Vol 18 No 12, December 2009
1674-1056/2009/18(12)/5179-10

Chinese Physics B

© 2009 Chin. Phys. Soc.
and IOP Publishing Ltd

Entanglement of two atoms in two-mode Raman coupled model with intrinsic decoherence*

Zhang Jian(张 剑)†, Shao Bin(邵 彬), and Zou Jian(邹 健)

Department of Physics, School of Sciences, Beijing Institute of Technology, Beijing 100081, *China*

(Received 21 February 2009; revised manuscript received 8 June 2009)

Considering intrinsic decoherence, the two-atom two-mode Raman coupled model is investigated in this paper. Utilizing the constants of motion in this model, we obtain the analytic expressions of the density operator of the system for investigating the entanglement of two atoms. The speed of entanglement decay increases with the increasing of the coupling coefficient of one atom. The difference between the oscillation periods when the initial state parameter of atomic subsystem belongs to two intervals becomes smaller with the increasing of the coupling coefficient of one atom. The increasing of the initial photon number of the second field can hasten the vanishing of entanglement of atomic subsystem. The robustness of atomic entanglement against decoherence depends on the interval of the initial state parameter of atomic subsystem.

Keywords: entanglement, intrinsic decoherence, two-atom two-mode Raman coupled model
PACC: 0367, 3280

1. Introduction

Since the early years of quantum mechanics, entanglement has been of central importance in the discussions on the fundamentals of the mechanics.[1,2] The rapid development in quantum information and computation has stimulated the studies on the entanglement in various candidates of quantum information devices such as atom, trapped ion, liquid nuclear magnetic resonance (NMR), quantum dot, etc. Entanglement is considered to be the key feature that allows quantum information and computation to overcome some of the limitations in the classical counterpart. Quantum decoherence is recognized as one of main formidable obstructions in using some of these candidates to design quantum information devices. The deep understanding of the role of decoherence in the entanglement dynamics of qubit candidates is desirable and has been extensively discussed.[3−8] The phenomenon of entanglement sudden death (ESD) has attracted much attention and many interesting results are discovered in the recent progress.

Theoretically, there are several approaches to explaining the decoherence phenomena. The environment-induced decoherence approach[9−11] is to ascribe the decoherence of quantum system to the reservoirs with many degrees of freedom. The interaction between quantum system and a reservoir causes the coherence of quantum system to disappear. Another approach[12−15] is to modify the Schrödinger equation so that quantum coherence is damaged as the scale of physical system becomes large. In particular, Milburn[15] has proposed a simple model on the assumption that the system evolves in a stochastic sequence of identical unitary transformations on sufficiently short time steps. This model makes a feature of constants of the motion remaining unchanged. The methods of solving the Milburn model in JC model and its generalizations[16−22] have been developed. The effect of intrinsic decoherence in various systems[23−26] has been studied.

As a generalization of the JC model, the two-mode Raman coupled model[27] consists of two fields and a three-level Rydberg atom which the top level is eliminated adiabatically provided that the condition of large detuning is satisfied. Gerry *et al*[28] also extended this model to the two-atom case. Xu *et al*[21] studied the atom inversion, photon number distribution and squeezing of the radiation field in the single-atom two-mode Raman coupled model. We[29] investigated the entropy squeezing of the atom in the single-atom two-mode Raman coupled model. The entanglement between two atoms[30] and between field and atom[31] in the two-atom two-mode Raman coupled model has also been studied. The Raman coupled model has been widely applied to study the dynam-

*Project supported by the National Natural Science Foundation of China (Grant No 10374007).
†Corresponding author.

ics of systems and realize some quantum operations in quantum information processing.[32] These motivate us to investigate the entanglement dynamics of two atoms in the two-atom two-mode Raman coupled model in which the effect of the intrinsic decoherence is considered.

In this paper, we investigate the evolution of two atoms' entanglement in the two-atom two-mode Raman coupled model in which the effect of intrinsic decoherence is considered. The rest of the present paper is organized as follows. In Section 2, we give a brief review about the two-atom two-mode Raman coupled model and the Milburn equation. In Section 3, firstly, we derive the projection of Hamiltonian of the two-atom two-mode Raman coupled model on invariant subspace. We derive and discuss the entanglement dynamics of two atoms in asymmetric coupling coefficient case in Subsection 3.1 and the entanglement dynamics of two atoms in different two fields' initial photon numbers in Subsection 3.2. The summaries and conclusions are presented in Section 4.

2. Model and Milburn equation

For simplification, we set the Planck constant $\hbar = 1$ throughout this paper. We consider the situation that two atoms are interacting with two-mode fields through Raman coupling. The Hamiltonian for this system is:[28]

$$\begin{aligned} H = {} & \omega_{f1} a_1^\dagger a_1 + \omega_{f2} a_2^\dagger a_2 + \frac{\omega_{a1}}{2}\sigma_z^1 + \frac{\omega_{a2}}{2}\sigma_z^2 \\ & + g_1\left(a_1 a_2^\dagger \sigma_+^1 + a_1^\dagger a_2 \sigma_-^1\right) \\ & + g_2\left(a_1 a_2^\dagger \sigma_+^2 + a_1^\dagger a_2 \sigma_-^2\right), \end{aligned} \tag{1}$$

where σ_z^i , $\sigma_\pm^i$ are the spin flip operators characterizing the effective two-level atom with transition frequencies ω_{ai} and a_i ($a_i^\dagger$) is the annihilation (creation) operator of the the field mode i of frequencies ω_{fi} , and g_i is the coupling coefficient of the atom i. To reduce the complexity, we assume $\omega_{ai} = \omega$ and $\omega_{f1} - \omega_{f2} = \omega$. The configuration of the two two-level atoms is illustrated in Fig.1.

