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Energy spectrum and superfluidity of spin-2 ultracold bosons in optical lattices*

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This paper studies the superfluidity of ultracold spin-2 Bose atoms with weak interactions in optical lattices by calculating the excitation energy spectrum using the Bogoliubov approach. The energy spectra exhibit the characteristics of the superfluid-phase explicitly and it finds the nonvanishing critical speeds of superfluid. The obtained results display that the critical speeds of superfluid are different for five spin components and can be controlled by adjusting the lattice parameters in experiments. Finally it discusses the feasibilities of implementing and measuring superfluid.

Keywords: ultracold spin bosons, energy spectrum, superfluidity

PACC: 3280P, 0530J, 6750F

1. Introduction

The ultracold Bose atoms (UBA) in optical lattices have provided a controllable and versatile platform to investigate the condensed-matter-like properties.[1] Therefore, the UBA in optical lattices has attracted considerable interest recently both experimentally and theoretically.[2−5] In studies of ultracold bosonic gases, elementary excitations play an important role in understanding the various properties of the UBA like dynamics, superfluidity and thermodynamics, etc. Experimentally elementary excitations of Bose–Einstein condensate (BEC) have been observed, including the Bogoliubov excitation spectrum and collective modes with Bragg spectroscopy.[6−9] Furthermore, the spinor condensates optically trapped for $S = 1$ sodium atoms were first realized at MIT,[10] where an optical trap was applied to confine the BEC to weak-seeking magnetic states as magnetic trapping. The rich dynamics were also investigated in spin-2 BEC.[11−13] Recently, the experiment of $S = 3$ chromium atoms condensates has also been implemented.[14] Hence, the experiment paved the way for researching into the physical phenomenology of spinor BEC. The spin degree of freedom for UBA can be liberated in optical lattices due to its controllable potential depths and lattice constants.[15,16] Because the spin degree freedom is released, we have to consider the spin interaction between atoms when we research into the physical characters of the BECs in the optical lattices. Tin-Lun Ho, T. Ohmi and K. Machida[17,18] firstly accomplished the theory of spin-1 BEC respectively in 1998. Then Koashi and Ueda studied the theory and magnetic response of spin-2 BEC.[19−21] Based on the theory of spin-2, some physical properties are found, such as dynamics, soliton, vortex and superfluid-Mott insulator quantum phase transition of spin-2.[11,22−27] Recently, ground state and dynamical properties of $S = 3$ chromium atoms condensates have also been discussed in connection to experiment.[28]

The energy spectrum is a very important physical character of UBA in optical lattices. In this paper, we study some physical properties of the spin-2 BEC loaded in optical lattices at zero temperature. The Bogoliubov approach is used to obtain the explicit excitation energy spectra of weak interacting spin-2 Bose atoms in optical lattices. The superfluidity is explained by means of the energy spectra. We investigate the critical speeds of superfluid and find that the critical speeds are dependent on spin component and controllable by tuning the optical lattice parameters. We find the relation of the critical velocity and the on-site interaction matrix element which are associated to the scatter lengths.

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2. Bogoliubov approach and energy spectrum

Considering the hyperfine spin $F = 2$ UBA, such as ^{23}Na, ^{87}Rb, or ^{85}Rb, loaded in optical lattices, the general model Hamiltonian in second quantized form can be given as[8,17,18]

