fall abruptly to zero (which is the so-called ESD), and will remain zero for a period of time before it recovers. The length of time interval for the zero entanglement is dependent on the initial degree of entanglement between two atoms. The smaller the initial degree of entanglement is, the longer the state will stay in the disentangled separable state. In addition, the maximum of the concurrence $C^{\mathrm{AB}}(t)$ increases with the initial degree of entanglement between two atoms.

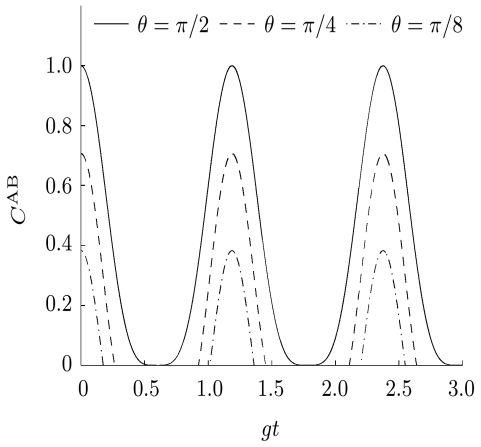


Fig.5. The time evolution of the concurrence C^{AB} with the initial atomic state $|\Phi(0)\rangle$ when $k/g = 1$ for different θ.

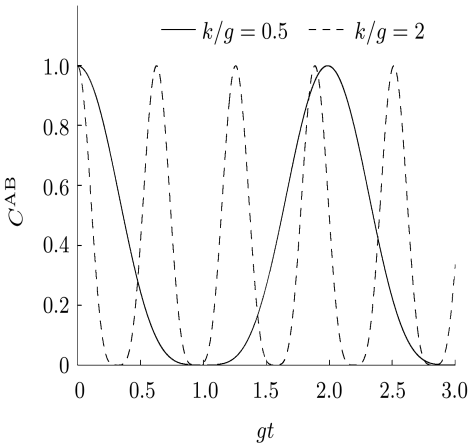


Fig.6. The time evolution of the concurrence C^{AB} with the initial atomic state $|\Phi(0)\rangle$ when $\theta = \pi/2$ for the case of symmetric atom–field couplings.

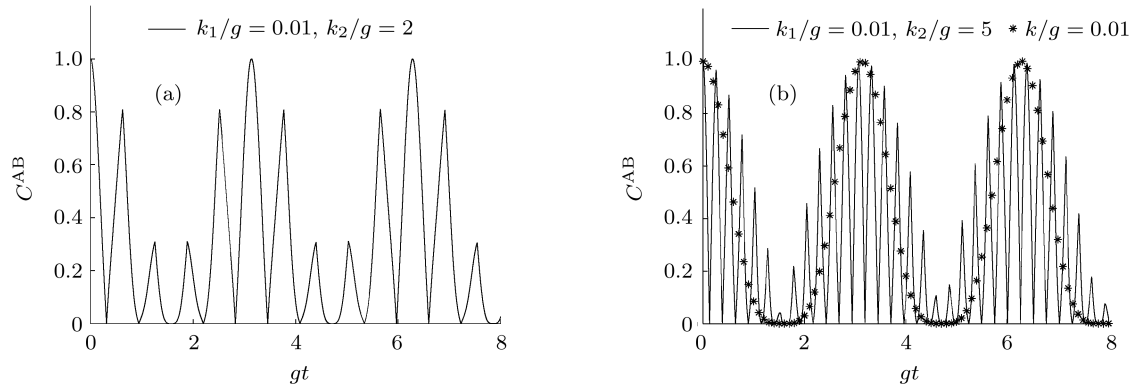


Fig.7. The time evolution of the concurrence C^{AB} with the initial atomic state $|\Phi(0)\rangle$ when $\theta = \pi/2$ for the case of asymmetric atom–field couplings.

We let $\theta = \pi/2$, and discuss the dependence of atom–atom entanglement on the relative coupling strength between the atom and the corresponding cavity field in Fig.6 where $k_1 = k_2 = k,\ g_1 = g_2 = g$ and in Fig.7 where $k_1 \neq k_2,\ g_1 = g_2 = g$. For the case of symmetric atom–field couplings, e.g., $k/g = 0.5$, which means the three-photon transition is much weaker than the single-photon transition in both subsystems, the atom–atom entanglement suffers ESD (solid line in Fig.6). In contrast with the other curve in Fig.6 and the solid line in Fig.5, it is evident that the length of time interval for the zero entanglement is dependent on the relative coupling strength of the atom–field interaction. The stronger the relative coupling strength is, the shorter the state will stay in the disentangled separable state. This implies that the ESD effect can be weakened by enhancing the mode–mode competition between the two different transitions, which stems from the fact that the increase in k_j/g_j $(j = 1, 2)$ leads to a stronger interaction between the atom and the corresponding field.

