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Finite-time disentanglement induced by single-mode thermal field*

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In this paper, we study the entanglement dynamics of atoms locally coupled to a cavity field. By studying two different models within the framework of cavity QED, we show that the so-called atomic entanglement sudden death always occurs if initially the cavity field is in the thermal state, in clear contrast with that in the vacuum state where the same entanglement decay is in infinite time.

Keywords: entanglement sudden death, thermal fields

PACC: 0365, 0367

Entanglement is not only a fundamental notion to distinguish between the quantum and classical worlds, but also an indispensable resource in quantum information processing and quantum computing.[1−6] However, entanglement is very fragile in practical application due to the unavoidable interaction with the environment. Therefore, it is of increasing importance to understand the dynamics for an entangled system in realistic situations and to find various conditions which would influence its dynamical behaviours. Recently, Yu and Eberly[7] discovered that the entanglement of a pair of qubits exposed to local noisy environments can vanish completely in a finite time, which is in striking contrast with decoherence of an individual qubit that decays only asymptotically. Such a finite-time disentanglement was termed entanglement sudden death (ESD). Because of its intrinsic and practical interests, ESD has attracted much attention from both theoretical[8−21] and experimental[22] points of view. In Ref.[13], the authors dealt with a double Jaynes–Cummings model (JCM)[23] in which two initially entangled two-level atoms A and B are independently coupled with vacuum cavity fields a and b, respectively, but there are no interactions at all between the subsystems Aa and Bb. They considered two types of Bell-like states $|\phi_{AB}\rangle \sim |ee\rangle + |gg\rangle$ and $|\psi_{AB}\rangle \sim |eg\rangle + |ge\rangle$ (with $|g(e)\rangle$ the atomic ground (excited) state) as atomic initial states and found out that $|\phi_{AB}\rangle$ undergoes ESD but $|\psi_{AB}\rangle$ is not in the time-evolving process.[13] Their results revealed that in one and the same kind of environment, the disentanglement dynamics might be dependent on the type of initial entangled state. An entangled state with ESD in evolution is less robust than that without it, since ESD sets a limitation on the application time of entanglement. Therefore, the study of ESD, especially conditions and parameter domains for its occurrence, is important from both theoretical and practical points of view. In Ref.[24] the double JCM is considered again and it is found that if the cavity fields are initially in Fock states with nonzero photon numbers then both atomic states $|\phi_{AB}\rangle$ and $|\psi_{AB}\rangle$ would suffer from ESD. The double JCM was also investigated from other perspectives and it was shown that the entanglement evolution of atoms is closely related to their energy variation[25] and there is a natural entanglement invariant demonstrating the entanglement transfer among all the system's degrees of freedom.[14] In distributed quantum computing/information processing the entangled qubits belong to different remote locations. At a location the qubit independently interacts with its own environment and is operated by appropriate quantum gates determined by a certain quantum algorithm/protocol. There may be lengthy intervals before any operation of the gates or between operations of two successive gates. The study interest is the environment-induced evolution of entanglement between remote qubits in these intervals. This motivates the consideration of the JCM which can be realized in the laboratory in several ways.[26,27]

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In addition to the type of initial entangled state, it is shown that the initial condition of noisy environment is also responsible for the occurrence of ESD.[24] Therefore, exploring the environment that may induce ESD seems to be necessary, in order to avoid this type of environment in practical application of entanglement. In this paper, we shall study the effects of thermal fields on atomic entanglement dynamics in both the single and double JCM. We consider the $|\psi_{AB}\rangle$-type Bell-like state that does not exhibit ESD in initially vacuum cavity fields in the double JCM[13] as the initial state of atoms AB. Besides the case in which both atoms are embedded in two cavities, i.e., the double JCM, we shall also consider the case in which only one atom coupled to a cavity while another atom is isolated from any environment. Our purpose is to explore the influencing extent of the thermal fields on the disentanglement dynamics of atoms. We shall show that in both the double and single JCM, atomic $|\psi_{AB}\rangle$ initial entangled state would suffer from ESD in the thermal field(s).

In two-qubit domains, there exist a number of good measures of entanglement such as concurrence[28] and negativity.[29] Although the various entanglement measures may be somewhat different quantitatively, they are qualitatively equivalent to each other in the sense that all of them are equal to zero for unentangled states. Here we adopt Wootters' concurrence[28] because of its convenience in definition, normalization and calculation. The concurrence $C(\rho)$ for any (reduced) density matrix ρ of two qubits is defined as

$$C(\rho)=\max\left\{0,\sqrt{\lambda_1}-\sqrt{\lambda_2}-\sqrt{\lambda_3}-\sqrt{\lambda_4}\right\}, \quad (1)$$