$$\begin{aligned}\hat{H} = &\sum_{\alpha}\int \mathrm{d}^3X\hat{\psi}^{\dagger}_{\alpha(X)}\left(-\frac{\nabla^2}{2m}+V_{0(X)}+V_{T(X)}\right)\hat{\psi}_{\alpha(X)}+\frac{C_0}{2}\sum_{\alpha,\beta}\int \mathrm{d}^3X\hat{\psi}^{\dagger}_{\alpha(X)}\hat{\psi}^{\dagger}_{\beta(X)}\hat{\psi}_{\beta(X)}\hat{\psi}_{\alpha(X)}\\ &+\frac{C_1}{2}\sum_{\alpha,\beta,\alpha',\beta'}\int \mathrm{d}^3X\hat{\psi}^{\dagger}_{\alpha(X)}\hat{\psi}^{\dagger}_{\beta(X)}F_{\alpha\alpha'}F_{\beta\beta'}\hat{\psi}_{\beta(X)}\hat{\psi}_{\alpha'(X)}+C_2\sum_{\alpha,\beta,\alpha',\beta'}\int \mathrm{d}^3X\hat{\psi}^{\dagger}_{\alpha(X)}\hat{\psi}^{\dagger}_{\alpha'(X)}\\ &\times\langle 2\alpha;2\alpha'|00\rangle\langle 00|2\beta;2\beta'\rangle\hat{\psi}_{\beta(X)}\hat{\psi}_{\beta'(X)},\end{aligned}\tag{1}$$

where m is the mass of a single atom, $\hat{\psi}_{(X)}$ is the atomic field annihilation operator associated with atoms in the hyperfine spin state $|F = 2,\ m_F = \alpha\rangle$ and the indices α, β, α', β' label the five spin components (α, β, α', $\beta' = 0, \pm1, \pm2$), the optical potential $V_{0(X)} = V_0(\sin^2 kx + \sin^2 ky + \sin^2 kz)$ is formed by a standing light wave, which is assumed to be the same for all five spin components, where $k = 2\pi/\lambda$ is the wave number with the wavelength λ of the laser and V_0 is the tunable depth of the potential well and the lattice constant $d = \lambda/2$. The physical quantity $V_{T(X)}$ is an additional external trapping potential, which varies slowly. C_0, C_1, and C_2 are related to s-wave scattering length a_0, a_2, and a_4 of the two collided bosons with total angular momentum 0, 2, and 4 respectively and given by

$$\begin{aligned}C_0 &= 4\pi\hbar^2(3a_4+4a_2)/7m,\\ C_1 &= 4\pi\hbar^2(a_4-a_2)/7m,\\ C_2 &= 4\pi\hbar^2(3a_4-10a_2+7a_0)/7m,\end{aligned}\tag{2}$$

and $\langle 2\alpha;2\alpha'|00\rangle$, $\langle 00|2\beta;2\beta'\rangle$ are Clebsch–Gordan coefficients with nonvanishing values:

$$\begin{aligned}&\alpha = 2, \quad \alpha' = -2, \quad \langle 2\alpha;2\alpha'|00\rangle = \sqrt{1/5},\\ &\alpha = -2, \quad \alpha' = +2, \quad \langle 2\alpha;2\alpha'|00\rangle = \sqrt{1/5},\\ &\alpha = 1, \quad \alpha' = -1, \quad \langle 2\alpha;2\alpha'|00\rangle = -\sqrt{1/5},\\ &\alpha = -1, \quad \alpha' = +1, \quad \langle 2\alpha;2\alpha'|00\rangle = -\sqrt{1/5},\\ &\alpha = 0, \quad \alpha' = 0, \quad \langle 2\alpha;2\alpha'|00\rangle = +\sqrt{1/5}.\end{aligned}\tag{3}$$

$F^i_{\alpha\alpha'}$ ($i = x, y, z$) are the (α, α') components of 5×5 spin-2 matrices satisfying the commutation relation $[F^{\alpha}, F^{\beta}] = \varepsilon_{\alpha\beta\gamma}F^{\gamma}$ with[20]

$$F^x = \frac{1}{2}\begin{pmatrix}0&2&0&0&0\\2&0&\sqrt{6}&0&0\\0&\sqrt{6}&0&\sqrt{6}&0\\0&0&\sqrt{6}&0&2\\0&0&0&2&0\end{pmatrix},$$

$$F^y = \frac{\mathrm{i}}{2}\begin{pmatrix}0&-2&0&0&0\\2&0&-\sqrt{6}&0&0\\0&\sqrt{6}&0&-\sqrt{6}&0\\0&0&\sqrt{6}&0&-2\\0&0&0&2&0\end{pmatrix},$$

$$F^z = \begin{pmatrix}2&0&0&0&0\\0&1&0&0&0\\0&0&0&0&0\\0&0&0&-1&0\\0&0&0&0&-2\end{pmatrix}.\tag{4}$$