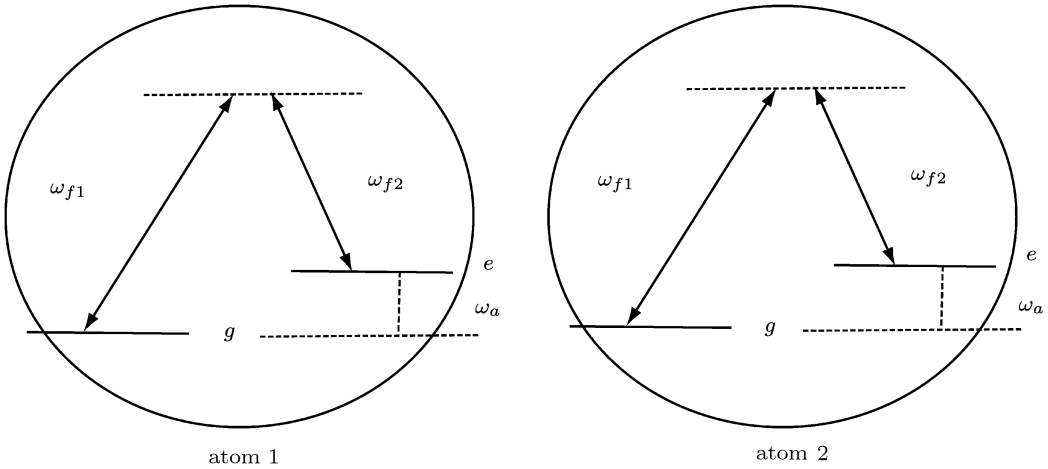


Fig.1. Configuration of atomic levels. $\omega_{ai} = \omega_{f1} - \omega_{f2} = \omega$.

Here, we are interested in the effects of the intrinsic decoherence and some system parameters on the entanglement evolution of two two-level atoms. The intrinsic decoherence approach has been studied by Milburn[15] who has proposed a simple model based on the assumption that on sufficiently short time steps the system does not evolve continuously under unitary evolution but rather in a stochastic sequence of identical unitary transformations. This assumption leads to a modification of the Schrödinger equation which contains a term responsible for the decay of quantum coherence in the energy eigenstate bases, without the intervention of a reservoir and the usual energy dissipation associated with normal decay. The stochastic behaviour in the time evolution of the system implies that the density operator satisfies, to the first order,

the following equation of motion[15]

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = -\,\mathrm{i}[H,\rho(t)] - \frac{1}{2\gamma}\,[H,[H,\rho(t)]]\,, \qquad (2)$$

where γ is the intrinsic decoherence parameter. In the limit $\gamma \to \infty$, the Milburn equation reduces to the standard von Neumann equation describing the Schrödinger evolution. The formal solution of Eq.(2) can be written explicitly as

$$\rho(t) = \sum_{k=0}^{\infty} \left(\frac{t}{\gamma}\right)^{k} \frac{1}{k!} M_k(t)\rho(0)M_k^{\dagger}(t), \qquad (3)$$

where $\rho(0)$ is the initial state of the system and the Kraus operators $M_k(t)$ $(k = 0, 1, \ldots)$ contain all the information of any quantum-computational process and are given by

$$M_k(t) = H^k \exp\left(-\,\mathrm{i}\,Ht - \frac{t}{2\gamma}H^2\right) \quad k = 0, 1, \ldots . \qquad (4)$$

It is worth while to mention that the form of Milburn equation (2) is the same as the form of the master equation derived from phase damping[33,34] and the formula of phase damping has been used to discuss the entanglement in the field-atom systems;[35,36] therefore, the derivation and discussion in our paper are also useful for phase damping.

In this paper, we adopt the concurrence as a measure to calculate the entanglement of two atoms. The concurrence[37] of a 2×2 system is defined by $C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$ where the λ_i, (i=1, 2, 3, 4) are the square roots of the eigenvalues of the operator $R = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$, where the asterisk indicates complex conjugation and λ_1 is the largest one among four eigenvalues.

3. The two-atom entanglement evolution

We concentrated on the investigation of entanglement dynamics in the two-atom case by means of invariant subspace. We find that there exists two constants of motion in the Hamiltonian (1)

$$K_1 = a_1^{\dagger}a_1 + 1 + \frac{\sigma_z^1 + \sigma_z^2}{2},$$
$$K_2 = a_2^{\dagger}a_2 + 1 - \frac{\sigma_z^1 + \sigma_z^2}{2}, \qquad (5)$$

which commute with the Hamiltonian (1). It is not difficult to find the four eigenvectors in the subspace of $K_1 = n_1 + 1$ and $K_2 = n_2 + 1$

$$|n_1, n_2, e, g\rangle\,; \quad |n_1, n_2, g, e\rangle\,;$$
$$|n_1 + 1, n_2 - 1, g, g\rangle\,;$$
$$|n_1 - 1, n_2 + 1, e, e\rangle\,. \qquad (6)$$

The projection of Hamiltonian (1) on the invariant subspace (6) has the matrix form in the representation spanned by the eigenvectors $\{|n_1 + 1, n_2 - 1, g, g\rangle;\quad |n_1, n_2, g, e\rangle;\quad |n_1, n_2, e, g\rangle;\ |n_1 - 1, n_2 + 1, e, e\rangle\}$

$$H_{n_1+1,n_2+1} = \begin{pmatrix} \Omega & g_2\Gamma_2 & g_1\Gamma_2 & 0 \\ g_2\Gamma_2 & \Omega & 0 & g_1\Gamma_1 \\ g_1\Gamma_2 & 0 & \Omega & g_2\Gamma_1 \\ 0 & g_1\Gamma_1 & g_2\Gamma_1 & \Omega \end{pmatrix}, \qquad (7)$$

where

$$\Omega = \omega_1 n_1 + \omega_2 n_2,$$
$$\Gamma_1 = \sqrt{n_1(n_2 + 1)},$$
$$\Gamma_2 = \sqrt{(n_1 + 1)n_2}. \qquad (8)$$

3.1. The effect of asymmetric coupling coefficients on the entanglement

In this subsection, we concentrate on the effect of two atom-field coupling coefficients on entanglement evolution of two atoms. For simplification, we set two constants of motion $K_1 = 2, K_2 = 2$; therefore, the expression (7) becomes

$$H_{2,2} = \begin{pmatrix} \Omega & g_2\sqrt{2} & g_1\sqrt{2} & 0 \\ g_2\sqrt{2} & \Omega & 0 & g_1\sqrt{2} \\ g_1\sqrt{2} & 0 & \Omega & g_2\sqrt{2} \\ 0 & g_1\sqrt{2} & g_2\sqrt{2} & \Omega \end{pmatrix}, \qquad (9)$$

where $\Omega = \omega_1 + \omega_2$.