where λ_i ($\lambda_1\geq\lambda_2\geq\lambda_3\geq\lambda_4$) are the eigenvalues of the matrix $\zeta=\rho(\sigma_y\otimes\sigma_y)\rho^*(\sigma_y\otimes\sigma_y)$, σ_y is Pauli matrix and ρ^* denotes the complex conjugation of ρ in the standard basis. For separate states $C(\rho)=0$, whereas for maximally entangled states $C(\rho)=1$. In particular, if ρ is of the X-form,

$$\rho=\begin{pmatrix}\varrho_{11}&0&0&\varrho_{14}\\0&\varrho_{22}&\varrho_{23}&0\\0&\varrho_{32}&\varrho_{33}&0\\\varrho_{41}&0&0&\varrho_{44}\end{pmatrix}, \quad (2)$$

where ϱ_{kk} are real positive and $\varrho_{kl}=(\varrho_{lk})^*$ are generally complex, then the concurrence (1) simplifies to

$$\begin{aligned}C(\rho)=&\,2\max\{0,|\varrho_{23}|-\sqrt{\varrho_{11}\varrho_{44}},|\varrho_{14}|\\&-\sqrt{\varrho_{22}\varrho_{33}}\}. \quad (3)\end{aligned}$$

The initial cavity fields are assumed to be in the thermal state and its mean photon number $\overline{n}$ is a weighted mixture of Fock states whose density operator $\rho_f(0)$ can be represented as

$$\rho_f(0)=\sum_{n=0}^{\infty}P_n|n\rangle\langle n| \quad (4)$$

with $|n\rangle$ being the Fock state of n photons and P_n is given by

$$P_n=\frac{\overline{n}^n}{(1+\overline{n})^{n+1}}. \quad (5)$$

The state we choose as the initial state of atoms A and B is a Bell-like state

$$|\psi(0)\rangle_{AB}=\sin\alpha\,|eg\rangle_{AB}+\cos\alpha\,|ge\rangle_{AB}. \quad (6)$$

Here, the single JCM refers to the case in which atom A is coupled to a single-mode cavity field a, while another atom B interacts with an ideal environment. The Hamiltonian of the system can be written as ($\hbar=1$)

$$H=\frac{\omega_0}{2}(\sigma_z^A+\sigma_z^B)+\omega a^\dagger a+g_A(a^\dagger\sigma_-^A+a\sigma_+^A), \quad (7)$$

where ω is the field frequency, ω_0 is the energy difference between the two levels of an atom, g_A is the coupling constant between atom A and the cavity mode, $a^\dagger$ (a) is the creation (annihilation) operator for the cavity field, $\sigma_z^X=|e\rangle_{XX}\langle e|-|g\rangle_{XX}\langle g|$, $\sigma_+^X=|e\rangle_{XX}\langle g|$ and $\sigma_-^X=|g\rangle_{XX}\langle e|$, where $X=A,B$. For simplicity, in this paper we concentrate our analysis only on the resonant case with $\omega_0=\omega$.

The time evolution operator $U_{Aa}(t)=\exp(-\mathrm{i}Ht)$ for the local interaction of atom A with cavity a was derived exactly in Ref.[30]. At any time $t>0$ the total initial state $\rho_{ABa}(0)\equiv\rho_a(0)\otimes|\psi(0)\rangle_{AB}\langle\psi(0)|$ evolves into $\rho_{ABa}(t)=U_{Aa}(t)\rho_{ABa}(0)U_{Aa}^\dagger(t)$. The reduced density matrix $\rho^{AB}(t)$ of the atomic subsystem can be obtained by tracing out $\rho_{ABa}(t)$ over the cavity field a, i.e., $\rho^{AB}(t)=\mathrm{Tr}_a\rho_{ABa}(t)$, which exhibits the X-form as in Eq.(2) with,

$$\begin{aligned}\rho_{11}&=\sum_{m=0}^{\infty}\sin^2\alpha\sin^2(\sqrt{m+1}g_At),\\\rho_{22}&=\sum_{m=0}^{\infty}\cos^2\alpha\cos^2(\sqrt{m}g_At),\\\rho_{33}&=\sum_{m=0}^{\infty}\sin^2\alpha\cos^2(\sqrt{m+1}g_At),\\\rho_{44}&=\sum_{m=0}^{\infty}\cos^2\alpha\sin^2(\sqrt{m}g_At),\end{aligned}$$

$$\rho_{14} = \rho_{41} = 0,$$
$$\rho_{23} = \rho_{32} = \sum_{m=0}^{\infty} \sin\alpha\cos\alpha\cos(\sqrt{m}g_A t)\cos(\sqrt{m+1}g_A t). \tag{8}$$

With the formulae derived above we can calculate the concurrence $C_{AB}(t)$ of atoms AB by virtue of Eq.(3) and analyse their entanglement evolution.