We can transform the field operator $\hat{\psi}_{(X)}$ with the Wannier basis, which is a superposition of Bloch states:

$$\hat{\psi}_{\alpha(X)} = \sum_i \hat{a}_{\alpha i}w_{(X-X_i)},\tag{5}$$

where $w_{(X-X_i)}$ are the Wannier functions localized in the lattice site i and $\hat{a}_{\alpha i}$ counts for the bosonic annihilation operator on the lattice site i. With Eq.(5), the general Hamiltonian Eq.(1) can be reduced into the Bose–Hubbard Hamiltonian

$$\begin{aligned}\hat{H} = &-J\sum_{\langle i,j\rangle}\sum_{\alpha}\hat{a}^{\dagger}_{\alpha i}\hat{a}_{\alpha j}+\sum_i\sum_{\alpha}\varepsilon_i\hat{a}^{\dagger}_{\alpha i}\hat{a}_{\alpha i}+\frac{1}{2}U_0\sum_i\sum_{\alpha\beta}\hat{a}^{\dagger}_{\alpha i}\hat{a}^{\dagger}_{\beta i}\hat{a}_{\beta i}\hat{a}_{\alpha i}+\frac{1}{2}U_1\sum_i\sum_{\alpha\beta}\hat{a}^{\dagger}_{\alpha i}\hat{a}^{\dagger}_{\beta i}F_{\alpha\alpha'}F_{\beta\beta'}\hat{a}_{\beta i}\hat{a}_{\alpha i}\\ &+U_2\sum_i\sum_{\alpha\beta\alpha'\beta'}\hat{a}^{\dagger}_{\alpha i}\hat{a}^{\dagger}_{\alpha' i}\langle 2\alpha;2\alpha'|00\rangle\langle 00|2\beta;2\beta'\rangle\hat{a}_{\beta i}\hat{a}_{\beta' i},\end{aligned}\tag{6}$$

where

$$J = -\int \mathrm{d}^3X w^*_{(X-X_i)}\left[-\frac{\nabla^2}{2m} + V_0(X)\right] w_{(X-X_j)},$$
$$\varepsilon_i = \int \mathrm{d}^3X w^*_{(X-X_i)} V_{T(X)} w_{(X-X_i)},$$
$$U_{0(1,2)} = C_{0(1,2)} \int |w_{(x-x_i)}|^4 \mathrm{d}^3x.$$

The constant J is the hopping matrix element between adjacent sites i and j, ε_i is assumed to be of the same value ε for all lattice sites in this paper, and the $U_{0(1,2)}$ is the on-site interaction matrix element. The first term in Eq.(6) describes the strength of the spin-symmetric tunneling. The second term denotes energy offset of the lattice site i due to the external confinement. The third, fourth and fifth terms characterize three repulsion interactions between two Bose atoms on a single lattice site i respectively.

With creation and annihilation operators $\hat{a}^\dagger_k$ and $\hat{a}_k$, we transform the coordinate space to momentum space as follows:

$$\hat{a}_{\alpha i} = \frac{1}{\sqrt{N_\mathrm{S}}}\sum_k \hat{a}_{k,\alpha}\,\mathrm{e}^{\mathrm{i}kx_i},$$
$$\hat{a}^\dagger_{\alpha i} = \frac{1}{\sqrt{N_\mathrm{S}}}\sum_k \hat{a}^\dagger_{k,\alpha}\,\mathrm{e}^{-\mathrm{i}kx_i}, \tag{7}$$

where N_S is the total number of the lattice sites and x_i is the coordinate of site i. The atoms are on single orbit levels, and have a weak repulsion between bosons so that double occupancies are allowed, and the average occupation number is far from the Mott regime. The wave vector k localizes only in the first Brillouin zone. In this paper we limit our description to simple cubic lattice for tight-binding approximation (TBA). Substituting Eq.(7) into Eq.(6), one has the following Hamiltonian in momentum space:

$$\hat{H} = \sum_\alpha\sum_k \varepsilon(k)\hat{a}^\dagger_{k,\alpha}\hat{a}_{k,\alpha} + \hat{H}_\mathrm{int},$$

$$\begin{aligned}\hat{H}_\mathrm{int} = {}& \frac{U_0}{2N_\mathrm{S}}\sum_{\alpha\beta}\sum_{kpk'p'}\delta_{k+p,k'+p'}\hat{a}^\dagger_{k,\alpha}\hat{a}^\dagger_{p,\beta}\hat{a}_{p',\beta}\hat{a}_{k',\alpha}\\ &+ \frac{U_1}{2N_\mathrm{S}}\sum_{\alpha\beta\alpha'\beta'}\sum_{kpk'p'}\delta_{k+p,k'+p'}\\ &\times \hat{a}^\dagger_{k,\alpha}\hat{a}^\dagger_{p,\beta}F_{\alpha\alpha'}F_{\beta\beta'}\hat{a}_{p',\beta'}\hat{a}_{k',\alpha'}\\ &+ \frac{U_2}{N_\mathrm{S}}\sum_{\alpha\beta\alpha'\beta'}\sum_{kpk'p'}\delta_{k+p,k'+p'}\hat{a}^\dagger_{k\alpha}\hat{a}^\dagger_{p\alpha'}\langle 2\alpha;2\alpha'|00\rangle\\ &\times \langle 00|2\beta;2\beta'\rangle\hat{a}_{k',\beta}\hat{a}_{p',\beta'},\end{aligned} \tag{8}$$

where $\varepsilon(k) = \varepsilon - Jz\cos(kd)$ with z the number of nearest neighbours of each site and d is the distance between two nearest sites.

Since the number of atoms condensed in the zero-momentum state is much larger than one, we can use the approximation:

$$\begin{aligned}\sum_\alpha \hat{a}_{0,\alpha}\hat{a}^\dagger_{0,\alpha} &= \sum_\alpha \hat{a}^\dagger_{0,\alpha}\hat{a}_{0,\alpha} + 1 \approx \sum_\alpha N_{0\alpha}\\ &= N_0 \gg 1,\end{aligned} \tag{9}$$

where $N_{0\alpha}$ is the number of condensed atoms of spin-α component and N_0 is the total number of condensed atoms in the zero-momentum state. So we can replace the operator $\hat{a}_{0\alpha}$ and $\hat{a}^\dagger_{0\alpha}$ with a "c" number $\sqrt{N_{0\alpha}}$. Then we obtain

$$N_{0\alpha} = N_\alpha - \sum_{k\neq 0}\hat{a}^\dagger_{k,\alpha}\hat{a}_{k,\alpha}, \tag{10}$$

where N_α is the total number of atoms with spin-α and the second term in Eq.(10) is the small fluctuation, thus products of four boson operators are approximated as quadratic and we can ignore higher-order small terms, for example,

$$\hat{a}^\dagger_{0,\alpha}\hat{a}^\dagger_{0,\alpha}\hat{a}_{0,\alpha}\hat{a}_{0,\alpha} = N^2_{0\alpha} \approx N^2_\alpha - 2N_\alpha\sum_{k\neq 0}\hat{a}^\dagger_{k,\alpha}\hat{a}_{k,\alpha}, \tag{11}$$

$$\sum_{\alpha,\beta,k\neq 0}\hat{a}^\dagger_{0,\alpha}\hat{a}^\dagger_{k,\beta}\hat{a}_{k,\beta}\hat{a}_{0,\alpha} = \sum_{\alpha,\beta,k\neq 0}\hat{a}^\dagger_{k,\beta}\hat{a}_{k,\beta}N_{\alpha 0} = \left[\sum_\alpha\left(N_\alpha - \sum_{k\neq 0}\hat{a}^\dagger_{k,\alpha}\hat{a}_{k,\alpha}\right)\right]\sum_{\beta,k\neq 0}\hat{a}^\dagger_{k,\beta}\hat{a}_{k,\beta}. \tag{12}$$