We assume that at $t = 0$ the two atoms and two fields are described by the product state

$$\rho\,(0) = \rho_f\,(0) \otimes \rho_a\,(0) = |\psi\rangle_{f\,f}\,\langle\psi| \otimes |\varphi\rangle_{a\,a}\,\langle\varphi|\,, \qquad (10)$$

where the two atoms initially are prepared in the state

$$|\varphi\rangle_a = \cos\alpha|e, g\rangle + \sin\alpha|g, e\rangle, \quad \alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \qquad (11)$$

and two-mode fields are initially in the Fock states

$$|\psi\rangle_f = |1\rangle_{f1}|1\rangle_{f2}. \qquad (12)$$

Without losing generality, we assume $g_1 > g_2$. Diagonalizing the Hamiltonian (9), we obtain the eigenvectors expressed in terms of the subspace vectors (6) and the corresponding eigenvalues

$$\begin{aligned}
|E_1\rangle &= \frac{1}{2}(-|2,0,g,g\rangle - |1,1,g,e\rangle + |1,1,e,g\rangle + |0,2,e,e\rangle), \quad \lambda_1 = \Omega - \sqrt{2}(g_1 - g_2),\\
|E_2\rangle &= \frac{1}{2}(-|2,0,g,g\rangle + |1,1,g,e\rangle - |1,1,e,g\rangle + |0,2,e,e\rangle), \quad \lambda_2 = \Omega + \sqrt{2}(g_1 - g_2),\\
|E_3\rangle &= \frac{1}{2}(|2,0,g,g\rangle - |1,1,g,e\rangle - |1,1,e,g\rangle + |0,2,e,e\rangle), \quad \lambda_3 = \Omega - \sqrt{2}(g_1 + g_2),\\
|E_4\rangle &= \frac{1}{2}(|2,0,g,g\rangle + |1,1,g,e\rangle + |1,1,e,g\rangle + |0,2,e,e\rangle), \quad \lambda_4 = \Omega + \sqrt{2}(g_1 + g_2).
\end{aligned} \tag{13}$$

Expanding the initial state (10) of the total system in terms of the eigenvectors (13) and substituting $\rho(0)$ into Eq.(3), we obtain the density operator of the total system as

$$\begin{aligned}
\rho(t) = \frac{1}{4}\{ & A_1^2\left[|E_1\rangle\ \langle E_1| + |E_2\rangle\langle E_2| - \chi_{21}|E_2\rangle\langle E_1| - \chi_{12}|E_1\rangle\langle E_2|\right]\\
& + A_2^2\left[|E_3\rangle\langle E_3| + |E_4\rangle\langle E_4| - \chi_{34}|E_3\rangle\langle E_4| - \chi_{43}|E_4\rangle\langle E_3|\right]\\
& - A_1A_2\left[\chi_{13}\left(|E_1\rangle\langle E_3| + |E_4\rangle\langle E_2|\right) + \chi_{31}\left(|E_3\rangle\langle E_1| + |E_2\rangle\langle E_4|\right)\right.\\
& \left.-\chi_{14}\left(|E_1\rangle\langle E_4| + |E_3\rangle\langle E_2|\right) - \chi_{41}\left(|E_4\rangle\langle E_1| + |E_2\rangle\langle E_3|\right)\right]\},
\end{aligned} \tag{14}$$

where

$$\begin{aligned}
&A_1 = \cos\alpha - \sin\alpha, \quad A_2 = \cos\alpha + \sin\alpha,\\
&\chi_{ij} = \exp[-\mathrm{i}(\lambda_i - \lambda_j)t]\exp\left[-\frac{t}{2\gamma}(\lambda_i - \lambda_j)^2\right].
\end{aligned} \tag{15}$$

By tracing over two fields' variables, we obtain the reduced density operator describing the subsystem containing only two atoms as

$$\begin{aligned}
\rho_{a1a2}(t) = \frac{1}{16}\{ & [A_1^2(2-\tilde{\chi}_{12}) + A_2^2(2-\tilde{\chi}_{34}) + 2A_1A_2(\tilde{\chi}_{13} - \tilde{\chi}_{14})]\,|g,g\rangle\langle g,g|\\
& + [A_1^2(2+\tilde{\chi}_{12}) + A_2^2(2+\tilde{\chi}_{34}) - 2A_1A_2(\tilde{\chi}_{13} + \tilde{\chi}_{14})]\,|g,e\rangle\langle g,e|\\
& + [-A_1^2(2+\tilde{\chi}_{12}) + A_2^2(2+\tilde{\chi}_{34})](|g,e\rangle\langle e,g| + |e,g\rangle\langle g,e|)\\
& + [A_1^2(2+\tilde{\chi}_{12}) + A_2^2(2+\tilde{\chi}_{34}) + 2A_1A_2(\tilde{\chi}_{13} + \tilde{\chi}_{14})]\,|e,g\rangle\langle e,g|\\
& + [A_1^2(2-\tilde{\chi}_{12}) + A_2^2(2-\tilde{\chi}_{34}) - 2A_1A_2(\tilde{\chi}_{13} - \tilde{\chi}_{14})]\,|e,e\rangle\langle e,e|\},
\end{aligned} \tag{16}$$

where

$$\tilde{\chi}_{ij} = 2\cos[(\lambda_i - \lambda_j)t]\exp\left[-\frac{t}{2\gamma}(\lambda_i - \lambda_j)^2\right]. \tag{17}$$

On the basis of the analytical solution (16), we investigate the influence of asymmetric coupling coefficients on the entanglement evolution of two atoms. From Eq.(16), the concurrence for the two atoms is obtained as

$$C = \mathrm{Max}\{0, 2\left(\sqrt{\rho_{eg,ge}\rho_{ge,eg}} - \sqrt{\rho_{ee,ee}\rho_{gg,gg}}\right)\}, \tag{18}$$

where

$$\begin{aligned}
\sqrt{\rho_{eg,ge}\rho_{ge,eg}} - \sqrt{\rho_{ee,ee}\rho_{gg,gg}} = \frac{1}{16}\Bigg\{ & \sqrt{[-A_1^2(2+\tilde{\chi}_{12}) + A_2^2(2+\tilde{\chi}_{34})]^2}\\
& - \sqrt{[A_1^2(2-\tilde{\chi}_{12}) + A_2^2(2-\tilde{\chi}_{34})]^2 - [2A_1A_2(\tilde{\chi}_{13} - \tilde{\chi}_{14})]^2}\Bigg\}.
\end{aligned} \tag{19}$$

The analytic expression (19) shows that the entanglement evolution of atomic subsystem depends on the two coupling coefficients, the initial state parameter of atomic subsystem and the decoherence strength. In Fig.2, we plot the entanglement as the function of time t and the coupling coefficient of the first atom g_1 for two values of the initial state parameter, where we fix the coupling coefficient of the second atom $g_2 = 1$, and decoherence

$\gamma = 100$. From Figs.2(a) and 2(b), we observe that the oscillation frequency of the entanglement monotonically increases with the increasing of the coupling coefficient of the first atom g_1. The disappearance of entanglement becomes faster with the larger value of g_1. We present the analytical expression in the case where $\alpha = \pi/4$ to demonstrate them. When $\alpha = \pi/4$, we obtain the following expression

$$\sqrt{\rho_{eg,ge}\rho_{ge,eg}} - \sqrt{\rho_{ee,ee}\rho_{gg,gg}} = \frac{1}{2}\Big\{ \exp\Big[- \frac{4t(g_1+g_2)^2}{\gamma}\Big] \cos[2\sqrt{2}t(g_1+g_2)]\Big\}; \tag{20}$$

therefore, the condition of entanglement is

$$\cos[2\sqrt{2}t(g_1+g_2)] > 0. \tag{21}$$

From the above expression, the period of entanglement oscillation decreases with the increasing of g_1.

