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Generation of arbitrary four-atom entangled decoherence-free states*

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This paper proposes a scheme to generate arbitrary four-atom entangled decoherence-free states by using simple linear optical elements, four one-sided cavities in which four atoms are confined respectively. By conveniently tuning the titled angle of one half-wave plate, it can obtain arbitrary four-atom entangled decoherence-free states with a successful probability of 1 as long as there is no photon loss.

Keywords: one-sided cavity, linear optical element, four-atom entangled decoherence-free states
PACC: 0367

1. Introduction

Entanglement has played a key role as a valuable resource for quantum communication and quantum computation.[1,2] Recently, schemes for preparing various entangled states have been proposed by many authors.[3−8] However, the decoherence precesses caused by the influence of the environment often easily destroy entanglement and furthermore cause noise in the communication or errors in the outcomes of computation. If logical qubits are encoded into a subspace of the whole Hilbert space, a decoherence-free (DF) space, the noise and errors mentioned above can be resisted efficiently. A DF state is invariant under any identical unitary transformation on each qubit in it, which makes it useful for quantum information processing between parties who do not even share a common reference frame.[9−14] Moreover, its immunity against collective decoherence has been demonstrated experimentally in different physical systems.[15−19] Several optical schemes for generating four-photon polarization-entangled decoherence-free states have been proposed recently.[20−22] In Ref.[20], using spontaneous parametric down conversions (SPDC), Bourennane *et al* have prepared a four-photon polarization-entangled DF state which has the form

$$|\Psi\rangle = \alpha|\Phi_0\rangle + \beta|\Phi_1\rangle), \tag{1}$$

in which

$$|\Phi_0\rangle = \frac{1}{2}(|HVHV\rangle - |HVVH\rangle - |VHHV\rangle + |VHVH\rangle), \tag{2}$$

and

$$|\Phi_1\rangle = \frac{1}{2\sqrt{3}}(2|VVHH\rangle + 2|HHVV\rangle - |VHVH\rangle - |VHHV\rangle - |HVVH\rangle - |HVHV\rangle), \tag{3}$$

where H and V denote horizonal and vertical linear polarization, respectively, they are two orthogonal states of a DF subspace with two dimensions and four qubits. In their scheme, they must first use different methods to prepare the states $|\Phi_0\rangle$ and $|\Phi_1\rangle$ from SPDC, then coherently overlaps the two states to prepare superposition state of Eq.(1), which is difficult to realize. Lately, Zou *et al* proposed a scheme[21] to directly generate four-photon polarization-entangled DF states from SPDC by using only a tunable beam splitter, but only a few particular superposition states could be generated and the successful probability of generating four-photon polarization-entangled DF states is far lower than unit. Therefore Gong *et al* proposed another scheme[22] to generate arbitrary states using two SPDC sources and linear optical elements, but they need to tune the phase shifter and reflectivities of two beam splitters to obtain arbitrary four-photon polarization-entangled DF states. In this paper, we propose a scheme for generating four-atom

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entangled DF states in a system consisting of four one-sided cavities, a conventional photon detector and some linear optical elements. In the scheme, in order to obtain arbitrary four-atom entangled DF states we only need to conveniently tune the titled angle of one half-wave plate. Furthermore, we can generate four-atom entangled DF states with probability 1 as long as there is not photon loss. The scheme is easy to realize due to the fact that quantum state of light is robust against the decoherence and photons are ideal carriers for transmitting quantum information over a long distance. In addition, we only need a conventional photon detector to distinguish between the vacuum and nonvacuum Fock number states instead of four photons coincidence detection. These advantages make the scheme more feasible in experiments.

2. Generation of four-atom entangled DF states

The basic building model[23,24] involved in our scheme is shown in Fig.1. A single-photon pulse with horizontal (H) polarization enters a one-sided cavity in which a Λ-type three-level atom is confined. Levels $|0\rangle$ and $|e\rangle$ are resonantly coupled with the H component of the input photon. Suppose the input photon is of H polarization, it will have a resonant interaction with the cavity if the atom is in state $|1\rangle$. When $\kappa T \gg 1$ is satisfied (where T is the duration of input photon pulse and κ is the cavity decay rate), the pulse will be reflected by the cavity with its pulse shape almost unchanged but its phase added by π. On the contrary, when the atom is in state $|0\rangle$, the pulse will be reflected by the cavity with both its shape and phase unchanged. If the pulse is of vertical (V) polarization, it will be reflected by the mirror without any change.