With the above approximation, we can obtain

$$\hat{H} = \hat{H}_\mathrm{C} + \hat{H}_\mathrm{NC}, \tag{13}$$

here $\hat{H}_\mathrm{C}$ is a constant Hamiltonian while $\hat{H}_\mathrm{NC}$ is non-constant term. In order to diagonalize the Hamiltonian $\hat{H}_\mathrm{NC}$ in Eq.(13), we introduce the Bogoliubov transformation,

$$\hat{a}_{k,\alpha} = u_{k,\alpha}\hat{b}_{k,\alpha} - v_{k,\alpha}\hat{b}^\dagger_{-k,\alpha}, \qquad \hat{a}^\dagger_{k,\alpha} = u_{k,\alpha}\hat{b}^\dagger_{k,\alpha} - v_{k,\alpha}\hat{b}_{-k,\alpha}. \tag{14}$$

This transformation introduces a new set of operators. The new creation and annihilation operators obey $[\hat{b}_{k\alpha}, \hat{b}^{\dagger}_{k'\beta}] = \delta_{k,k'}\delta_{\alpha\beta}$, so that the coefficients satisfy $u^2_{k,\alpha} - v^2_{k,\alpha} = 1$. The auxiliary parameters $u_{k,\alpha}$ and $v_{k,\alpha}$ are chosen so that the coefficients of the nondiagonal terms $b^{\dagger}_{k\alpha}b^{\dagger}_{-k\alpha}$ and $b_{k\alpha}b_{-k\alpha}$ vanish in the Hamiltonian. In virtue of Eq.(14), we finally obtain the diagonalizable Hamiltonian

$$\hat{H} = E_{\rm c} + \sum_{k\neq 0,\alpha} E_{k,\alpha}\hat{b}^{\dagger}_{k,\alpha}\hat{b}_{k,\alpha} \tag{15}$$

with constant energy

$$\begin{aligned} E_c =\ & (\varepsilon - Jz)N + \frac{U_0}{2N_{\rm S}}N^2 \\ & + \frac{2U_2}{5N_{\rm S}}\left[(N_0 - 2\sqrt{N_{-1}}\sqrt{N_1} - 2\sqrt{N_{-2}}\sqrt{N_2})^2 + 8N_{00}\sqrt{N_{-2}}\sqrt{N_2} - 8N_{00}\sqrt{N_{-1}}\sqrt{N_1}\right] \\ & + \frac{U_1}{2N_{\rm S}}\left[(2N_2 - 2N_{-2} + N_1 - N_{-1})^2 + (2\sqrt{N_{-1}}\sqrt{N_{-2}} - \sqrt{6}\sqrt{N_{-1}}\sqrt{N_0}\right. \\ & + \sqrt{6}\sqrt{N_1}\sqrt{N_0} - 2\sqrt{N_1}\sqrt{N_2})^2 + 8\sqrt{6}N_{-10}\sqrt{N_{-2}}\sqrt{N_0} + 8\sqrt{6}N_{10}\sqrt{N_2}\sqrt{N_0} \\ & \left. + 24N_{00}\sqrt{N_1}\sqrt{N_{-1}} + 16\sqrt{N_1}\sqrt{N_2}\sqrt{N_{-1}}\sqrt{N_{-2}}\right], \end{aligned}$$

and the energy spectra $E_{k,\alpha}$ ($\alpha = 0, \pm 1, \pm 2$) of the quasiparticles are

$$E_{k,\pm 2} = \sqrt{\bar{\varepsilon}(k)\left[2n_{\pm 2}(U_0 + 4U_1) + \bar{\varepsilon}(k)\right]}, \tag{16}$$

$$E_{k,\pm 1} = \sqrt{\bar{\varepsilon}(k)\left[\bar{\varepsilon}(k) + 2n_{\pm 1}(U_0 + U_1) + 4\sqrt{6}U_1\sqrt{n_0}\sqrt{n_{\pm 2}}\right]}, \tag{17}$$

