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M-ary information processing via an optimal nonlinear detector*

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This paper presents a novel approach of M-ary baseband pulse amplitude modulated signal processing via a parameter-optimized nonlinear dynamic system. This nonlinear system usually shows the phenomenon of stochastic resonance by adding noise. To thoroughly discuss the signal processing performance of the nonlinear system, we tune the system parameters to obtain a nonlinear detector with optimal performance. For characterizing the output of the nonlinear system, the derivation of the probability of detection error is given by the system response speed and the probability density function of the nonlinear system output. By varying the noise intensity with fixed system parameters, the phenomenon of stochastic resonance is shown and by tuning the system parameters with fixed noise, the probability of detection error is minimized and the nonlinear system is optimized. The detection performance of the two cases is compared with the theoretical probability of detection error, which is validated by numerical simulation.

Keywords: nonlinear detector, stochastic resonance, tuning parameters, probability of detection error
PACC: 0540, 0250, 0547

1. Introduction

Nonlinear dynamical systems that originate from physical fields present rich potentialities for signal and information processing. One of the interesting phenomena which attracts great attention of specialists on nonlinear dynamics is stochastic resonance (SR) (see Refs.[1] and [2] for overviews), where noise plays a constructive role in a variety of systems. One of the earliest and most systems studied for SR, is the nonlinear bistable system governed by a double-well potential. Since it was originally put forward in 1981, many researchers have further analysed the potentialities of such nonlinear systems in various fields.[1,2] Particularly, in signal processing some interesting results were obtained,[3−14] which are important for engineers, especially in situations where the possibilities of linear systems have been exhausted. Recently, the system parameters have been viewed as an important tunable element for the study of SR, and parameter-induced SR was shown.[15,16] To thoroughly discuss the detection performance of the nonlinear system, in this paper the system parameters are tuned to optimize the detection performance, which is somehow the classical way of optimizing a tunable filter according to the criterion (such as minimum mean square error).

Detection of binary information via a nonlinear detector was discussed in many papers, and some researchers gave quite good theoretical and numerical results in measuring the detection performance of the nonlinear system.[8,11,17] However, for M-ary signal detection, few work has been reported, because the derivation of the quantity for measuring the detection performance of the nonlinear system is more complicated.

In this paper, we use the nonlinear dynamic system as a nonlinear detector. The derivation of the probability of detection error is given by using the system response speed and the probability density function (PDF) of the nonlinear system output, the latter is used to characterize the output of the nonlinear system with the input M-ary baseband pulse amplitude modulated (PAM) signal and band-limited Gaussian white noise. To discuss in detail the detection performance of the nonlinear dynamic system, instead of tuning the level of the noise with the system parameters fixed, we tune the system parameters to optimize the detection performance at a fixed noise level. Compared with tuning noise, tuning parameters may be more realizable, especially when the input noise intensity exceeds the resonance level, or when we do

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not know the characteristic of the noise (generally, it is non-Gaussian). Moreover, the detection performance of SR and the optimal detector are compared with each other. Such a comparison is important for the application of nonlinear systems to signal processing. Finally, the theoretical results are validated by numerical simulation.

2. Problem formulation

Consider the conventional system in which SR was revealed, i.e. a nonlinear dynamic system governed by the quartic potential. With the presence of input signal and noise, it can be described as

$$\dot{x} = ax - bx^3 + s(t) + \Gamma(t), \quad a, b > 0, \tag{1}$$

where a and b are usually given in terms of system parameters, $s(t)$ is the input pseudo-random M-ary baseband PAM signal. $\Gamma(t)$ denotes a zero mean Gaussian white noise with correlation function

$$\langle \Gamma(0)\Gamma(t)\rangle = 2D\delta(t), \tag{2}$$

in which D is the noise intensity. For an M-ary baseband PAM signal, we introduce hypotheses

$$H_m : s(t) = A_m, \quad \text{with prior probability } P_m, \quad m = 1, 2, \ldots, M, \tag{3}$$

where $s(t)$, for $(n-1)T_b \le t \le nT_b$ and $n = 1, 2, \ldots, N$, has the bit interval T_b and amplitude values $A_m = (2m - 1 - M)d$. At each time multiple of the pulse duration T_b, the output of the system $x(t)$ is read by making a decision

$$D_m : s(t) = A_m, \quad \text{if} \quad x(t) \in R_m, \quad m = 1, 2, \ldots, M, \tag{4}$$

where R_m is the accepted region corresponding to A_m, and is divided by decision levels. The performance of the nonlinear system is measured by the probability of detection error[18]

$$P_{\text{er}} = \sum_{j=1}^{M} \sum_{\substack{i=1 \\ i\neq j}}^{M} P_i P(D_j|H_i). \tag{5}$$

