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One-step implementing three-qubit phase gate via manipulating rf SQUID qubits in the decoherence-free subspace with respect to cavity decay*

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We present a scheme for implementing a three-qubit phase gate via manipulating rf superconducting quantum interference device (SQUID) qubits in the decoherence-free subspace with respect to cavity decay. Through appropriate changes of the coupling constants between rf SQUIDs and cavity, the scheme can be realized only in one step. A high fidelity is obtained even in the presence of decoherence.

Keywords: decoherence-free subspaces, cavity quantum electrodynamics

PACC: 0367, 4250

1. Introduction

A quantum computer, based on the principle of quantum mechanics, would perform certain tasks faster than a classical computer, such as factorizing a number and searching for data in an array.[1,2] This matter of fact has triggered in the past years many studies on the theoretical and the practical aspects of quantum computing. So far, a lot of substantial efforts have been dedicated to the field of quantum computation and a number of significant progresses have been made. Quantum logic gate, as a basic element of quantum computer, has been realized with different physical models in recent years such as ion-trap system,[3] cavity quantum electrodynamics (QED),[4,5] nuclear magnetic resonance (NMR) system[6] and linear optics.[7]

The developing technology of cavities provides a way to coherently exchange quantum information between material qubits, so cavity QED can be considered as an ideal system to implement quantum information processing. It should be noted that in many physical carriers of qubits, the solid-state qubits are better for quantum computation because they are scalable and can be easily fabricated to implement a large-scale quantum computing. As one kind of the solid-state systems, the superconducting qubits placed in a superconducting cavity own the relatively long decoherence time and can be considered as a good candidate. Up to the present, many schemes for realizing quantum computation have been advanced.[8−14] However, it is still a great challenge for quantum computation to be put into practice since the random errors arising during gate operations. In 2001, Duan *et al* proposed a scheme for holonomic quantum computation with trapped ions in microcavity.[15] This scheme is relying on the geometric quantum phase which is expected to be particularly robust against noise. Subsequently, many schemes were put forward based on the similar principle.[16−19] In 2005, Zheng proposed a novel scheme for quantum computing by adiabatic evolution of dark eigenstates of the considered system.[20] This method is based on the adiabatic evolution of the dark eigenstates itself, which is robust against certain error and totally different from normal dynamical and geometric gates. Afterward, several schemes were presented in the light of this work.[21−23] Nevertheless, the operations of schemes we mentioned above are complicated when a two-qubit gate wanted.

Multiqubit gate can be decomposed into conditional z gate and one-qubit gate,[24] but the procedure will get more complicated as the number of the qubits increases. Hence, researchers are trying to find effective methods to implement the multiqubit gate directly. Although several schemes for one-

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step realizing multiqubit quantum phase gate have been proposed,[25−29] they are all hard to be put into practice on the current technology. From 2000, Beige *et al* have proposed a series of elegant schemes for realizing quantum computation only in one step via manipulating atoms or ions in a decoherence-free subspace.[30−33] These schemes take advantages of dissipation and tuning weak classical fields coupled to the qubit states to implement the quantum gate in decoherence-free subspace. Enlightened by the works of Beige *et al*, we construct a theoretical model for implementing a three-qubit phase gate with rf SQUIDs coupled to superconducting cavity by using nonidentical SQUID-cavity couplings. The advantage of our scheme is that the qubits are encoded in the decoherence-free subspace and the three-qubit phase gate can be achieved only in one step, which greatly simplifies the experiment.

The three-qubit phase gate can be defined by the operator

$$\begin{aligned} U = & |0,0,0\rangle_{123}\langle 0,0,0| + |0,0,1\rangle_{123}\langle 0,0,1| \\ & + |0,1,0\rangle_{123}\langle 0,1,0| + |0,1,1\rangle_{123}\langle 0,1,1| \\ & + |1,0,0\rangle_{123}\langle 1,0,0| + |1,0,1\rangle_{123}\langle 1,0,1| \\ & - |1,1,0\rangle_{123}\langle 1,1,0| + |1,1,1\rangle_{123}\langle 1,1,1|, \quad (1) \end{aligned}$$

from Eq.(1) we can see a minus appearing when three qubits stay in the state $|110\rangle_{123}$. This three-qubit phase gate plus two Hadamard gates can construct a Toffoli gate, which is a universal gate with a very high computational power. In the following we mainly discuss the implementation of the case.