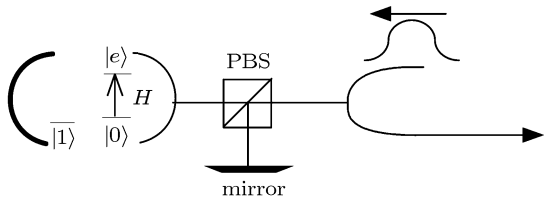


Fig.1. Schematic setup to implement the controlled phase flip gate with atom and single-photon pulse.

Therefore, the controlled phase flip (CPF) gate between the atom and the photon can be described by the unitary operator $U = \mathrm{e}^{(\mathrm{i}\,\pi|1\rangle\langle 1|\otimes|H\rangle\langle H|)}$.

We assume that the input photon is initially in the state $|H\rangle$ and the four atoms trapped in different cavities are in the state $|+\rangle$, where $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle$. The setup is schematically shown in Fig.2. The 45°-titled half-wave plate (HWP) rotates the photon polarization as $|H\rangle \leftrightarrow |V\rangle$. The $\theta/2$-titled HWP (H_1), whose titled angle is tunable, rotates the photon polarization as $|H\rangle \rightarrow \cos\theta|H\rangle + \sin\theta|V\rangle$. The $\arccos(\sqrt{2/3})/2$-titled half-wave plate (H_2) rotates the photon polarization as $|H\rangle \rightarrow \sqrt{2/3}|H\rangle + \sqrt{1/3}|V\rangle$. The 22.5°-titled HWP ($\mathrm{H}_i$, $i = 3, 4, \ldots, 9$) rotates the photon polarization as $|H\rangle \rightarrow 1/\sqrt{2}(|H\rangle + |V\rangle)$, $|V\rangle \rightarrow 1/\sqrt{2}(|H\rangle - |V\rangle)$. We will see that the photon in the vertical polarization mode $|V\rangle$ will be reflected by polarizing beam splitter (PBS), while the photon in the horizontal polarization mode $|H\rangle$ will pass through PBS. After the input photon passes through H_1, the state of the whole system is

$$(\cos\theta|H\rangle + \sin\theta|V\rangle)|++++\rangle_{1234}. \tag{4}$$

The two components $\cos\theta|H\rangle$ and $\sin\theta|V\rangle$ evolve along different paths a and b, respectively, as shown in Fig.2, after passing through PBS_1.

Now, let us describe the evolution of $\cos\theta|H\rangle$ in detail. After passing through H_2, PBS_2, H_3, H_4, PBS_3, HWP_1, H_5, H_p (phase shifter, transforming $|H\rangle \rightarrow -|H\rangle$), and H_6, $\cos\theta|H\rangle$ evolves into

$$\cos\theta\Big[\frac{1}{\sqrt{3}}(|H\rangle + |V\rangle)_{\mathrm{a1}} + \frac{1}{2\sqrt{3}}(-|H\rangle - |V\rangle)_{\mathrm{a2}} + \frac{1}{2\sqrt{3}}(-|H\rangle - |V\rangle)_{\mathrm{a3}}\Big], \tag{5}$$

where the subscript a1, a2, a3 indicate different paths shown in Fig.2. Then the photon goes through PBS_4, HWP_2, PBS_5, HWP_3, PBS_6, HWP_4, the state of Eq.(5) is transformed into

$$\cos\theta\Big[\frac{1}{\sqrt{3}}(|H\rangle_{\mathrm{a11}} + |H\rangle_{\mathrm{a12}}) + \frac{1}{2\sqrt{3}}(-|H\rangle_{\mathrm{a21}} - |H\rangle_{\mathrm{a22}}) + \frac{1}{2\sqrt{3}}(-|H\rangle)_{\mathrm{a31}} - |H\rangle_{\mathrm{a32}})\Big], \tag{6}$$

where the subscript a11, a12, ..., a32 indicate next different branch-pathes shown in Fig.2.