3. Quantitative observations of the nonlinear detector with M-ary PAM signal

Assume the signal levels are statistically independent and equiprobable (i.e., $P_i = 1/M, i = 1, 2, \ldots, M$), so equation (5) becomes

$$P_{\text{er}} = \frac{1}{M} \sum_{j=1}^{M} \sum_{\substack{i=1 \\ i\neq j}}^{M} P(D_j|H_i). \tag{6}$$

In each bit interval T_b, system (1) is subjected to the constant amplitude A_m, with an additive Gaussian white noise (the band-limited noise can be approximated to white noise as the noise bandwidth is much larger than the bit rate $1/T_b$). Under these conditions, the statistically equivalent description for the corresponding probability density $\rho(x, t|A_m)$ is governed by the Fokker–Planck equation (FPE)[19]

$$\frac{\partial \rho(x,t|A_m)}{\partial t} = -\frac{\partial}{\partial x}[V'_{A_m}(x)\rho(x,t|A_m)] + \frac{D}{2}\frac{\partial^2 \rho(x,t|A_m)}{\partial x^2}, \quad m = 1, 2, \ldots, M, \tag{7}$$

where $V'_{A_m}(x)$ is the derivative of $V_{A_m}(x)$ with respect to x, and

$$V_{A_m}(x) = \frac{a}{2}x^2 - \frac{b}{4}x^4 + A_m x. \tag{8}$$

In Eq.(7), the PDF $\rho(x, t|A_m)$ can be approximately calculated from the FPE, which is discussed in the Appendix.

Figure 1 shows the sketch of probability density functions for $A_m, m = 1, 2, \ldots, M$. According to the maximum likelihood ratio decision rule,[18] the optimal decision levels $x_m, m = 1, 2, \ldots, M-1$ can be evaluated by solving the equations

$$\rho(x, t|A_m) = \rho(x, t|A_{m+1}), \quad m = 1, 2, \ldots, M-1. \tag{9}$$

Because of the symmetry, $x_m = x_{M-m}$ and $\rho(x, t|A_m) = \rho(x, t|A_{M-m})$ for $m < M/2$. Assume $P(D_j|H_i)$ with $|i-j| \ge 2$ (in this case, $|A_i - A_j| \ge 4d$) is very small and can be neglected. Hence, the probability of detection error P_{er} in Eq.(6) can be expressed as

$$P_{\text{er}} = \frac{1}{M}\Big\{ P_{A_1}(x_1 \to +\infty) + \sum_{m=2}^{M-1} [P_{A_m}(-\infty \to x_{m-1}) + P_{A_m}(x_m \to +\infty)] + P_{A_M}(x_{M-1} \to +\infty)\Big\}, \tag{10}$$

where

$$P_{A_m}(A \to B) = \int_A^B \rho(x, t|A_m)\mathrm{d}x, \quad m = 1, 2, \ldots, M, \tag{11}$$

in which A and B denote the lower limit and the upper limit, respectively.[18] For example, when $M = 2$, P_{er} is

$$P_{\mathrm{er}} = \int_0^{+\infty} \rho(x, i \cdot T_b|A_1)\mathrm{d}x, \tag{12}$$

where $A_1 = -A_2 = -d$. When $M = 4$, P_{er} is

$$\begin{aligned} P_{\mathrm{er}} &= \frac{1}{4}\{2P_{A_1}(x_1 \to +\infty) + 2[P_{A_2}(-\infty \to x_1) \\ &\quad + P_{A_2}(x_2 \to +\infty)]\} \\ &= \frac{1}{2}[P_{A_1}(x_1 \to +\infty) + P_{A_2}(-\infty \to x_1) \\ &\quad + P_{A_2}(x_2 \to +\infty)], \end{aligned} \tag{13}$$

where $x_2 = 0$ is easily obtained according to the likelihood ratio criterion.