2. Decoherence-free subspace with respect to cavity decay

We consider three rf SQUIDs with Λ-type configuration simultaneously interaction with a single-mode vacuum cavity field. The transition between the levels $|1\rangle_i \leftrightarrow |2\rangle_i$ is coupled to the cavity mode resonantly with the frequency $\omega_{\rm c}$, as shown in Fig.1. The Hamiltonian of the system in the interaction picture can be written as

$$H_{\rm I} = \sum_{i=1}^{3} g_i (a|2\rangle_{ii}\langle 1| + |1\rangle_{ii}\langle 2|a^{\dagger}), \quad (2)$$

where a and $a^{\dagger}$ are the annihilation and creation operators, respectively, for the cavity mode,

$$g_i = \frac{1}{L_i}\sqrt{\frac{\omega_{\rm c}}{2\mu_0\hbar}}_i\langle 1|\Phi_i|2\rangle_i \int_{S_i} \boldsymbol{B}_{\rm c}^{i}(\boldsymbol{r})\cdot {\rm d}\boldsymbol{S} \quad (3)$$

with L_i being the inductance of the i-th SQUID loop, μ_0 being the vacuum permeability, Φ_i being the externally applied biased flux, $\boldsymbol{B}_{\rm c}^{i}(\boldsymbol{r})$ being the magnetic component of the cavity mode in the superconducting loop of the i-th rf SQUID, and S being the surface bounded by the SQUID ring. For the sake of convenience, we assume $g_1 = g_2 = g$ and g_3 to be real and consider the evolution of the system in the following three states

$$\begin{aligned} |a,0\rangle = & \frac{1}{\sqrt{2g_3^2+4g^2}}\big[g_3(|211\rangle + |121\rangle) \\ & - 2g|112\rangle\big]_{123}\otimes|0\rangle_{\rm cav}, \quad (4) \\ |a_1,0\rangle = & \frac{1}{\sqrt{g^2+g_3^2}}(g|102\rangle \\ & - g_3|201\rangle)_{123}\otimes|0\rangle_{\rm cav}, \quad (5) \\ |a_2,0\rangle = & \frac{1}{\sqrt{g^2+g_3^2}}(g|012\rangle \\ & - g_3|021\rangle)_{123}\otimes|0\rangle_{\rm cav}. \quad (6) \end{aligned}$$

From Eq.(2) we can see that the states $|a,0\rangle$, $|a_1,0\rangle$ and $|a_2,0\rangle$ are the dark states of the system since $H_{\rm I}|\chi,0\rangle = 0$ $(\chi = a, a_1, a_2)$, furthermore, they are robust against cavity decay because the non-Hermitian term $\mathrm{i}\kappa a^{\dagger}a$ will not affect the evolution of the states including vacuum state of the cavity. Besides, we find a completed basis which suits for being encoded as qubits because they are also the decoherence-free states with respect to cavity decay. In our scheme, we encode the quantum information in the subspace $\{|0\rangle_1, |1\rangle_1, |0\rangle_2, |1\rangle_2, |0\rangle_3, |1\rangle_3\}$ of the rf SQUID qubits with leaving the cavity in the vacuum state $|0\rangle_{\rm cav}$. However, it is difficult for us to manipulate these dark states without any additional field. Thus to realize our phase gate, we need a weak classical field driving the transition resonantly between the level $|0\rangle$ and the level $|2\rangle$ with Rabi frequency Ω for the third rf SQUID,

$$H_{\rm Laser} = \Omega(|2\rangle_{33}\langle 0| + |0\rangle_{33}\langle 2|) \quad (7)$$

with

$$\Omega = \frac{1}{L_3\hbar}{}_3\langle 0|\Phi_3|2\rangle_3 \int_{S_3} \boldsymbol{B}_{\mu w}^{3}(\boldsymbol{r},t)\cdot {\rm d}\boldsymbol{S}, \quad (8)$$

where $\boldsymbol{B}_{\mu w}^{3}(\boldsymbol{r})$ is the magnetic component of the classical microwave in the superconducting loop of the third rf SQUID. While the introduction of the classical field will drive the system out of the decoherence-free states. If the condition $\{\Omega \ll g, g_3\}$ is satisfied, the evolution of the system is always in the decoherence-free state in the presence of the classical field of Eq.(2) to a very good approximation.[33]