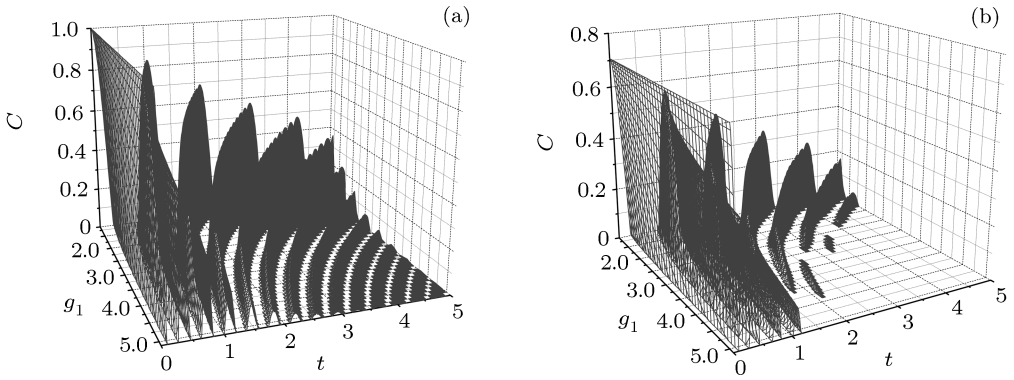


Fig.2. The entanglement C as the function of time t and the coupling coefficient of the first atom $g_1 \in [1.1, 5.1]$. The coupling coefficient of the second atom $g_2 = 1$; decoherence $\gamma = 100$; the initial photon numbers of two fields $n_1 = 1, n_2 = 1$; the initial state parameter of two atoms (a) $\alpha = \pi/4$; (b) $\alpha = \pi/8$.

The evolution of entanglement as the function of time t and the initial state parameter α for two values of g_1 is plotted in Fig.3, when the other parameters are the same as those in Fig.2. From Fig.3(a), we observe that at the beginning the atomic entanglement falls abruptly to zero and remains zero for a period of time before recovering to a smaller value. During the evolution, the interval of entanglement recovering becomes small. From Fig.3, we find that the peaks of entanglement evolution in the case of $\alpha = \pi/4$ are the largest among $\alpha \in [0,\ \pi/2]$, and the values of peaks decrease with the increasing of $|\alpha - \pi/4|$. In the interval $[-\pi/2,\ 0]$, we can draw the similar conclusion on the entanglement evolution when $\alpha = -\pi/4$ from Fig.3. Compared with these in the case where the initial parameter $\alpha = \pi/4$, the frequencies of oscillations and speeds of entanglement vanishing are relatively small in the case of $\alpha = -\pi/4$. The larger the coupling coefficient of the first atom, the smaller the difference between the oscillation periods of two cases. We calculate the expression $\sqrt{\rho_{eg,ge}\rho_{ge,eg}} - \sqrt{\rho_{ee,ee}\rho_{gg,gg}}$ in the case of $\alpha = -\pi/4$

$$\sqrt{\rho_{eg,ge}\rho_{ge,eg}} - \sqrt{\rho_{ee,ee}\rho_{gg,gg}} = \frac{1}{2}\Big\{ \exp\Big[- \frac{4t(g_1-g_2)^2}{\gamma}\Big] \cos[2\sqrt{2}t(g_1-g_2)]\Big\}. \tag{22}$$

Comparing the expression (20) with the expression (22), the oscillation frequency $g_1 - g_2$ in the case of $\alpha = -\pi/4$ is smaller than the frequency $g_1 + g_2$ in the case of $\alpha = \pi/4$. When g_1 is increasing, the ratio between two frequencies $(g_1+g_2)/(g_1-g_2)$ becomes small as shown in Fig.3. The entanglement of two atoms when the initial atom parameter $\alpha = -\pi/4$ is more robust against decoherence than that when $\alpha = \pi/4$, and the difference between abilities of two cases decreases with the increasing of g_1. In a specific value of $\alpha \in [0,\ \pi/2]$, the larger the value of $|\alpha - \pi/4|$ is, the shorter the time of entanglement disappearance is. But the oscillation frequency is not monotonic function of α, because the frequencies $2\sqrt{2}g_1$, $2\sqrt{2}g_2$ appear when $\alpha \neq \pi/4$.

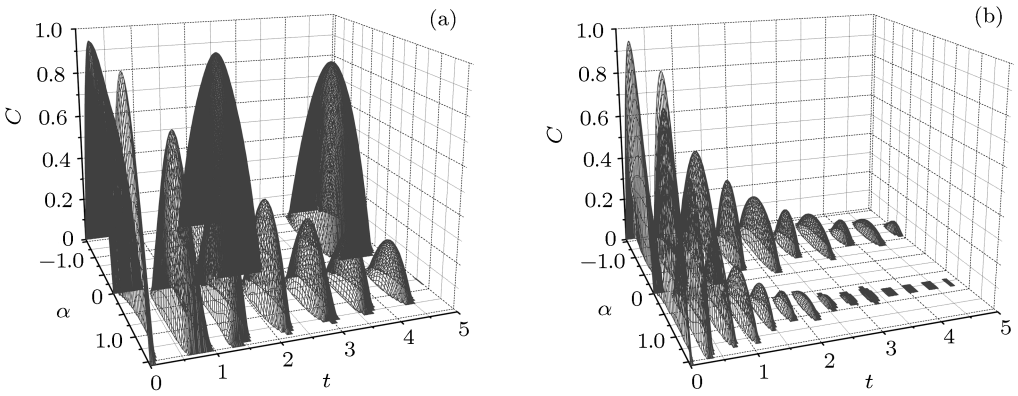


Fig.3. The entanglement C as the function of time t and the initial state parameter of two atoms $\alpha \in [-\pi/2,\ \pi/2]$. The second coupling coefficient $g_2 = 1$; decoherence $\gamma = 100$; the initial photon numbers of two fields $n_1 = 1$, $n_2 = 1$; the coupling coefficient of the first atom (a) $g_1 = 2$; (b) $g_1 = 5$.