In Fig.1, we plot the dynamical evolution of $C_{AB}(t)$ for the initially vacuum field (a) and thermal field with mean photon numbers $\overline{m} = 0.1$ (b) and $\overline{m} = 1$ (c) for different initial entanglement degrees in terms of α. From the figure we can see that if the cavity is initially in the vacuum, atomic entanglement vanishes only at discrete moments $t = (2l+1)\pi/2g$ with $l = 0, 1, 2, \ldots$. This implies the non-existence of ESD. If, however, the cavity field is initially in the thermal state, atomic entanglement non-smoothly vanishes and remains zero for a finite time before reviving, implying the occurrence of ESD phenomenon. Thus

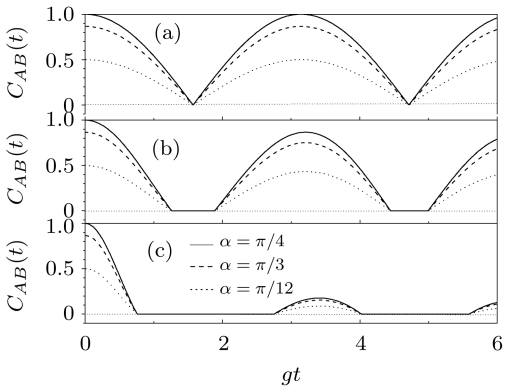


Fig.1. The concurrence $C_{AB}(t)$ as a function of rescaled time gt for the vacuum of cavity field (a) and the thermal state of cavity field with the mean photon numbers $\overline{m} = \overline{n} = 0.1$ (b) and $\overline{m} = \overline{n} = 1$ (c) for different initial entanglement degrees in terms of α.

we can conclude that the thermal field can induce ESD for certain entanglement type which may be more robust (i.e., without ESD) in the vacuum field. Moreover, we can see by comparing Fig.1(b) with Fig.1(c) that the more the mean photon numbers, the faster the entanglement vanishes, and the longer the time for the entanglement remains to be zero before recovering.

Next, we consider the double JCM where two identical two-level atoms A and B coupled with two spatially separate single-mode cavities a and b, respectively. Initially, the atoms A, B are prepared in the Bell-like entangled state (6) while the cavities a, b in a factorable state. The interaction of the system consists of two local interactions in the sense that atom A (B) is coupled only to cavity a (b), implying that the subsystems Aa and Bb are absolutely independent. The local atom-cavity field interaction is described by the well-known JCM, so the total Hamiltonian is of the form

$$H = \sum_{k=A,B} H^k, \tag{9}$$

where ($\hbar = 1$)

$$H^k = \omega a_k^\dagger a_k + \frac{\omega_0}{2}\sigma_z^k + g_k(a_k^\dagger \sigma_-^k + a_k \sigma_k^k). \tag{10}$$

The time evolution operator $U_{Aa(Bb)}(t) \equiv \exp(-\mathrm{i}H^{A(B)}t)$ is for the local interaction of atoms A (B) with cavity a (b). At any time $t > 0$ the state $\rho_{ABab}(0)$ evolves into $\rho_{ABab}(t) = U_{Aa}(t)\,U_{Bb}(t)\rho_{ABab}(0)U_{Aa}^\dagger(t)U_{Bb}^\dagger(t)$. The reduced density matrix $\rho^{AB}(t)$ of the atomic subsystem can be obtained by tracing out $\rho_{ABab}(t)$ over the cavity fields, i.e., $\rho^{AB}(t) = \mathrm{Tr}_{ab}\rho_{ABab}(t)$, which exhibits the X-form as in Eq.(2) with

$$\rho_{11} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}[\cos^2\alpha\cos^2(\sqrt{m}g_A t)\sin^2(\sqrt{n+1}g_B t) + \sin^2\alpha\sin^2(\sqrt{m+1}g_A t)\cos^2(\sqrt{n}g_B t)],$$
$$\rho_{22} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}[\cos^2\alpha\cos^2(\sqrt{m}g_A t)\cos^2(\sqrt{n+1}g_B t) + \sin^2\alpha\sin^2(\sqrt{m+1}g_A t)\sin^2(\sqrt{n}g_B t)],$$
$$\rho_{33} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}[\cos^2\alpha\sin^2(\sqrt{m}g_A t)\sin^2(\sqrt{n+1}g_B t) + \sin^2\alpha\cos^2(\sqrt{m+1}g_A t)\cos^2(\sqrt{n}g_B t)],$$
$$\rho_{44} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}[\cos^2\alpha\sin^2(\sqrt{m}g_A t)\cos^2(\sqrt{n+1}g_B t) + \sin^2\alpha\cos^2(\sqrt{m+1}g_A t)\sin^2(\sqrt{n}g_B t)],$$
$$\rho_{14} = \rho_{41} = 0,$$

$$\rho_{23} = \rho_{32} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}[\sin\alpha\cos\alpha\cos(\sqrt{m}g_A t)\cos(\sqrt{m+1}g_A t)\cos(\sqrt{n+1}g_B t)\cos(\sqrt{n}g_B t)]. \quad (11)$$

With the formulae derived above we can calculate the concurrence $C_{AB}(t)$ of atoms AB by virtue of Eq.(3) and analyse their entanglement evolution.