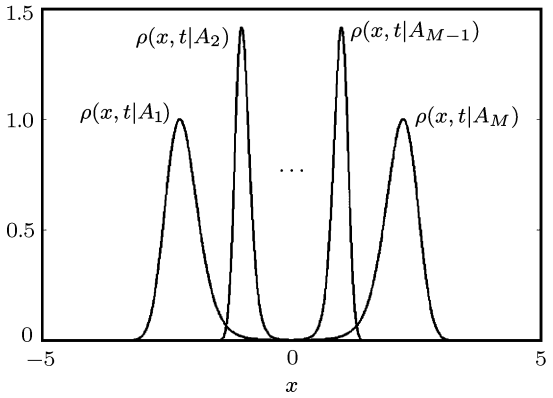


Fig.1. Sketch of probability density functions for $A_m, m = 1, 2, \ldots, M$.

For a pseudo-random M-ary PAM signal, assume the current symbol takes the amplitude A_m. If the number of the successive amplitudes with the same symbol is K ($K = 0, 1, \ldots$), and the amplitude of the $(K+1)$-th symbol (after $(K+1)T_b$ time interval) is $A_n, m \neq n$, the probability of detection error for the i-th (here, $0 < i \leq (K+1)$) amplitude is

$$P_i(A_l|A_m) = \int_A^B \rho(x, i \cdot T_b|A_m)\mathrm{d}x, \quad l = m-1 \ \text{ or } \ l = m+1. \tag{14}$$

Since the input amplitudes are statistically independent and equiprobable, the probability that the same symbol continuously transmits i times is $1/M^i$. For example, with $M = 2$ equation (12) can be expressed as

$$\begin{aligned} P_{\mathrm{er}} &= \frac{1}{2}\int_0^{\infty} \rho(x, T_b|A_1)\mathrm{d}x + \frac{1}{4}\int_0^{\infty} \rho(x, 2T_b|A_1)\mathrm{d}x \\ &\quad + \frac{1}{8}\int_0^{\infty} \rho(x, 3T_b|A_1)\mathrm{d}x + \cdots \\ &= \sum_{i=1}^{\infty} \frac{1}{2^i}\int_0^{\infty} \rho(x, i \cdot T_b|A_1)\mathrm{d}x. \end{aligned} \tag{15}$$

In the following calculation of P_{er}, we take $i = 1, 2, 3$ (i.e., consider the probability that the same symbol continuously transmits $i \leq 3$ times), is

$$\begin{aligned} P_{\mathrm{er}} &= \frac{1}{2}\int_0^{\infty} \rho(x, T_b|A_1)\mathrm{d}x + \frac{1}{4}\int_0^{\infty} \rho(x, 2T_b|A_1)\mathrm{d}x \\ &\quad + \frac{1}{4}\int_0^{\infty} \rho(x, 3T_b|A_1)\mathrm{d}x, \end{aligned} \tag{16}$$

because the prior probability and the probability of detection error with $i > 3$ are small and the corresponding terms could be neglected. For example, as $i = 4$, the prior probability is $1/8$ and the decaying exponential in $\rho(x, 4T_b|A_1)$ is $\exp(-4\lambda_1 T_b)$ (see Appendix A), which leads to very low contribution of the term $\frac{1}{8}\int_0^{\infty} \rho(x, 4T_b|A_1)\mathrm{d}x$ to P_{er}. In the above equation, the coefficient of the third term on the right is taken as $1/4$ so that the sum of all the coefficients is 1. Similarly, P_{er} for $M = 4$ derived from Eq.(13) is

$$\begin{aligned} P_{\mathrm{er}} &= \sum_{i=1}^{\infty} \frac{1}{2^{i+1}}\int_{x_1}^{\infty} \rho(x, i \cdot T_b|A_1)\mathrm{d}x \\ &\quad + \sum_{i=1}^{\infty} \frac{1}{2^{i+1}}\int_{-\infty}^{x_1} \rho(x, i \cdot T_b|A_2)\mathrm{d}x \\ &\quad + \sum_{i=1}^{\infty} \frac{1}{2^{i+1}}\int_{x_2}^{\infty} \rho(x, i \cdot T_b|A_2)\mathrm{d}x. \end{aligned} \tag{17}$$