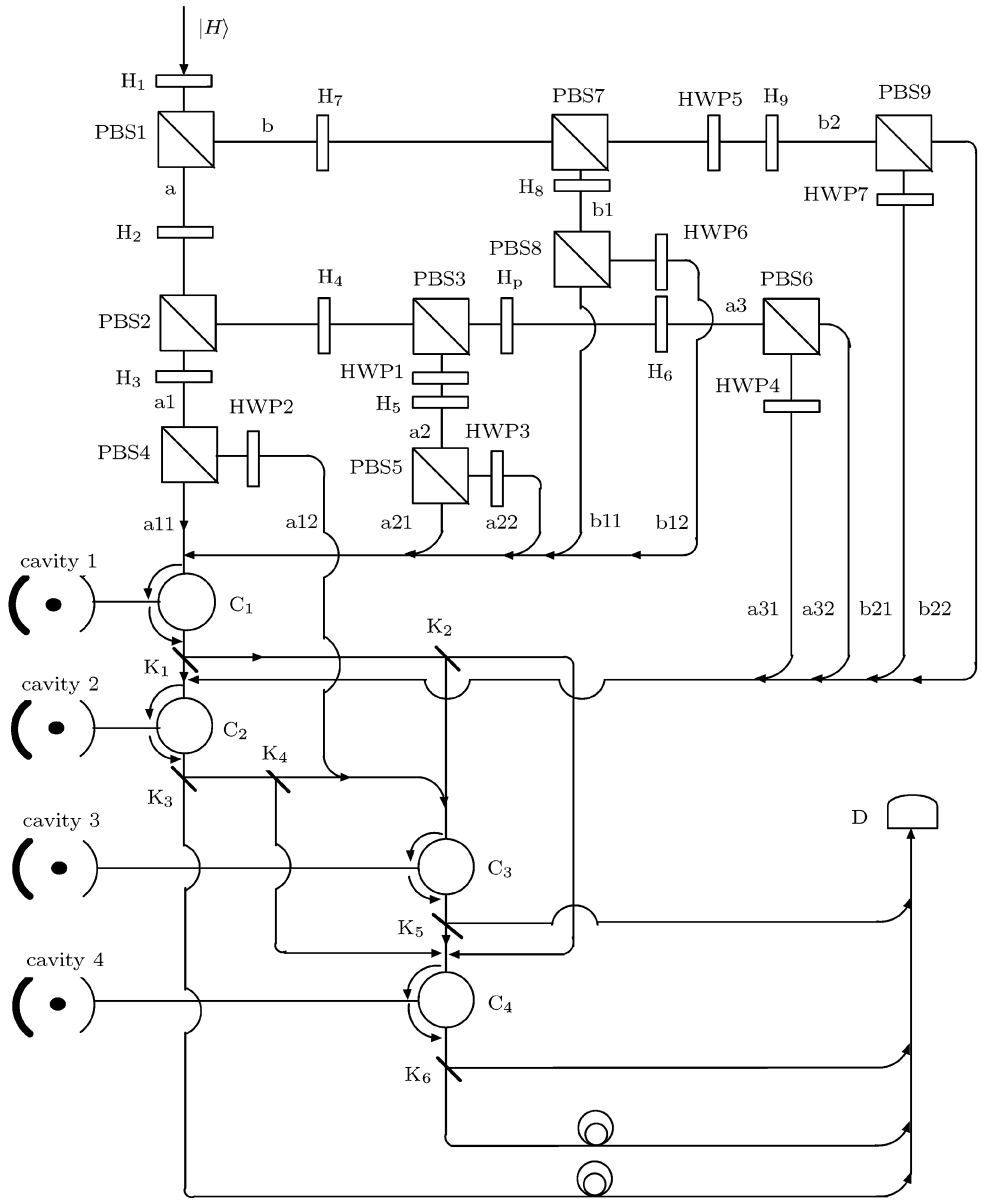


Fig.2. Schematic setup to generate arbitrary four-atom entangled DF states. The 45°-titled HWP rotates the photon polarization as $|H\rangle \leftrightarrow |V\rangle$. The $\theta/2$-titled half-wave plate (H_1) rotates the photon polarization as $|H\rangle \rightarrow \cos\theta|H\rangle + \sin\theta|V\rangle$. The $\arccos(\sqrt{2/3})/2$-titled HWP (H_2) rotates the photon polarization as $|H\rangle \rightarrow \sqrt{2/3}|H\rangle + \sqrt{1/3}|V\rangle$. The phase shifter H_p transforms $|H\rangle$ to $-|H\rangle$. The 22.5°-titled HWP (H_i, $i = 3, 4, \ldots, 9$) rotates the photon polarization as $|H\rangle \rightarrow 1/\sqrt{2}(|H\rangle + |V\rangle)$, $|V\rangle \rightarrow 1/\sqrt{2}(|H\rangle - |V\rangle)$. The polarizing beam splitter (PBS) transmits the horizontal polarization mode $|H\rangle$ and reflects the vertical polarization mode $|V\rangle$. D is the conventional photon detector.