$$E_{k,0} = \sqrt{\bar{\varepsilon}(k)\left[\bar{\varepsilon}(k) + 2n_0U_0 + \frac{4}{5}n_0U_2 + 12U_1\sqrt{n_1}\sqrt{n_{-1}} + \frac{8}{5}U_2\sqrt{n_2}\sqrt{n_{-2}} - \frac{8}{5}U_2\sqrt{n_1}\sqrt{n_{-1}}\right]}, \tag{18}$$

where $\bar{\varepsilon}(k) = zJ[1 - \cos(kd)]$ and $n_\alpha = N_\alpha/N_{\rm S}$ is the average atom number of spin-α component per lattice site. If $U_2 = 0$ and $n_{\pm 2} = 0$, equations (17) and (18) reduce the energy spectra given in Ref.[29] in the case of spin-1, except that there is a different constant factor of U_1 term in Eq.(18), mainly because there are different matrix elements of spin-2 corresponding to spin-1.

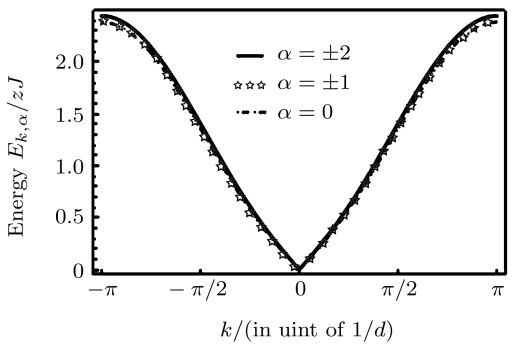


Fig.1. The spectra $E_{k,\alpha}$ of the spin-α component in unit of zJ. The parameters are $U_0 = 0.1zJ$, $U_1 = 0.01zJ$, $U_2 = 0.01zJ$, $n_0 = 3$, $n_{\pm 1} = 3$, and $n_{\pm 2} = 3$ respectively.

The scattering lengths of ^{23}Na atoms are $a_0 = 34.9 \pm 1.0$ a.u., $a_2 = 45.8 \pm 1.0$ a.u. and $a_4 = 64.5 \pm 1.0$ a.u.,[19] therefore U_0 is one order of magnitude larger than U_1 or U_2 for ^{23}Na atom of spin-2. Choosing the typical values $U_0 = 0.1zJ$, $U_1 = 0.005zJ$ and $U_2 = -0.001zJ$, we give the diagrams of energy spectra of quasi-particles in Figs.1 and 2. The average particle number of each spin component is $n_0 = 3, n_{\pm 1} = 3$, and $n_{\pm 2} = 3$ respectively in Fig.1. From Fig.1, one finds that for the same average atom number n_α of different spin-α component per lattice site, the relation

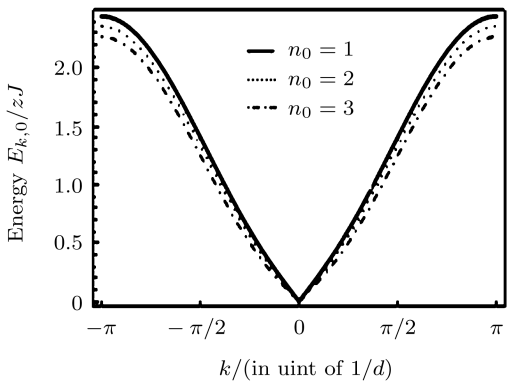


Fig.2. The spectra $E_{k,0}$ of the spin $\alpha = 0$ component in unit of zJ. The parameters n_0 are 1, 2 and 3 respectively, the other parameters are given in Fig.1.

of the different energies $E_{k,\alpha}$ ($\alpha = 0, \pm 1, \pm 2$) of the quasiparticle is $E_{k,0} > E_{k,,\pm 1} > E_{k,,\pm 2}$; for different spin-$\alpha$ component, however, the difference of energies is small at given momentum k. When k approaches zero, the energies $E_{k,,\alpha}$ ($\alpha = 0, \pm 1, \pm 2$) of the quasiparticle approach zero. The energy spectra $E_{k,\alpha}$ with increase of the wave vector k. The energy spectra $E_{k,0}$ of the quasiparticle of spin-0 component increase with increasing the average atom number of per lattice site n_0 (see Fig.2).