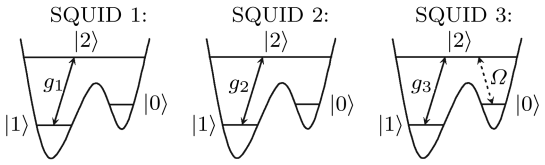


Fig.1. The level configuration of three rf SQUIDs. The levels $|1\rangle_i$ and $|2\rangle_i$ are coupled resonantly to the cavity mode with the coupling constants g_i and the level $|0\rangle_3$ and $|2\rangle_3$ are coupled to the classical microwave pulse with the coupling strength Ω. The qubits are encoded in the states $|0\rangle_i$ and $|1\rangle_i$.

3. Effective Hamiltonian

In this section, we focus on deducing the effective Hamiltonian which acts on the decoherence-free states of the system. To obtain a complete basis, we introduce other three states

$$\begin{aligned}|s,0\rangle &= \frac{1}{\sqrt{g_3^2+2g^2}}[g(|211\rangle+|121\rangle)\\ &\quad+g_3|112\rangle]_{123}\otimes|0\rangle_{\rm cav}, \qquad (9)\\ |s_1,0\rangle &= \frac{1}{\sqrt{g^2+g_3^2}}(g_3|102\rangle\\ &\quad+g|201\rangle)_{123}\otimes|0\rangle_{\rm cav}, \qquad (10)\\ |s_2,0\rangle &= \frac{1}{\sqrt{g^2+g_3^2}}(g_3|012\rangle\\ &\quad+g|021\rangle)_{123}\otimes|0\rangle_{\rm cav}. \qquad (11)\end{aligned}$$

If we prepare the initial state of the system in the qubit state, then the general state after time t can be written as a superposition of the form

$$|\Psi(t)\rangle=\sum_{x,n}C_{x,n}|x,n\rangle, \qquad (12)$$

where $|x,n\rangle$ represents the SQUIDs prepared in $|x\rangle$ and n photons in the cavity. The evolution of probability amplitudes is governed by the Schrödinger equation:

$$\mathrm{i}\frac{\partial|\Psi(t)\rangle}{\partial t}=(H_{\rm I}+H_{\rm Laser})|\Psi(t)\rangle. \qquad (13)$$

We first calculate the time evolution of the coefficients of the decoherence-free states $|000,0\rangle_{123,\rm cav}$, $|010,0\rangle_{123,\rm cav}$, $|100,0\rangle_{123,\rm cav}$, $|110,0\rangle_{123,\rm cav}$, $|a,0\rangle$, $|a_1,0\rangle$ and $|a_2,0\rangle$

$$\begin{aligned}\mathrm{i}\dot{C}_{000,0} &= \Omega C_{002,0},\\ \mathrm{i}\dot{C}_{010,0} &= \frac{\Omega}{\sqrt{g^2+g_3^2}}(g_3C_{s_2,0}+gC_{a_2,0}),\\ \mathrm{i}\dot{C}_{100,0} &= \frac{\Omega}{\sqrt{g^2+g_3^2}}(g_3C_{s_1,0}+gC_{a_1,0}),\\ \mathrm{i}\dot{C}_{110,0} &= -\Omega\left(\frac{\sqrt{2}g}{\sqrt{g_3^2+2g^2}}C_{a,0}-\frac{g_3}{\sqrt{g_3^2+2g^2}}C_{s,0}\right),\\ \mathrm{i}\dot{C}_{a,0} &= -\frac{\sqrt{2}g\Omega}{\sqrt{g_3^2+2g^2}}C_{110,0},\\ \mathrm{i}\dot{C}_{a_1,0} &= \frac{g\Omega}{\sqrt{g^2+g_3^2}}C_{100,0},\\ \mathrm{i}\dot{C}_{a_2,0} &= \frac{g\Omega}{\sqrt{g^2+g_3^2}}C_{010,0}. \qquad (14)\end{aligned}$$