3.2. The effect of two fields' initial photon numbers on the entanglement evolution

In Subsection 3.2, we focus our attention on the effect of the initial photon numbers of two fields on the entanglement evolution of the atomic subsystem. We set the coupling coefficients of two atoms to be equal to each other, i.e. $g_1 = g_2 = g$. The projection of Hamiltonian (7) becomes

$$H_{n_1+1,n_2+1} = \begin{pmatrix} \Omega & g\Gamma_2 & g\Gamma_2 & 0 \\ g\Gamma_2 & \Omega & 0 & g\Gamma_1 \\ g\Gamma_2 & 0 & \Omega & g\Gamma_1 \\ 0 & g\Gamma_1 & g\Gamma_1 & \Omega \end{pmatrix}, \quad (23)$$

where Γ_1, Γ_2 and Ω are defined in Eq.(8). We then obtain the eigenvectors and corresponding eigenvalues of the expression (23) as

$$\begin{aligned}
|E_1\rangle &= \frac{\Gamma_2}{\sqrt{\Gamma_1^2+\Gamma_2^2}}\left[-\frac{\Gamma_1}{\Gamma_2}|n_1+1,n_2-1,g,g\rangle + |n_1-1,n_2+1,e,e\rangle\right], \quad \lambda_1 = \Omega;\\
|E_2\rangle &= \frac{1}{\sqrt{2}}[-|n_1,n_2,g,e\rangle + |n_1,n_2,e,g\rangle], \quad \lambda_2 = \Omega;\\
|E_3\rangle &= \frac{\Gamma_1}{\sqrt{2(\Gamma_1^2+\Gamma_2^2)}}\left[\frac{\Gamma_2}{\Gamma_1}|n_1+1,n_2-1,g,g\rangle - \frac{\sqrt{\Gamma_1^2+\Gamma_2^2}}{\sqrt{2}\Gamma_1}|n_1,n_2,g,e\rangle\right.\\
&\quad \left. - \frac{\sqrt{\Gamma_1^2+\Gamma_2^2}}{\sqrt{2}\Gamma_1}|n_1,n_2,e,g\rangle + |n_1-1,n_2+1,e,e\rangle\right], \quad \lambda_3 = \Omega - \sqrt{2}g\sqrt{\Gamma_1^2+\Gamma_2^2};\\
|E_4\rangle &= \frac{\Gamma_1}{\sqrt{2(\Gamma_1^2+\Gamma_2^2)}}\left[\frac{\Gamma_2}{\Gamma_1}|n_1+1,n_2-1,g,g\rangle + \frac{\sqrt{\Gamma_1^2+\Gamma_2^2}}{\sqrt{2}\Gamma_1}|n_1,n_2,g,e\rangle\right.\\
&\quad \left. + \frac{\sqrt{\Gamma_1^2+\Gamma_2^2}}{\sqrt{2}\Gamma_1}|n_1,n_2,e,g\rangle + |n_1-1,n_2+1,e,e\rangle\right], \quad \lambda_4 = \Omega + \sqrt{2}g\sqrt{\Gamma_1^2+\Gamma_2^2}. \qquad (24)
\end{aligned}$$

We assume the two atoms initially are prepared in the same state as Eq.(11) and two-mode fields are initially in the Fock states

$$|\psi\rangle_f = |n_1\rangle_{f1}|n_2\rangle_{f2}. \quad (25)$$

The atomic subsystem is separable from field subsystem initially

$$\rho(0) = \rho_f(0) \otimes \rho_a(0) = |\psi\rangle_{f\ f}\langle\psi| \otimes |\varphi\rangle_{a\ a}\langle\varphi|. \quad (26)$$

Following the method described in previous subsection, we obtain the dynamics of the total system as

$$\rho(t)=\frac{1}{2}\Big\{A_1^2|E_2\rangle\langle E_2|-\frac{A_1A_2}{\sqrt{2}}[\chi_{23}(|E_2\rangle\langle E_3|-|E_4\rangle\langle E_2|)+\chi_{32}(|E_3\rangle\langle E_2|-|E_2\rangle\langle E_4|)]$$
$$+\frac{A_2^2}{2}[|E_3\rangle\langle E_3|+|E_4\rangle\langle E_4|-\chi_{43}|E_4\rangle\langle E_3|-\chi_{34}|E_3\rangle\langle E_4|]\Big\}, \tag{27}$$

where $A_1,\ A_2$ and χ_{ij} are defined in Eq.(15).

The reduced density operator of the two atoms is obtained by tracing over the variables of two fields

$$\rho_{a1a2}(t)=\frac{1}{2}\Big\{\frac{A_2^2}{4}(2-\tilde{\chi}_{34})(\Gamma_1')^2\Big(\frac{\Gamma_2}{\Gamma_1}\Big)^2|g,g\rangle\langle g,g|+\Big[\frac{A_1^2}{2}+\frac{A_2^2}{8}(2+\tilde{\chi}_{34})-\frac{A_1A_2}{2}\tilde{\chi}_{23}\Big]|g,e\rangle\langle g,e|$$
$$+\Big[\frac{A_2^2}{8}(2+\tilde{\chi}_{34})-\frac{A_1^2}{2}\Big](|g,e\rangle\langle e,g|+|e,g\rangle\langle g,e|)+\Big[\frac{A_1^2}{2}+\frac{A_2^2}{8}(2+\tilde{\chi}_{34})+\frac{A_1A_2}{2}\tilde{\chi}_{23}\Big]|e,g\rangle\langle e,g|$$
$$+\frac{A_2^2}{4}(2-\tilde{\chi}_{34})(\Gamma_1')^2|e,e\rangle\langle e,e|\Big\}, \tag{28}$$

where $\tilde{\chi}_{ij}$ is defined as Eq.(17) and

$$\Gamma_1'=\frac{\Gamma_1}{\sqrt{\Gamma_1^2+\Gamma_2^2}}. \tag{29}$$

Utilizing Eq.(28), we obtain the corresponding concurrence as

$$C=\mathrm{Max}\{0,2\left(\sqrt{\rho_{eg,ge}\rho_{ge,eg}}-\sqrt{\rho_{ee,ee}\rho_{gg,gg}}\right)\}, \tag{30}$$

where

$$\sqrt{\rho_{eg,ge}\rho_{ge,eg}}-\sqrt{\rho_{ee,ee}\rho_{gg,gg}}=\frac{1}{2}\Big\{\sqrt{\Big(\frac{A_2^2}{8}(2+\tilde{\chi}_{34})-\frac{A_1^2}{2}\Big)^2}-\frac{A_2^2}{4}(2-\tilde{\chi}_{34})\frac{\Gamma_1\Gamma_2}{\Gamma_1^2+\Gamma_2^2}\Big\}. \tag{31}$$