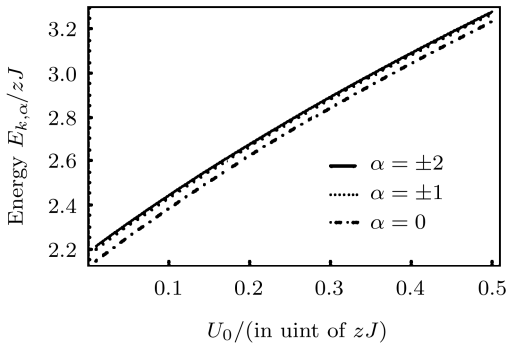


Fig.3. The diagram of spectra $E_{k,\alpha}$ of the spin-α component vs. U_0 in units of zJ. Here $kd = \pi$, the other parameters are given in Fig.1.

Figures 3 and 4 show the diagrams of the spectra $E_{k,\alpha}$ vs. U_0 and U_1 respectively. One sees that the excitation energy increases with increasing interacting strength U_0 and U_1 respectively. From Fig.4, we find that the difference of the energies between spin-2 component and spin-1 or spin-0 component becomes larger with increasing U_1, but the difference between spin-1 and 0 components is small.

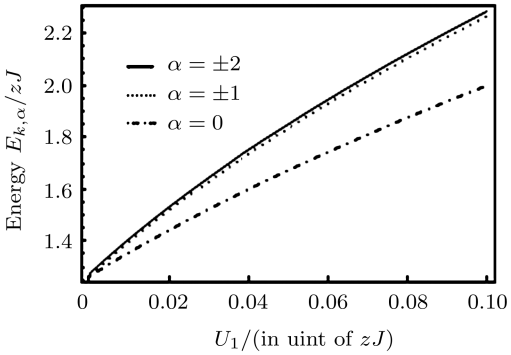


Fig.4. The diagram of spectra $E_{k,\alpha}$ of the spin-α component vs. U_1 in units of zJ. Here $kd = \pi$, the other parameters are given in Fig.1.

3. Critical velocity of superfluid

The energy spectra Eqs.(16)–(18) have typical superfluidity. When $k \to 0$, one can obtain the critical speeds of superfluid from the energy spectra:

$$v_{s,\pm 2} = \left(\frac{\partial E_{k,\pm 2}}{\partial k}\right)_{k\to 0} = \sqrt{zJd^2(n_{\pm 2}U_0 + 4n_{\pm 2}U_1)}, \tag{19}$$

$$v_{s,\pm 1} = \left(\frac{\partial E_{k,\pm 1}}{\partial k}\right)_{k\to 0} = \sqrt{zJd^2\left[n_{\pm 1}U_0 + n_{\pm 1}U_1 + 2\sqrt{6}U_1\sqrt{n_0}\sqrt{n_{\pm 2}}\right]}, \tag{20}$$

$$v_{s,0} = \left(\frac{\partial E_{k,0}}{\partial k}\right)_{k\to 0} = \sqrt{zJd^2\left[n_0U_0 + \frac{2}{5}n_0U_2 + 6U_1\sqrt{n_1}\sqrt{n_{-1}} + \frac{4}{5}U_2\sqrt{n_2}\sqrt{n_{-2}} - \frac{4}{5}U_2\sqrt{n_1}\sqrt{n_{-1}}\right]}. \tag{21}$$

According to the Landau criterion, the nonvanishing critical velocity indicates the existence of superfluid phases. From the above Eqs.(19)–(21), one finds that the critical speeds of superfluid is determined by J and $U_{0(1,2)}$which are related to the strength of optical lattice potential and interaction. We can control J and $U_{0(1,2)}$ by adjusting the laser parameters or scattering length by Feshbach resonance to satisfy $U_0 \ll J$ which is the requirement for using Bogliubov transformation and realizing superfluidity. Since U_1 or U_2 is much less than U_0, it can ensure the existence of nonvanishing critical speeds.