Then we consider the case of the states $|s,0\rangle$, $|s_1,0\rangle$, $|s_2,0\rangle$ and $|002,0\rangle$

$$\begin{aligned}\mathrm{i}\dot{C}_{s,0} &= \frac{g_3\Omega}{\sqrt{g_3^2+2g^2}}C_{110,0}+\sqrt{g_3^2+2g^2}C_{111,1},\\ \mathrm{i}\dot{C}_{s_1,0} &= \frac{g_3\Omega}{\sqrt{g^2+g_3^2}}C_{100,0}+\sqrt{g^2+g_3^2}C_{101,1},\\ \mathrm{i}\dot{C}_{s_2,0} &= \frac{g_3\Omega}{\sqrt{g^2+g_3^2}}C_{010,0}+\sqrt{g^2+g_3^2}C_{011,1},\\ \mathrm{i}\dot{C}_{002,0} &= \Omega C_{000,0}+g_3C_{001,1}, \qquad (15)\end{aligned}$$

and the states $|001,1\rangle$, $|011,1\rangle$, $|101,1\rangle$ and $|111,1\rangle$

$$\begin{aligned}\mathrm{i}\dot{C}_{001,1} &= g_3C_{002,0},\\ \mathrm{i}\dot{C}_{011,1} &= \sqrt{g^2+g_3^2}C_{s_2,0},\\ \mathrm{i}\dot{C}_{101,1} &= \sqrt{g^2+g_3^2}C_{s_1,0},\\ \mathrm{i}\dot{C}_{111,1} &= \sqrt{g_3^2+2g^2}C_{s,0}. \qquad (16)\end{aligned}$$

There are two different time scales for the evolution of states, i.e., $1/\Omega$ for DFS and $1/g$ ($1/g_3$) for outside the DFS. If we prepare the system in the qubit state initially, we can easily eliminate the amplitudes that change on the fast time scaled defined by the cavity coupling constants g (g_3). After setting the derivatives of the non-decoherence-free states equal zero, we obtain from Eq.(16)

$$C_{002,0}=C_{s_2,0}=C_{s_1,0}=C_{s,0}=0. \qquad (17)$$

Substituting this result into Eq.(14), we acquire

$$\begin{aligned}\mathrm{i}\dot{C}_{000,0} &= 0,\\ \mathrm{i}\dot{C}_{010,0} &= \frac{g\Omega}{\sqrt{g^2+g_3^2}}C_{a_2,0},\\ \mathrm{i}\dot{C}_{100,0} &= \frac{g\Omega}{\sqrt{g^2+g_3^2}}C_{a_1,0},\\ \mathrm{i}\dot{C}_{110,0} &= -\frac{\sqrt{2}g\Omega}{\sqrt{g_3^2+2g^2}}C_{a,0},\\ \mathrm{i}\dot{C}_{a,0} &= -\frac{\sqrt{2}g\Omega}{\sqrt{g_3^2+2g^2}}C_{110,0},\\ \mathrm{i}\dot{C}_{a_1,0} &= \frac{g\Omega}{\sqrt{g^2+g_3^2}}C_{100,0},\end{aligned}$$

$$\mathrm{i}\dot{C}_{a_2,0} = \frac{g\Omega}{\sqrt{g^2+g_3^2}}C_{010,0}. \tag{18}$$

From Eq.(18), we can conclude that the time evolution of above system can be summarized as an effective Hamiltonian in the form

$$H_{\text{eff}} = \Omega\Big[\frac{g}{\sqrt{g_3^2+g^2}}(|100,0\rangle\langle a_1,0| + |010,0\rangle\langle a_2,0|) - \frac{\sqrt{2}g}{\sqrt{g_3^2+2g^2}}|110,0\rangle\langle a,0| + \text{H.c.}\Big]. \tag{19}$$

It should be noted that during the gate operation, the population accumulates unintentionally in the states $|111,1\rangle$, $|001,1\rangle$, $|101,1\rangle$ and $|011,1\rangle$. We can calculate the effect by setting the derivatives of Eq.(15) equal zero and obtain

$$\begin{aligned} C_{111,1} &= -\frac{g_3\Omega}{g_3^2+2g^2}C_{110,0}, \\ C_{001,1} &= -\frac{\Omega}{g_3}C_{000,0}, \\ C_{101,1} &= -\frac{g_3\Omega}{g_3^2+g^2}C_{100,0}, \\ C_{011,1} &= -\frac{g_3\Omega}{g_3^2+g^2}C_{010,0}. \end{aligned} \tag{20}$$

Nevertheless, this error can be minimized via adiabatic passage by tuning the laser field slowly enough such that $\frac{\mathrm{d}}{\mathrm{d}t}\Omega(T) = 0$ with T being the gating time. Then the system can adapt to the changing parameters and the unwanted amplitudes vanish adiabatically at time T.[33]