However, the maximum of the concurrence $C^{\mathrm{AB}}(t)$ remains the same value, 1. For the case of asymmetric atom–field couplings, the two atoms are periodically entangled and disentangled, as shown in Fig.7. Comparing Fig.7 with Fig.4, we can see that the time evolution properties of the concurrence $C^{\mathrm{AB}}(t)$ are similar to those for the initial atomic state $|\Psi(0)\rangle$ except for the abrupt disappearance of entanglement. Further study shows that the same entanglement dynamics appears when the values of k_1/g_1 and k_2/g_2 are exchanged in the case of asymmetric atom–field couplings for both initial atomic states $|\Psi(0)\rangle$ and $|\Phi(0)\rangle$.

4. Conclusion

In this paper, we have proposed a double mode–mode competition model, and studied the entanglement dynamics of two isolated atoms by means of concurrence. We find that the atom–atom entanglement is highly dependent on the initial state and the relative coupling strength between the atom and the corresponding cavity field. Firstly, the initial degree of entanglement between two atoms and the relative coupling strength can control the amplitude and the period of the atom–atom entanglement evolution, respectively. Secondly, the atom–atom entanglement in the case of symmetric atom–field couplings is modulated by the asymmetric atom–field couplings. Furthermore, the atom–atom entanglement presents exactly the same time evolution properties when the values of k_1/g_1 and k_2/g_2 are exchanged in the case of asymmetric atom–field couplings. Thirdly, so-called entanglement sudden death occurs when the atoms are initially in the state $|\Phi(0)\rangle$, although it is absent for the state $|\Psi(0)\rangle$. Finally, the ESD effect can be weakened by enhancing the mode–mode competition between the two different transitions. We expect that our work will be helpful for studying the entanglement dynamics of qubit pairs in practical experiments.

References

[1] Turchette Q A, Wood C S, King B E, Myatt C J, Leibfried D, Itano W M, Monroe C and Wineland D J 1998 *Phys. Rev. Lett.* **81** 3631

[2] Julsgaard B, Kozhekin A and Polzik E S 2001 *Nature* **413** 400

[3] Aspect A, Grangier P and Roger G 1981 *Phys. Rev. Lett.* **47** 460

[4] Yönac M, Yu T and Eberly J H 2006 *J. Phys.* B **39** S621

[5] Yönac M, Yu T and Eberly J H 2007 *J. Phys.* B **40** S45

[6] Santos M F, Milman P, Davidovicn L and Zagury N 2006 *Phys. Rev.* A **73** 040305

[7] Yu T and Eberly J H 2004 *Phys. Rev. Lett.* **93** 140404

[8] Yu T and Eberly J H 2006 *Phys. Rev. Lett.* **97** 140403

[9] Yu T and Eberly J H 2006 *Opt. Commum.* **264** 393

[10] Yu T and Eberly J H 2007 *Quantum Inf. Comput.* **7** 459

[11] Eberly J H and Yu T 2007 *Science* **316** 555

[12] Ficek Z and Tanaś R 2006 *Phys. Rev.* A **74** 024304

[13] Cui H T, Li K and Yi X X 2007 *Phys. Lett.* A **365** 44

[14] Sainz I and Björk G 2007 *Phys. Rev.* A **76** 042313

[15] Cao X and Zheng H 2008 *Phys. Rev.* A **77** 022320

[16] Ficek Z and Tanas R 2008 *Phys. Rev.* A **77** 054301

[17] Hu Y H, Fang M F, Cai J W and Jiang C L 2008 *Int. J. Theor. Phys.* **47** 2554

[18] Zhang G F 2007 *Chin. Phys.* **16** 1855

[19] Guo L and Liang X T 2009 *Acta Phys. Sin.* **58** 50 (in Chinese)

[20] Man Z X and Xia Y J 2008 *Chin. Phys.* B **17** 3198

[21] Zheng Q, Zhang X P and Ren Z Z 2008 *Chin. Phys.* B **17** 3553

[22] Almeida M P, Melo de, Hor-Meyll M, Salles A, Walborn S P, Souto Ribeiro P H and Dacidocich L 2007 *Science* **316** 579

[23] Salles A, Melo F, Almeida M P, Hor-Meyll M, Walborn S P, Souto Ribeiro P H and Davidovich L 2008 *Phys. Rev.* A **78** 022322

[24] Agarwal G S and Nayak N 1986 *Phys. Rev.* A **33** 391

[25] Zhu Y 1990 *Phys. Rev.* A **41** 6574

[26] Agarwal G S and Nayak N 1982 *Phys. Rev.* A **25** 2056

[27] Bloembergen N 1965 *Nonlinear Optics* (New York: Benjamin)

[28] Jyotsna I V and Agarwal G S 1994 *Phys. Rev.* A **50** 1770

[29] Wu Q and Fang M F 2004 *Chin. Phys.* **13** 1432

[30] Wu Q and Fang M F 2005 *Commun. Theor. Phys.* **43** 515

[31] Wu Q and Fang M F 2006 *Acta Quantum Opt.* **12** 8 (in Chinese)

[32] Wu Q and Fang M F 2006 *Journal of Natural Science of Hunan Normal University* **29** 49 (in Chinese)

[33] Wu Q, Fang M F and Hu Y H 2007 *Chin. Phys.* **16** 1971

[34] Wu Q, Fang M F, Feng Y Z and Hu Y H 2008 *Chin. Phys.* B **17** 2963

[35] Wootters K 1998 *Phys. Rev. Lett.* **80** 2248