4. The optimal nonlinear detector with M-ary PAM signal

To discuss in detail the detection performance of the nonlinear system, instead of tuning the level of the noise with the system parameters fixed to produce SR, we tune the system parameters in order to optimize the detection performance at a fixed noise level. When the input noise intensity exceeds the resonance level, or when we do not know the statistical characteristic of the noise (generally, it is non-Gaussian), tuning system parameters a and b may be more realizable.

Assume that the signal is known and the noise intensity can be measured, so M, d, T_b, and D are fixed, and P_{er} is only a function of system parameters a and b. We can tune a and b to minimize P_{er} and

obtain the optimal system parameters. The probability of detection error for this detector with optimal parameters can be expressed as

$$\min_{\substack{a,b \\ M,d,T_b,D=\text{const.}}} P_{\text{er}}(M,d,a,b,T_b,D) = P_{\text{er}}^{\text{Op}}(M,d,T_b,D). \quad (18)$$

The method of tuning parameters a and b was discussed in detail in Ref.[20]. Figure 2 presents the quantitative influence of parameters a and b of the nonlinear detector on the probability of detection error, theoretically evaluated with the help of Eq.(16). P_{er} is plotted as a function of a and b, and the surface presents a valley shape. This shows that for a given a or b there exists a unique optimal b or a in minimizing P_{er}. By theoretical calculation, we find that in the range of $0 < a < 1$, P_{er} corresponding to optimal pairs of a and b varies slightly and is almost minimal. So, for simplifying calculation, generally we take $a = 0.1$ as one of the two optimal parameters of the nonlinear detector and then optimize b as the other one.

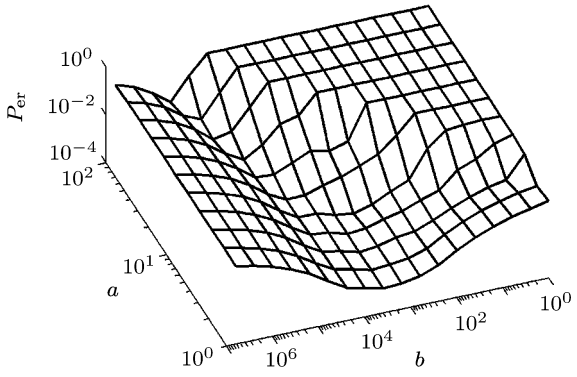


Fig.2. P_{er} as a function of a and b with $M = 2$, $d = 1$ mV, $T_b = 0.1$ mV^{-2}, and $D = 0.005$ mV^2, theoretically evaluated with the help of Eq.(16).

Figure 3 depicts $P_{\text{er}}^{\text{Op}}$ as a function of the noise intensity D with the input baseband quaternary PAM signal ($M = 4$, $d = 1$ mV and $T_b = 1$ s), where for each D, the corresponding $P_{\text{er}}^{\text{Op}}$ is optimized according to Eq.(18) and the theoretical results (solid lines with cross data points) are evaluated from Eq.(17). For comparison, the results of SR is also shown (solid lines with circle data points), where, to produce the phenomenon of SR, we take $a = 50$ mV and $b = 5000$ mV^{-2}. The theoretical results of SR is evaluated from the following formula,

$$\min_{\substack{D \\ M,d,T_b,a,b=\text{const.}}} P_{\text{er}}(M,d,a,b,T_b,D) = P_{\text{er}}^{\text{SR}}(M,d,T_b,a,b). \quad (19)$$

As presented in the figure, the detection performance of the nonlinear system with optimal parameters (a and b, under the condition of fixed D) is apparently better than that of the system for exhibiting SR with $a = 50$ mV and $b = 5000$ mV^{-2}.