Similarly, the evolution of $\sin\theta|V\rangle$ can be described as follows:

$$\xrightarrow{H_7,PBS_7,H_8,HWP_5,H_9} \sin\theta\frac{1}{2}[(-|H\rangle + |V\rangle|)_{b1} + (|H\rangle + (-|V\rangle)_{b2}], \tag{7}$$

$$\xrightarrow{PBS_8,HWP_6,PBS_9,HWP_7} \sin\theta\frac{1}{2}[-|H\rangle_{b11} + |H\rangle_{b12} - |H\rangle_{b21} + |H\rangle_{b22}], \tag{8}$$

where the subscript b1, b2, b11, b12, b21, b22 indicate different pathes shown in Fig.2.

At present, the state of the whole system is

$$\Big[\cos\theta\frac{1}{2\sqrt{3}}(2|H\rangle_{a11} + 2|H\rangle_{a12} - |H\rangle_{a21} - |H\rangle_{a22} - |H\rangle_{a31} - |H\rangle_{a32}), + \sin\theta\frac{1}{2}(-|H\rangle_{b11} + |H\rangle|_{b12} - |H\rangle_{b21} + |H\rangle_{b22})\Big]| + + + +\rangle_{1234}. \tag{9}$$

It can be seen that, after the photon passes through circulator C_1 and cavity 1, the state of the system becomes

$$\begin{aligned}\cos\theta\frac{1}{2\sqrt{3}}&[(2|H\rangle_{a11}-|H\rangle_{a21}-|H\rangle_{a22})|-+++\rangle_{1234}\\&+(2|H\rangle_{a12}-|H\rangle_{a31}-|H\rangle_{a32})|+++-\rangle_{1234}]\\&+\sin\theta\frac{1}{2}[(-|H\rangle_{b11}+|H\rangle|_{b12})|-++-\rangle\\&+(-|H\rangle_{b21}+|H\rangle_{b22})|+++-\rangle_{1234}.\end{aligned}\tag{10}$$

Subsequently, the photon arrives at the switch K_1, whose working state is first transmitting , then reflecting. We choose the lengths of optical fibers appropriately, so that the photon in state $2|H\rangle_{a11}$ passes through K_1, and the photon in the state $|H\rangle_{a21}$, or $|H\rangle_{a22}$, or $|H\rangle_{b11}$, or $|H\rangle_{b12}$ is reflected by K_1. Now, the photon in state $|H\rangle_{a21}$, or $|H\rangle_{a22}$, or $|H\rangle_{b11}$, or $|H\rangle_{b12}$ arrives at the switch K_2, which (and the switches $K_3, K_4, \ldots, K_6$ mentioned later) has the same function as the switch K_1. Thus, the photon in state $|H\rangle_{a21}$ or $|H\rangle_{b11}$ will pass through K_2, while $|H\rangle_{a22}$ and $|H\rangle_{b12}$ will be reflected by K_2, because the length of each optical fiber can be selected. After passing through circulator C_2 and cavity 2, the state of the system is

$$\begin{aligned}\cos\theta\frac{1}{2\sqrt{3}}&[2|H\rangle_{a11}|--++\rangle_{1234}+(-|H\rangle_{a21}-|H\rangle_{a22})|-+++\rangle_{1234}\\&+(2|H\rangle_{a12}|+++-\rangle_{1234}+(-|H\rangle_{a31}-|H\rangle_{a32})|+-++\rangle_{1234}]\\&+\sin\theta\frac{1}{2}[(-|H\rangle_{b11}+|H\rangle_{b12})|-++-\rangle_{1234}\\&+(-|H\rangle_{b21}+|H\rangle_{b22})|+-++\rangle_{1234}].\end{aligned}\tag{11}$$