4. Three-qubit phase gate

Although two-atom entanglement and four-atom entanglement and a series of two-qubit gates with the decoherence-free qubit have been reported in Refs.[30]–[33], the three-qubit phase gate cannot be achieved for the identical coupling strengths between atoms and the cavity. In order to implement our three-qubit phase gate with only one step, according to Eq.(19) we should choose

$$\begin{aligned} &\int_0^T \frac{g\Omega(t)}{\sqrt{g_3^2+g^2}}\,\mathrm{d}t = 2n\pi, \\ &\int_0^T \frac{\sqrt{2}g\Omega(t)}{\sqrt{g_3^2+2g^2}}\,\mathrm{d}t = (2n+1)\pi \end{aligned} \tag{21}$$

with n being an integer larger than one, which means

$$\frac{\sqrt{g_3^2+2g^2}}{\sqrt{2g_3^2+2g^2}} = \frac{2n}{2n+1}, \tag{22}$$

we obtain

$$g_3 = \frac{\sqrt{2+8n}}{\sqrt{4n^2-4n-1}}g. \tag{23}$$

In Fig.2 we plot the ratio g_3/g versus the integer in Eq.(23), from which we can see that the larger the value of n is, the more smaller the value of g_3/g is, i.e., g_3/g is a monotonous decreasing function of n. Meanwhile, the duration of the classical field will increase with the increase of n, thus the optimal value for this scheme is $n = 2$, then we obtain $g_3/g = \sqrt{18/7}$.

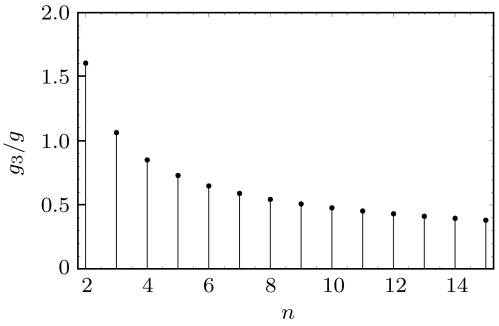


Fig.2. The ratio between g_3 and g versus the integer n.

In the following we choose the time-dependent classical microwave pulse as[33]

$$\Omega(t) = 2\Omega_{\max}\sin^2\left(\frac{\sqrt{7}}{20}\Omega_{\max}t\right) \tag{24}$$

with $T = 20\pi/(\sqrt{7}\Omega_{\max})$ and $\Omega_{\max} = g_3/10$. Therefore, a simple three-qubit phase gate can be easily achieved with the form of Eq.(1). In order to characterize the availability of Eq.(19), we plot the corresponding coefficients evolution of states vs. time $\int_0^T \Omega(t)\,\mathrm{d}t$ by numerical calculation with Eqs.(2) and (7) in Fig.3. The real parts of the coefficients of the qubit states and fidelity of the three-qubit phase obtained through the effective Hamiltonian H_{eff} compared with the ones obtained from the interaction Hamiltonian $H_{\text{I}} + H_{\text{Laser}}$ after the classical microwave pulse are given in Table 1. The result shows that the effective Hamiltonian in Eq.(19) is correct. Here we adopt the Truth table for characterizing this three-qubit phase gate, which is the simple measures of quantum gate.[34] A more sufficient method is the quantum process tomography, which is rather complicated since it needs $8^4 - 8^2$ parameters to give a full assessment of a three-qubit gate.[24]

Table 1. The real parts of the coefficients of the qubit states and fidelity obtained through the effective Hamiltonian $H_{\rm eff}$ compared with the ones obtained from the interaction Hamiltonian $H_2 + H_{\rm Laser}$ after the microwave pulse.