Firstly, we consider the effect of the initial photon numbers of two fields on entanglement evolution in the special case of $\alpha=\pi/4$. The expression (31) when $\alpha=\pi/4$ is reduced to

$$\sqrt{\rho_{eg,ge}\rho_{ge,eg}}-\sqrt{\rho_{ee,ee}\rho_{gg,gg}}=\frac{1}{4}\Big[\frac{1}{2}(2+\tilde{\chi}_{34})-(2-\tilde{\chi}_{34})\frac{\Gamma_1\Gamma_2}{\Gamma_1^2+\Gamma_2^2}\Big]. \tag{32}$$

Fixing the initial photon number of the first field n_1, we find that the oscillation frequency and the speed of entanglement decay become faster with the increasing of the initial photon number of the second field n_2 as shown in Figs.4(a) and 4(b). In the limit $t\to\infty$, the expression (32) becomes

$$\frac{1}{4}\Big[1-2\frac{\sqrt{n_1(1+n_1)n_2(1+n_2)}}{n_1+n_2+2n_1n_2}\Big]. \tag{33}$$

In Fig.4(a), the entanglement of two atoms exhibits different long-term behaviours with different values of n_2. When the photon number of the second field is small such as $n_2=1$, the entanglement decays to zero. When the initial photon number of the second field is large such as $n_2=10$, the entanglement decays to a stable value.

In the case of $n_1=5$, when the initial photon number of the second field is small such as $n_2=1$, the entanglement would decay to a stable value. When the initial photon number of the second field is large such as $n_2=10$, the entanglement would disappear completely.

Secondly, we investigate the effect of the initial photon numbers of two fields on entanglement evolution in the generic case where $\alpha\in[-\pi/2,\ \pi/2]$. In the case where the initial photon number of the first field is small such as $n_1=1$, we observe that the entanglement of two atoms has no obvious oscillation all time in the case where the initial state parameter of two atoms $\alpha\in[-\pi/2,\ 0)$ which is not found in the asymmetric coupling coefficient case discussed in the previous subsection. The entanglement of two atoms shows decay behaviour in the case where the initial state parameter of two atoms $\alpha\in[0,\pi/2]$. When the initial photon number of the second field is small such as $n_2=1$, the entanglement decays to zero finally in the case of $\alpha\in[0,\pi/2]$ as shown in Fig.5(a). Increasing the initial photon number of the second field, for instance, $n_2=5$, the entanglement

decreases more rapidly and turns to small stationary value when the initial state parameter of two atoms α belongs to a neighbourhood of $\pi/4$. When n_2 becomes large enough, such as $n_2 = 50$, the entanglement of two atoms shows no ESD when α belongs to a neighbourhood of $\pi/4$.

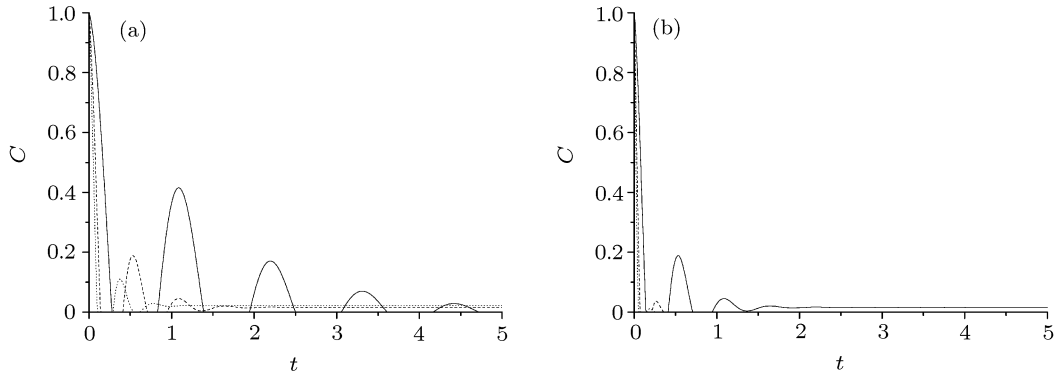


Fig.4. The time evolution of the entanglement C. The coupling coefficient $g = 1$; decoherence $\gamma = 20$; the initial parameter of two atoms $\alpha = \pi/4$; (a) $n_1 = 1$, $n_2 = 1$ (solid line), $n_2 = 5$ (dashed line), $n_2 = 10$ (dotted line); (b) $n_1 = 5$, $n_2 = 1$ (solid line), $n_2 = 5$ (dashed line), $n_2 = 10$ (dotted line).