Figures 5 and 6 show the diagrams of critical speed vs. U_0 and U_1 respectively. The critical speeds of spin components increase with the increase of interacting strengths U_0 and U_1. But the differences are small between three speeds when U_0 increases; the difference becomes larger between spin-2 component and spin-0 or 1 component when U_1 increases. From Figs.5 and 6, we find that U_1 has a larger influence than U_0 on the critical speeds.

For the same average atom number n_α of different spin-α component per lattice site, the relation of the different critical speeds is $v_{s,0} > v_{s,\pm 1,} > v_{s,\pm 2}$. The critical speeds of spin-α component increase with the increase of n_α. The parameters J and U_0 are determined by potential of the optical lattice essentially. The critical speeds of superfluid increase with the increase of parameter U_0/zJ, but, the superfluid vanishes when the parameter U_0/zJ increases to a critical value. The boson atoms in an optical lattice transform from superfluid phase to Mott insulator phase.

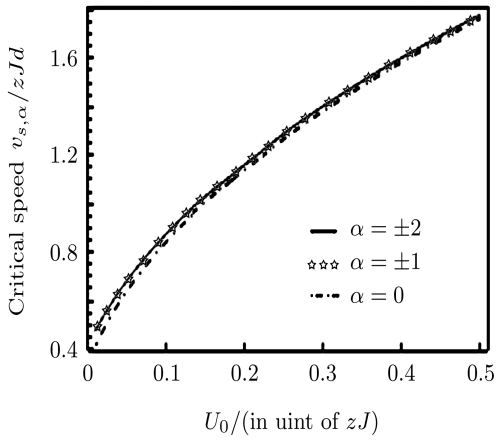


Fig.5. The diagram of critical speed $v_{s,\alpha}$ (in units of zJd) of the spin-α component vs. U_0 in units of zJ. The other parameters are given in Fig.1.

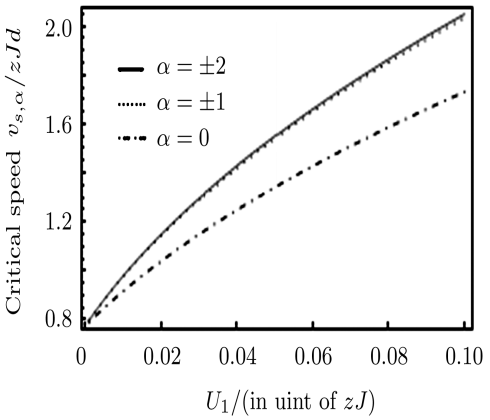


Fig.6. The diagram of critical speed $v_{s,\alpha}$ (in units of zJd) of the spin-α component vs. U_1 in units of zJ. The other parameters are given in Fig.1.

The critical speeds of superfluid are functions of densities for five spin components. Therefore, we can separate the different spin component atoms in experiment. The critical speeds of superfluid can be detected experimentally with a similar technique as used in Ref.[30] to measure the sound speed of a BEC in a trap or by counting the atom-number populations.[31] Another possible method is to employ Bragg spectroscopy[32,33] to the excitation spectrum. The critical speed can be extracted from the slope of the linear part of the excitation spectrum.

4. Conclusion

By means of the Bogoliubov method, we have investigated the energy spectra of spin-2 UBA in optical lattices. The effect of optical lattice on the Bloch energy and critical speeds of superfluid are studied. The excitation energy spectra of spin-2 ultracold bosons in optical lattices exhibits a typical superfluid phase from the viewpoint of the Landau criterion. The critical speeds of the superfluid depend on the spin-component and can be controlled by adjusting the laser standing beams. In addition, we found that the component separation of spin Bose atoms in optical lattices is possible because of the component dependence of speeds. According to the obtained results, one can estimate the critical speeds of the superfluid and energy spectra and moreover, the superfluid of spin-2 Bose atoms in optical lattices can be observed experimentally by modulating proper experimental parameters.

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