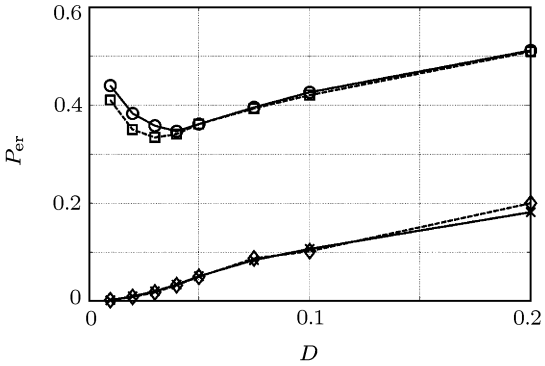


Fig.3. The probability of detection error P_{er} as a function of the noise intensity D with the input baseband quaternary PAM signal given ($M = 4$, $d = 1$ mV and $T_b = 1$ s). Solid line with circles: theoretical P_{er} with fixed parameters and varying D. Dashed line with squares: simulated P_{er} with fixed parameters and varying D. Solid line with crosses: theoretical $P_{\text{er}}^{\text{Op}}$ with optimal nonlinear detector for each fixed D. Dashed line with diamonds: simulated $P_{\text{er}}^{\text{Op}}$ with optimal nonlinear detector for each fixed D.

From the previous discussion we find that P_{er} depends on three parameters, i.e., a, b and D. Then, it is natural to tune the three parameters at the same time to achieve a minimal P_{er}. In this case, SR is produced by varying D and the system parameters a and b are optimal for the varying D. As is known the condition that SR can be realized is signal amplitude $A < \sqrt{4a^3/27b}$. In fact, the condition is not necessary to be strictly satisfied. Some researchers reported that system (1) could still produce residual SR[21,22] with the signal amplitude A being a little greater than $\sqrt{4a^3/27b}$. Therefore, with the above condition (SR is realized) it is possible to obtain the minimal P_{er} by tuning a, b and increasing D. Moreover, this P_{er} may be lower than the one obtained by tuning a and b with fixed D and it is valuable to discuss this problem in detail in the future work.

To validate the theoretical results, the numerical simulation is carried out by using simulation software MATLAB. The block scheme for simulating the process of the signal processing is presented in Fig.4. In the figure, the dashed-frame A simulates the nonlinear system, which was described in Ref.[1]. The input of the system is the pseudo-random PAM signal and

the band-limited Gaussian white noise. The relationship between the noise intensity D and the noise rms amplitude σ is[18]

$$2D = \sigma^2 \Delta t, \tag{20}$$

where Δt is the noise sampling time. The band-limited noise can be viewed as white noise on condition that the noise sampling time satisfies $\Delta t \ll T_b$. In the numerical simulation, Δt is taken to be 0.001 s, which satisfies $\Delta t \ll T_b$ ($T_b = 1$ s), so that the theoretical results based on the additive white noise input are effectively checked. The values of a, b and d are the same as those in the theoretical examples. The number of Monte Carlo simulation is 10000.

The results are also shown in Fig.4. It is seen from the figure that the theoretical results are almost in accordance with the numerical results. However, at small values of D, theoretical $P_{\rm er}$ with varying D (solid line with circles) is slightly greater, because at these values, the system response speeds are too low (less than three multiples of $1/T_b$. According to calculation, the system response speed increases with increasing D). In Eq.(15) we take the coefficient of the third term as $\frac{1}{4}$ (i.e., Eq.(16)). With very low system response speeds, this approximation will lead to large error, so more terms should be included.

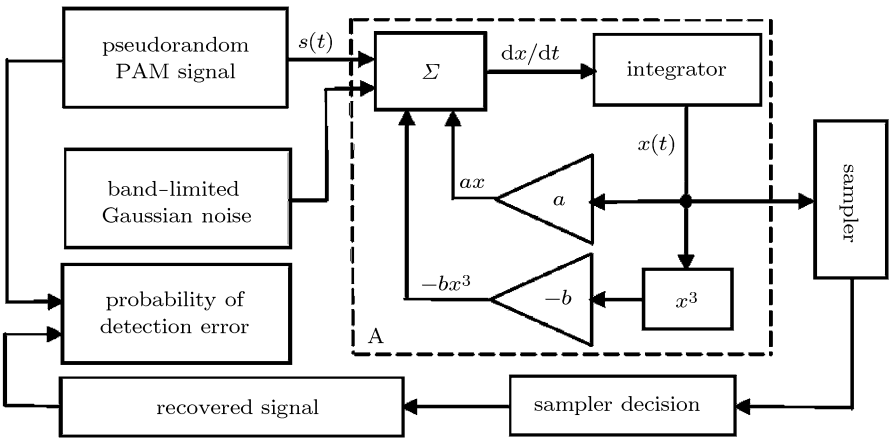


Fig.4. The block scheme for simulating the process of the baseband M-ary PAM signal processing.