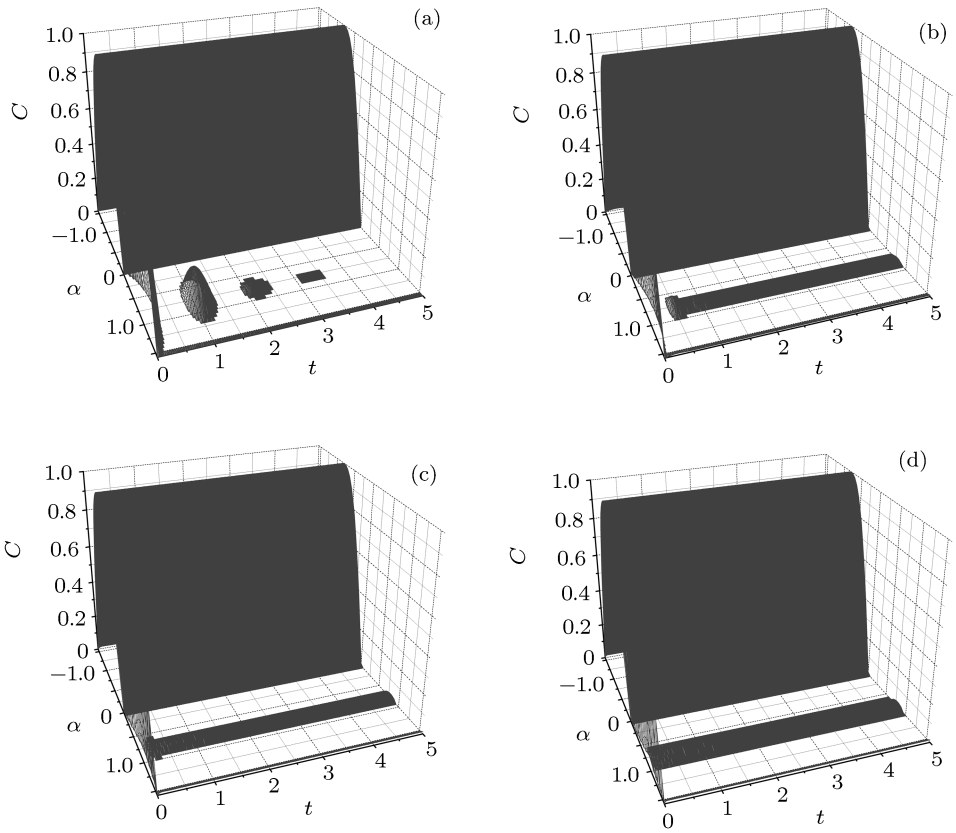


Fig.5. The entanglement C as the function of time t and the initial parameter of two atoms $\alpha \in [-\pi/2,\ \pi/2]$. The coupling coefficient $g = 1$; decoherence $\gamma = 10$; the initial photon number of the first field $n_1 = 1$; the initial photon number of the second field (a) $n_2 = 1$; (b) $n_2 = 5$; (c) $n_2 = 20$; (d) $n_2 = 50$.

We plot the entanglement as the function of time t and the initial state parameter of two atoms α when $n_1 = 5$ in Fig.6. The value of Eq.(31) maintains unchanged when exchanging the value of n_1 and n_2; therefore, we find the entanglement evolution in the case where the initial photon numbers of the second field $n_2 = 1$, is similar to that of the case $n_1 = 1$, $n_2 = 5$ as shown in Fig.6(a). The residual entanglement that can be observed

when α is a member of a neighbourhood of $\pi/4$ and n_2 is small and disappears with the increasing of the initial photon numbers of the second field as shown in Figs.6(a) and 6(b). In these cases, ESD occurs for every $\alpha \in [0,\ \pi/2]$. Continuing to increase the initial photon numbers of the second field, the entanglement turns to a stable value and shows no ESD when α is a member of the neighbourhood of $\pi/4$ as shown in Fig.6(c).

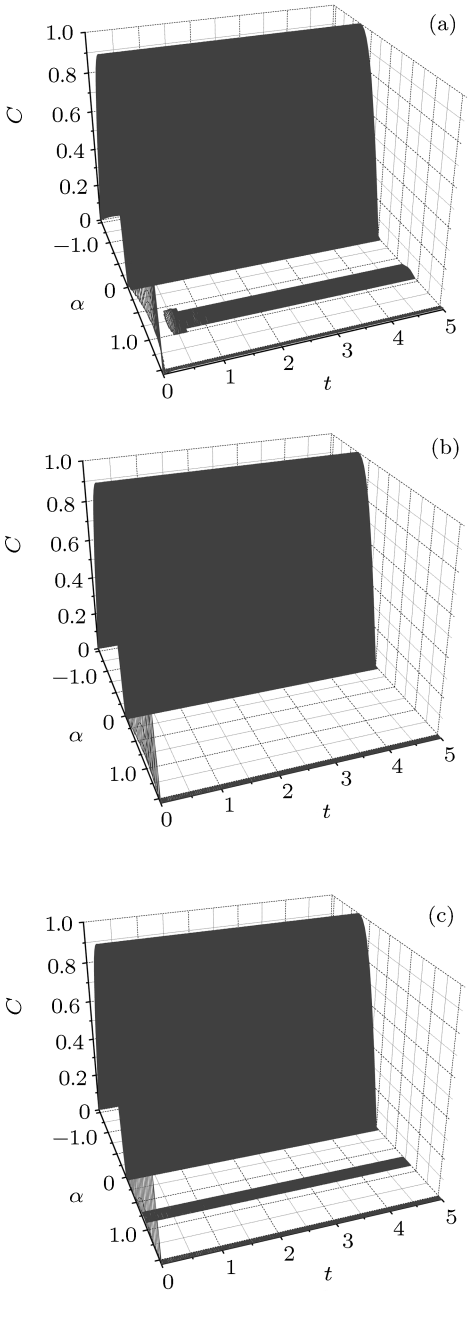


Fig.6. The entanglement C as the function of time t and the initial parameter of two atoms $\alpha \in [-\pi/2,\ \pi/2]$. The coupling coefficient $g = 1$; decoherence $\gamma = 10$; the initial photon number of the first field $n_1 = 5$; the initial photon number of the second field (a) $n_2 = 1$; (b) $n_2 = 10$; (c) $n_2 = 50$.

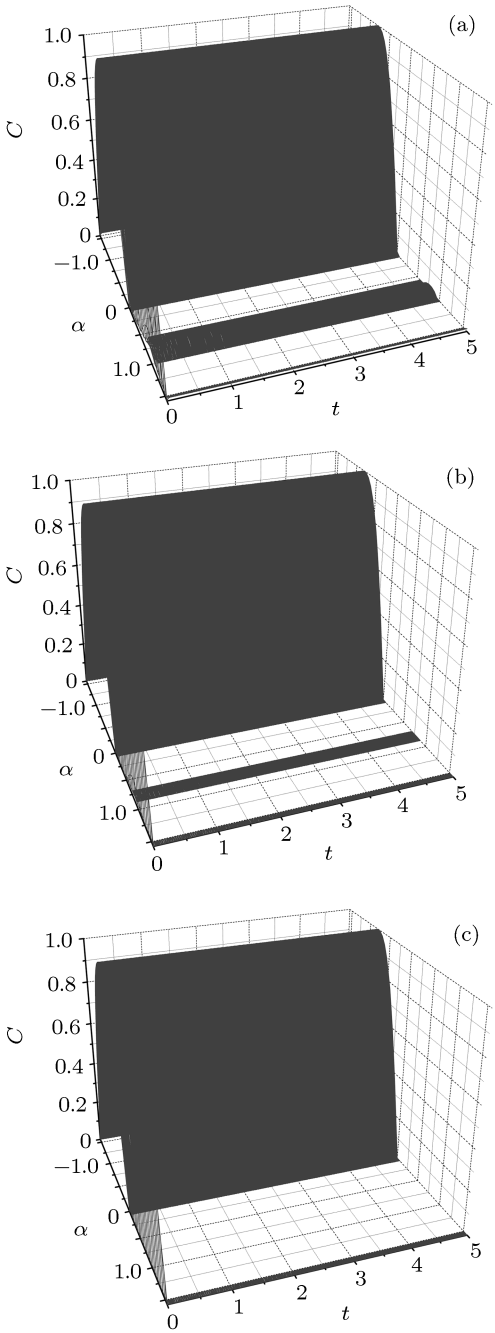


Fig.7. The entanglement C as the function of time t and the initial parameter of two atoms $\alpha \in [-\pi/2,\ \pi/2]$. The coupling coefficient $g = 1$; decoherence $\gamma = 10$; the initial photon number of the first field $n_1 = 50$; the initial photon number of the second field (a) $n_2 = 1$; (b) $n_2 = 5$; (c) $n_2 = 20$.