Then, the photon sees the switches K_3 and K_4. The photon in the state $2|H\rangle_{a11}$ passes through K_3, and the photon in state $|H\rangle_{a31}$ or $|H\rangle_{b21}$ is reflected by K_3 and then goes through K_4, while the photon in state $|H\rangle_{a32}$ or $|H\rangle_{b22}$ is reflected firstly by K_3, then by K_4. From Fig.2, we can see that, after the photon goes through circulator C_3 and cavity 3, the state of the system becomes

$$\begin{aligned}\cos\theta\frac{1}{2\sqrt{3}}&(2|H\rangle_{a11}|--++\rangle_{1234}+2|H\rangle_{a12}|++-+\rangle_{1234}-|H\rangle_{a21}|-+++\rangle_{1234}\\&-|H\rangle_{a22}|-+-+\rangle_{1234}-|H\rangle_{a31}|+--+\rangle_{1234}-|H\rangle_{a32}|+-++\rangle_{1234})\\&+\sin\theta\frac{1}{2}(-|H\rangle_{b11}|-+++\rangle_{1234}+|H\rangle_{b12}|-+-+\rangle_{1234}\\&-|H\rangle_{b21}|+--+\rangle_{1234}+|H\rangle_{b22}|+-++\rangle_{1234}).\end{aligned}\tag{12}$$

Similarly, if we choose the length of each optical fiber appropriately, the photon in $|H\rangle_{a21}$, or $|H\rangle_{a32}$, or $|H\rangle_{b11}$, or $|H\rangle_{b22}$ arrives at C_3 at the same time after the photon in path a12. Then the photon in the state $|H\rangle_{a12}$ will be transmitted by K_5, while the photon in the state $|H\rangle_{a21}$, or $|H\rangle_{a32}$, or $|H\rangle_{b11}$, or $|H\rangle_{b22}$ will be reflected by K_5. Finally, the photon goes through circulator C_4 (we may choose the length of each optical fiber appropriately, so that the photon in the states reflected by K_4 and the states transmitted by K_2 arrive at C_4 simultaneously after the photon transmitted by K_5), cavity 4 and K_6, and the state of the system is transformed into

$$\begin{aligned}\cos\theta\frac{1}{2\sqrt{3}}&(2|H\rangle_{a11}|--++\rangle_{1234}+2|H\rangle_{a12}|++--\rangle_{1234}-|H\rangle_{a21}|-++-\rangle_{1234}\\&-|H\rangle_{a22}|-+-+\rangle_{1234}-|H\rangle_{a31}|+--+\rangle_{1234}-|H\rangle_{a32}|+-+-\rangle_{1234})\\&+\sin\theta\frac{1}{2}(-|H\rangle_{b11}|-++-\rangle_{1234}+|H\rangle_{b12}|-+-+\rangle_{1234}\\&-|H\rangle_{b21}|+--+\rangle_{1234}+|H\rangle_{b22}|+-+-\rangle_{1234}).\end{aligned}\tag{13}$$

We appropriately select the lengths of optical fibers to make sure that the photon (in any path) arrives at a detector at one time. If the detector makes a click, the atoms 1, 2, 3, 4 will collapse to the state

$$\begin{aligned}|\psi\rangle&=\cos\theta\frac{1}{2\sqrt{3}}(2|--++\rangle+2|++--\rangle-|-+++-\rangle\\&\quad-|-+-+\rangle-|+--+\rangle-|+-+-\rangle)_{1234}\\&\quad+\sin\theta\frac{1}{2}(|-+-+\rangle+|+-+-\rangle-|-++-\rangle-|+--+\rangle)_{1234}\\&=\cos\theta|\phi_1\rangle+\sin\theta|\phi_0\rangle,\end{aligned}\tag{14}$$

where

$$|\phi_1\rangle = \frac{1}{2\sqrt{3}}(2|--++\rangle + 2|++--\rangle - |-++-\rangle - |-+-+\rangle - |+--+\rangle - |+-+-\rangle)_{1234} \quad (15)$$

and

$$|\phi_0\rangle = \frac{1}{2}(|-+-+\rangle + |+-+-\rangle - |-++-\rangle - |+--+\rangle)_{1234} \quad (16)$$

are the two orthogonal states of the DF subspace with two dimensions and four qubits. If we continuously tune the titled angle of the HWP H_1, we can obtain the arbitrary superposition states of $|\phi_1\rangle$ and $|\phi_0\rangle$.

3. Conclusions

In conclusion, we have generated arbitrary four-atom entangled DF states by using linear optical elements and four one-sided cavities in which four atoms are confined respectively. In our scheme, in order to obtain arbitrary four-atom entangled DF states we only need to conveniently tune the titled angle of one HWP and we can generate four-atom entangled DF states with a probability of 1 as long as there is no photon loss. The scheme is easy to realize due to the fact that the photon has weak interaction with the environment during the course of transmission. In addition, we only need a conventional photon detector to distinguish between the vacuum and nonvacuum Fock number states. So, the scheme is feasible in experiments.

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