	$C_{000,0}$	$C_{100,0}(C_{010,0})$	$C_{110,0}$	fidelity
$H_{\rm eff}$	1	1	-1	1
$H_{\rm I} + H_{\rm Laser}$	1	0.9967	-0.9980	0.9999

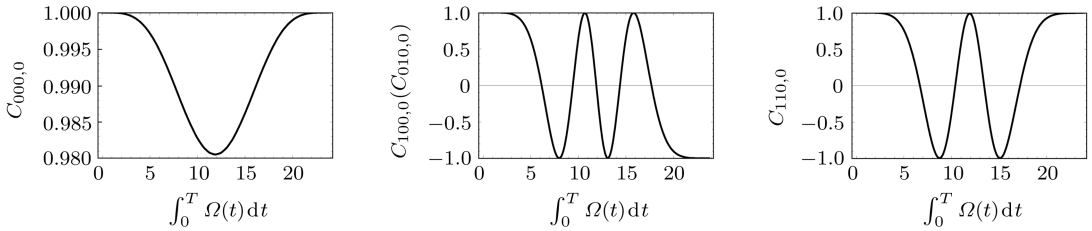


Fig.3. The evolution of the real parts of the probability amplitude of the qubit states versus $\int_0^T \Omega(t)\,\mathrm{d}t$.

5. Discussion and conclusion

Now we give a brief discussion about the experimental feasibility of our scheme. While the three-qubit phase gate coincides with the ideal one theoretically, we should consider the effect of decoherence on the scheme when it is put into experiment. The dissipation channels include the spontaneous emission of SQUIDs (at the rate γ) and the cavity decay (at the rate κ). Here we use the quantum jump method to give an assessment of the three-qubit gate in the presence of decoherence. Under the condition that no photon is detected either by the atomic spontaneous or by the leakage of a photon through the cavity mirror, the evolution of the system is governed by the conditional Hamiltonian

$$H_{\rm de} = \sum_{i=1}^{3} g_i(a|2\rangle_{ii}\langle 1| + |1\rangle_{ii}\langle 2|a^\dagger) + \Omega(|2\rangle_{33}\langle 0| + |0\rangle_{33}\langle 2|) - \sum_{i=1}^{3} \mathrm{i}\frac{\gamma}{2}|2\rangle_{ii}\langle 2| - \mathrm{i}\frac{\kappa}{2}a^\dagger a, \tag{25}$$

where κ is the cavity decay rate, and γ is the spontaneous emission rate of level $|2\rangle$. The last two terms in Eq.(25) can make the norm of the qubit states decrease with increase of time. To see these effects clearly, we utilize Eq.(25) to prepare an entanglement state $|\varphi_S\rangle = \frac{1}{2\sqrt{2}}(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle - |110\rangle + |111\rangle)_{123} \otimes |0\rangle_{\rm cav}$ from the initial state $|\varphi_i\rangle = \frac{1}{2\sqrt{2}}(|0\rangle + |1\rangle)_1(|0\rangle + |1\rangle)_2(|0\rangle + |1\rangle)_3 \otimes |0\rangle_{\rm cav}$, and plot the fidelity $F = \frac{|\langle\varphi_s|\exp[-\mathrm{i}H_{\rm de}20\pi/(\sqrt{7}\Omega_{\max})]|\varphi_i\rangle|^2}{P}$ and successful rate $P = \sum_{x,y,z=0}^{1}|\langle xyz|\exp[-\mathrm{i}H_{\rm de}20\pi/(\sqrt{7}\Omega_{\max})]|\varphi_i\rangle|^2$ of the three-qubit phase gate versus γ/g_3 and κ/g_3 in Figs.4 and 5, respectively. It can be seen that

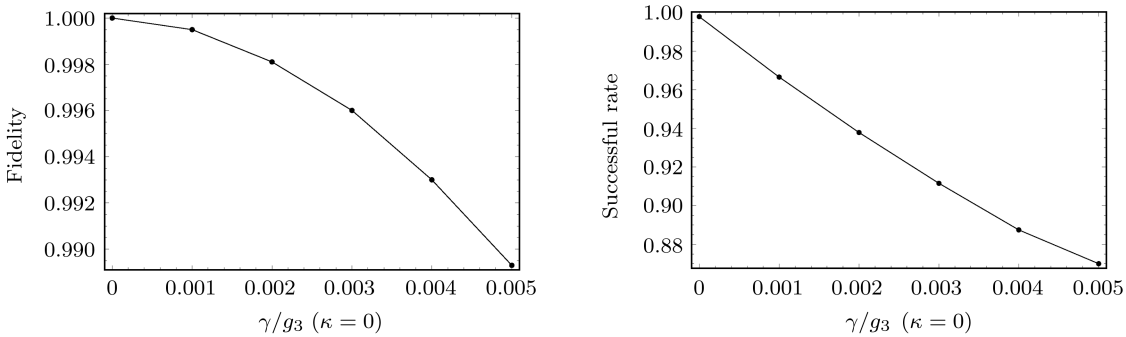


Fig.4. The fidelity and successful rate versus γ/g_3 with $\kappa = 0$.