5. Conclusions

We have investigated the optimal detection performance of a nonlinear dynamic system with the input M-ary PAM signal in the presence of Gaussian band-limited white noise. The theoretical quantity (the probability of detection error) used to evaluate the detection performance of the nonlinear system was given, which was deduced via the system response speed and the PDF of the nonlinear system output. Besides, to demonstrate the theoretical results, the numerical simulation was put forward. The figures show a good match between the theoretical and numerical simulation results.

By tuning system parameters a and b with fixed noise intensity D, the optimal detection performance of the nonlinear dynamic system $P_{\rm er}^{\rm Op}$ could be evaluated. By adjusting the noise intensity D with fixed system parameters a and b, the system shows the phenomenon of SR, where $P_{\rm er}^{\rm SR}$ can be obtained. Comparing $P_{\rm er}^{\rm Op}$ with $P_{\rm er}^{\rm SR}$ under the simulation condition, we found that the former was much lower than the latter. In fact, if more parameters are adjustable, with a parameter transformation, SR with adding noise and fixed parameters can be viewed as a special case of tuning parameters with fixed noise.[15,16] Moreover, tuning parameters may be more realizable, especially when the input noise intensity exceeds the resonance level, or when we do not know the statistical characteristic of the noise (generally, it is non-Gaussian).

Appendix: Derivation of the PDF in FPE

Assume that a solution of Eq.(7) is

$$\rho(x,t|A_m)=\psi(x,t)\exp\left[\frac{1}{2D}V_{A_m}(x)\right]. \tag{A1}$$

Substituting Eq.(A1) into Eq.(7) gives

$$\frac{1}{D}\frac{\partial\psi(x,t)}{\partial t}=\frac{\partial^2\psi(x,t)}{\partial x^2}-\left[\frac{1}{2D}V''_{A_m}(x)+\frac{1}{4D^2}V'^2_{A_m}(x)\right]\psi(x,t). \tag{A2}$$

Define $\psi(x,t)$ as

$$\psi(x,t)=u(x)\exp(-\lambda t). \tag{A3}$$

Putting $\psi(x,t)$ into Eq.(A2) yields

$$-\lambda\frac{u(x)}{D}=u''(x)-\left[\frac{1}{2D}V''_{A_m}(x)+\frac{1}{4D^2}V'^2_{A_m}(x)\right]u(x). \tag{A4}$$

This is an eigenvalue problem of the differential operator (A2). Obviously, the right-hand side of Eq.(A2) may be viewed as a self-conjugate differential operator acting on $\psi(x,t)$, so the eigenvalues of Eq.(A4) are real and the eigenfunctions are orthogonal with each other. Multiplying both sides of Eq.(A4) by $u(x)$ and integrating it from minus infinity to infinity with respect to $\lim_{x\to\pm\infty}u(x)=\lim_{x\to\pm\infty}u'(x)=0$, the equation can be transformed into a Rayleigh quotient form, i.e.,

$$\lambda=\underset{\neq 0}{\text{st.}}\frac{\displaystyle\int_{-\infty}^{+\infty}\left\{\left[\frac{1}{2D}V''_{A_m}(x)+\frac{1}{4D^2}V'^2_{A_m}(x)\right]u^2(x)+u'^2(x)\right\}\mathrm{d}x}{\displaystyle\int_{-\infty}^{+\infty}\frac{u^2(x)}{D}\mathrm{d}x}, \tag{A5}$$