When we simulate the entanglement evolution of the large initial photon numbers of the first field such as $n_1 = 50$, the entanglement keeps no great change all the time when $\alpha \in [-\pi/2,\ 0]$. When $\alpha \in (0, \pi/2]$, we also find that the entanglement turns to the stable value when the initial state parameter of two atoms α near

$\pi/4$ and otherwise, it decays to zero. Increasing the initial photon numbers of the second field, the residual entanglement decreases and turns to vanish. When n_2 is large enough, ESD occurs for all $\alpha \in [0, \pi/2]$.

Lastly, it is valuable to mention again, from Figs.5, 6 and 7, no matter what values of the initial photon numbers of two fields are, the entanglement of two atoms has no obvious oscillation and maintains the similar value as the initial state parameter of two atoms $\alpha \in [-\pi/2, 0]$. It means that if the initial state parameter of atomic subsystem is suitable, the entanglement of the atomic subsystem is robust against the intrinsic decoherence.

4. Summary and conclusion

In this paper, considering the intrinsic decoherence, we investigated the time evolution of the system consisting of two two-level atoms which interact with two quantized fields through Raman coupling. Utilizing the invariant subspace, we obtained the temporal evolution of density operator for discussing the entanglement evolution of two atoms. Considering the asymmetric coupling coefficients of two atoms, the oscillation frequency of entanglement and the speed of entanglement decay increase with the increasing of coupling coefficient of the first atom g_1 and the difference between the oscillation period when $\alpha \in [-\pi/2, 0]$ and the oscillation period when $\alpha \in [0, \pi/2]$ becomes smaller when g_1 becomes larger. Furthermore, we also investigated the effect of initial photon numbers of two fields on the two-atom entanglement evolution, and we find that the increasing of the initial photon number of the second field n_2 can hasten the oscillation and vanishing of the entanglement of atomic subsystem. When the initial atom parameter α is assumed in the interval $[-\pi/2,\ 0]$, the intrinsic decoherence has no great influence on the evolution of entanglement. When $\alpha \in [0,\ \pi/2]$, the entanglement of atomic subsystem decays rapidly. If the values of n_1 and n_2 are suitable, we find that the entanglement turns to stationary values when α is a member of a neighbourhood of $\pi/4$ the size of which changes with the different values of n_1 and n_2. The two-mode Raman coupled model has been applied to describe several systems which are used as candidates for qubit; therefore, our investigation and analytical expressions should be useful to the preparation of entangled state in quantum information processing.

References

[1] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777
[2] Schrödinger E 1935 *Naturwissenschaften* **23** 807
[3] Yu T and Eberly J H 2004 *Phys. Rev. Lett.* **93** 140404
[4] Yu T and Eberly J H 2006 *Opt. Commun.* **264** 393
[5] Yönaç, Yu T and Eberly J H 2006 *J. Phys.* B **39** S621
[6] Eberly J H and Yu T 2007 *Science* **316** 555
[7] Ficek Z and Tanaś R 2008 *Phys. Rev.* A **77** 054301
[8] López C E, Romero G, Lastra F , Solano E and Retamal J C 2008 *Phys. Rev. Lett.* **101** 080503
[9] Leggett A J, Chakravarty S, Dorsey A T, Fisher M P A, Garg A and Zweger W 1987 *Rev. Mod. Phys.* **59** 1
[10] Zurek W H 1981 *Phys. Rev.* D **24** 1516
[11] Zurek W H 1982 *Phys. Rev.* D **26** 1862
[12] Diosi L 1989 *Phys. Rev.* A **40** 1165
[13] Ghirardi G C, Pearle P and Rimini A 1990 *Phys. Rev.* A **42** 78
[14] Ellis J, Mohanty S and Nanapoulos D 1989 *Phys. Lett.* B **221** 113
[15] Milburn G J 1991 *Phys. Rev.* A **44** 5401
[16] Moya-Cessa H, Bužek V, Kim M S and Knight P L 1993 *Phys. Rev.* A **48** 3900
[17] Kuang L M, Chen X and Ge M L 1995 *Phys. Rev.* A **52** 1857
[18] Chen X, Kuang L M and Ge M L 1995 *J. Phys.* A **28** L267
[19] Obaday A S F, Abdel-Hafezz A M and Hessianx H A 1998 *J. Phys.* B **31** 5085.
[20] Xu J B and Zou X B 1999 *Phys. Rev.* A **60** 4743
[21] Xu J B, Zou X B and Yu J H 2000 *Eur. Phys. J.* D **10** 295
[22] Naderi M H 2007 *Can. J. Phys.* **85** 1071
[23] Guo D J, Shan C J and Xia Y J 2007 *Acta Phys. Sin.* **56** 2139 (in Chinese)
[24] Tan X, Zhang C Q and Xia Y J 2006 *Acta Phys. Sin.* **55** 2263 (in Chinese)
[25] Xu X B, Liu J M and Yu P F 2008 *Chin. Phys.* B **17** 456
[26] Hu X M and Liu J M 2009 *Chin. Phys.* B **18** 411
[27] Gerry C C and Eberly J H 1990 *Phys. Rev.* A **42** 6805
[28] Gerry C C and Huang H 1992 *Phys. Rev.* A **45** 8037
[29] Zhang J, Shao B and Zou J 2009 *Chin. Phys.* B **18** 1517
[30] Bashkirov E K 2006 *Laser Phys. Lett.* **3** 145
[31] Bashkirov E K and Rusakova M S 2008 *Opt. Commun.* **281** 4380
[32] Larson J and Garraway B M 2004 *J. Mod. Opt.* **51** 1691
[33] Kuang L M, Chen X, Chen G H and Ge M L 1997 *Phys. Rev.* A **56** 3139
[34] Hessian H A and Ritsch H 2002 *J. Phys.* B **35** 4619
[35] Li S B and Xu J B 2005 *Phys. Rev.* A **72** 022332
[36] Xu J B and Li S B 2005 *Eur. Phys. J.* D **35** 553
[37] Wootters W K 1998 *Phys. Rev. Lett.* **80** 2245