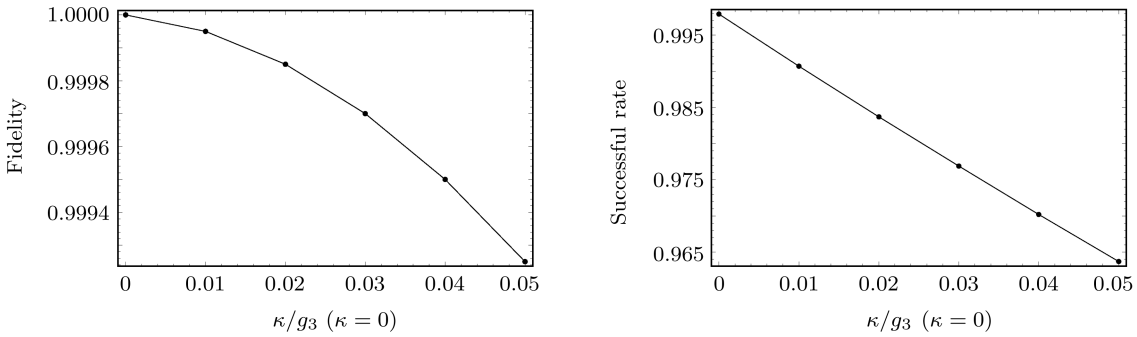


Fig.5. The fidelity and successful rate versus κ/g_3 with $\gamma = 0$.

the three-qubit phase gate is robust against the cavity decay because the fidelity is still higher than 0.999 even though $\kappa = 0.05g$, which coincides with that of the decoherence-free states are not affected by the factor of cavity decay. Although this gate is sensitive to the spontaneous decay of the excited flux state, the scheme is feasible on the strength of current technology.

The states $|0\rangle$, $|1\rangle$ and $|2\rangle$ are corresponding to the two lowest and an excited flux states of the SQUID, respectively. The strong-coupling limit of the cavity QED can be easily realized when rf SQUIDs are embedded in a cavity. We should make the distance between the two SQUIDs larger than the linear dimension of each SQUID, thus the direct coupling between the two SQUIDs is negligible. In our scheme, we require that the three SQUIDs have different coupling strengths with the cavity mode. The coupling between the SQUID and the cavity is determined by Eq.(3), and the magnetic component of the cavity mode can be expanded in the form[35]

$$\boldsymbol{B}_{\rm c}(z) = \mu_0\sqrt{2/V}\cos(kz), \tag{26}$$

where we have adopted a standing-wave form of the single-mode cavity. V is the volume of the cavity, k is wave vector of the cavity mode and z is the cavity axis, respectively. Because the linear dimension of SQUID is much less than the wavelength of the cavity mode, the factor $\cos(kz)$ is independent of the integral area with a good approximation and we can obtain the coupling between SQUID and the cavity as[36]

$$g = \frac{S}{L}\Phi_{\alpha\beta}\sqrt{\frac{\hbar\omega_{\rm c}}{\varepsilon_0 V c^2}}\cos(kz), \qquad (\alpha,\beta = 1,2) \tag{27}$$

with ε_0 being the permittivity of the vacuum. Thus we can satisfy the ratio $g_3/g = \sqrt{18/7}$ via locating the positions of the SQUIDs in the cavity. For the typical parameters,[35,37] we have the coupling strength between the SQUID and the superconducting cavity $g_3 = 1.8 \times 10^8$ s^{-1}, the spontaneous emission rate of the excited level $\gamma = 4\times10^5$ s^{-1}, and the cavity decay rate $\kappa = 1.32 \times 10^6$ s^{-1}. Then the operation time of our scheme is about 1.32 μs and we obtain a fidelity of 99.77% for three-qubit phase gate and a corresponding successful rate of 92.70%. Thus our scheme is a feasible one on the current technology.

In conclusion, we have proposed a scheme for one-step implementation of a three-qubit phase gate with SQUIDs interaction with a superconducting cavity by choosing nonidentical coupling strengths. The results show that the fidelity and the success rate in our scheme are relatively high, and it is easy to implement.

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