where st. denotes the stationary value in variational problems. The evaluation of λ was introduced in Ref.[20]. The minimum eigenvalue $\lambda_0=0$ and the corresponding eigenfunction $u_0(x)=\exp[V_{A_m}(x)/2D]$, so $\rho_0(x,t|A_m)=\exp[V_{A_m}(x)/D]$ is the steady state solution. The minimum nonzero eigenvalue λ_1 is the system response speed. The solution corresponding to eigenvalue λ_j for $j=1,2,\ldots$ is

$$\begin{aligned}\rho_j(x,t|A_m)&=u_j(x)\exp\left[\frac{1}{2D}V_{A_m}(x)\right]\exp(-\lambda_j t)\\&\triangleq\varphi_j^{A_m}(x)\exp(-\lambda_j t).\end{aligned} \tag{A6}$$

Therefore, the complete solution of Eq.(7) can be expressed as

$$\rho(x,t|A_m)=\sum_{j=0}^{\infty}c_j\varphi_j^{A_m}(x)\exp(-\lambda_j t), \tag{A7}$$

where $\varphi_0^{A_m}(x)=\exp[V_{A_m}(x)/D]$, c_0 can be obtained by the normalization condition of the steady state probability density and constant coefficients c_j's ($j=1,2,\ldots$) can be determined by the initial condition and the orthogonality of the eigenfunctions.

Assume a bit symbol takes the level A_1 and after $t=iT_b$, the level A_m ($m\neq 1$) is transmitted. Using Eq.(A7) we have

$$\begin{aligned}\rho(x,t|A_1)\simeq&\sum_{j=0}^{n}c_j\varphi_j^{A_1}(x)\exp(-\lambda_j t)\\&+Q(x)\exp(-\lambda_{n+1}t)\\&+O(\mathrm{e}^{-\lambda_{n+2}t}),\end{aligned} \tag{A8}$$

where $Q(x)\exp(-\lambda_{n+1}t)$ is the truncated term. Here, we assume $\lambda_1<\lambda_2<\cdots<\lambda_{n+2}$. The above equation fulfills the initial condition

$$\rho(x,0|A_1)=\rho_0(x|A_m), \tag{A9}$$

where A_m ($n\neq 1$) is the level before A_1, and $\rho_0(x|A_m)$ is the steady state solution with the input level A_m. Hence, omitting the trace quantity $O(\mathrm{e}^{-\lambda_{n+2}t})$, from Eqs.(A8) and (A9), $Q(x)$ can be obtained

$$Q(x)=\rho_0(x|A_m)-\sum_{j=0}^{n}c_j\varphi_j^{A_1}(x). \tag{A10}$$

Substituting Eq.(A10) into Eq.(A8) gives

$$\rho_n(x,t|A_1) \simeq \sum_{j=0}^{n} c_j \varphi_j^{A_1}(x) \exp(-\lambda_j t) + \left[\rho_0(x|A_m) - \sum_{j=0}^{n} c_j \varphi_j^{A_1}(x)\right] \exp(-\lambda_{n+1} t). \tag{A11}$$

As is assumed in Section 3, the input levels are statistically independent and equiprobable, so the input probability of the level A_m is $1/M$. Consequently, the approximate form of the PDF $\rho(x,t|A_1)$ is

$$\rho(x,t|A_1) \simeq \sum_{j=0}^{n} c_j \varphi_j^{A_1}(x) \exp(-\lambda_j t) + \sum_{m=2}^{M} \frac{1}{M} \left[\rho_0(x|A_m) - \sum_{j=0}^{n} c_j \varphi_j^{A_1}(x)\right] \exp(-\lambda_{n+1} t). \tag{A12}$$

Obviously, with the system approaching the steady state (i.e., $t \to +\infty$), the above equation is still satisfied. Similarly, the approximate expression of $\rho(x,t|A_i)$ with $i = 2, 3, \ldots, M$ is

$$\rho(x,t|A_i) \simeq \sum_{j=0}^{n} c_j \varphi_j^{A_i}(x) \exp(-\lambda_j t) + \sum_{m=1}^{M} \frac{1}{M} \left[\rho_0(x|A_m) - \sum_{j=0}^{n} c_j \varphi_j^{A_i}(x)\right] \exp(-\lambda_{n+1} t). \tag{A13}$$

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