In Fig.2, we plot the dynamical evolution of $C_{AB}(t)$ for the initial vacuum fields (a) and thermal fields with mean photon numbers $\overline{m} = \overline{n} = 0.1$ (b) and $\overline{m} = \overline{n} = 1$ (c) for different initial entanglement degrees in terms of α and the same coupling strengthes for Aa and Bb. From the figure we can see that if the cavities are initially in the vacuum, atomic entanglement vanishes only at discrete moments $t = (2l+1)\pi/2g$ with $l = 0, 1, 2, \ldots$. This implies non-existence of ESD, the same as shown in Ref.[13]. If, however, both cavities are initially in the thermal states, the atomic entanglement non-smoothly vanishes and remains to be zero for a finite time before reviving, implying the occurrence of ESD phenomenon. Thus we can conclude again that the thermal fields can induce ESD for a certain type of entanglement which may be more robust (i.e., without ESD) in the vacuum fields. In the double JCM, we can also see by comparing Fig.2(b) and Fig.2(c) that the more the mean photon numbers, the faster the entanglement vanishes, and the longer the time for the entanglement remains to be zero before recovering. From a practical view, one should control cavity fields to be in vacuum to avoid the ESD of atomic entanglement.

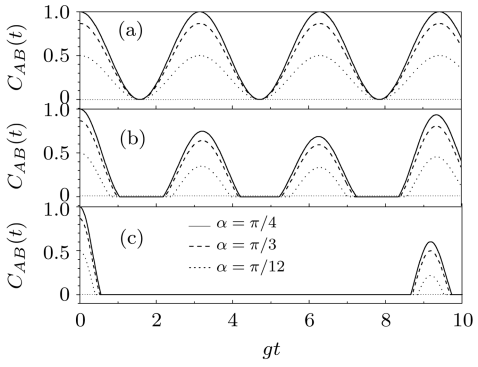


Fig.2. The concurrence $C_{AB}(t)$ as a function of rescaled time gt for the vacuum of cavity fields (a) and the thermal states of cavity fields with the mean photon numbers $\overline{m} = \overline{n} = 0.1$ (b) and $\overline{m} = \overline{n} = 1$ (c) for different initial entanglement degrees in terms of α.

The ESD phenomenon is essentially a representation of the fragility of quantum entanglement in a practical environment. An entangled state whether it exhibits ESD or not depends on the initial state of the entangled system, such as the initial entanglement degree and the entanglement type, as well as the conditions of the environment. For example, among two types of Bell-like states $|\phi_{AB}\rangle \sim |ee\rangle + |gg\rangle$ and $|\psi_{AB}\rangle \sim |eg\rangle + |ge\rangle$, the former undergoes ESD but the later dose not in the same vacuum fields.[13] However, we have shown that the $|\psi_{AB}\rangle$ type state Eq.(6) can instead exhibit ESD in the thermal fields. To understand deeply the phenomenon of ESD, one can resort to the theory of information erasure and recovery since the vanishing of entanglement is an information erasure process while the lost information can be recovered from the environment via necessary operations. In Ref.[31], Cai studied the fundamental theory of information erasure and recovery and constructed an operation that can erase and recover quantum information. The recovery of vanishing entanglement seems to be more interesting because the entanglement is an indispensable resource in quantum information processing. In addition to the double JCM, the ESD phenomenon is found and studied in other models, such as the Tavis–Cummings model (TCM)[32] and double TCM.[33] Some useful methods, e.g., Kerr medium[34] were also proposed to avoid the occurrence of ESD.

In conclusion, we have, by means of concurrence, investigated atomic entanglement dynamics in the thermal field(s). We consider both the single and double JCM. The results show that atomic initial Bell-like state $|\psi(0)\rangle$ (Eq.(6)) does not exhibit ESD in the vacuum field(s) but instead does exhibit in the thermal field(s). In other words the thermal field(s) can induce the finite-time disentanglement but the vacuum fields cannot. Also, the mean photon number is responsible for the time of entanglement vanishing and the time interval for the entanglement remaining to be zero before recovering. Since the ESD sets a limitation on the usefulness of entanglement, it is necessary to know various conditions for the occurrence of ESD and try to prevent it in practice. As we have shown, the thermal field is an important factor for the appearance of ESD. Therefore, keeping the cavities to be empty is a significant way for users to avoid the system suffering from ESD in any quantum information process and quantum computation in cavity QED.

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