Vol 18 No 12, December 2009
1674-1056/2009/18(12)/5133-06

Chinese Physics B

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Probabilistic and robust preparation of a GHZ-type state via atomic ensembles and linear optics*

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(Received 26 December 2008; revised manuscript received 13 January 2009)

This paper proposes two simple and robust schemes to generate an atomic-ensemble Greenberger–Horne–Zeilinger-type (GHZ-type) entangled state via linear optics and single photon detection. These schemes are based on two-photon Hong–Ou–Mandel-type interference, therefore they are insensitive to the phase fluctuation. This advantage will make the realizations of these two schemes easier. One scheme can scale efficiently with the number of ensembles because of the used quantum memory. Both schemes are also robust to the noise and within the reach of current technology.

Keywords: GHZ state, atomic ensemble, linear optics

PACC: 0365, 0367, 4250

Quantum entanglement is the key for many applications, such as the demonstration of the violation of Bell inequality[1] in quantum mechanics, quantum teleportation,[2] swapping,[3] quantum key distribution protocol[4] and quantum computation[5] in quantum information fields and high-precision spectroscopy[6] in metrology fields, and so on. Recently, there have been more efforts to get the entanglement among the multi-subsystem, since, quantum nonlocality becomes more striking with the increase of the subsystems,[7,8] and multi-subsystem entanglement can be used to realize the tele-cloning,[9] to beat the diffraction limit,[10] etc. To date, spontaneously parametric downconversion in a nonlinear optical crystal is a well-known way for generating an entangled two-photon state,[11] and this way can be used to generate a multiphoton entanglement.[12] Very recently, a six-photon entanglement was prepared experimentally.[13] Besides, there are some experiments and proposals to entangle atoms or other particles.[14,15]

Recently, Duan proposed a scheme to entangle many atomic ensembles in GHZ-type based on quantum memory via linear optics and single photon detection.[16] This scheme has the following advantages: it is robust because of built-in fault tolerance; it has a high efficiency because the efficiency is only slow-polynomial decreased with the number of ensembles due to the quantum memory used. In this scheme, an entanglement chain consisted of many two-atomic-ensemble entangled state pairs should be prepared first, then an N-ensemble GHZ-type state could be obtained by post-selection measurement, therefore N entangled ensemble pairs are needed to be prepared in order to generate an N-ensemble GHZ-type state. An entangled state between two atomic ensembles can be realized based on the idea given in Ref.[17]. Very recently, this kind of entangled state was realized experimentally.[18−20] Although this scheme is very simple in principle, it is very hard to be realized in practice. One main disadvantage is that this scheme is based on many phase-sensitive single photon Mach–Zehnder-type interferences. It is not easy to keep the phase of each interferometer stable, especially when two ensembles are far away from each other, because it needs to keep the relative length difference between two interfering beams to be submicrometer order in practice.[21,22] In Refs.[21] and [22], a more robust quantum repeater scheme is proposed to bypass the phase fluctuation over a long distance, which is based on a phase-insensitive Hong–Ou–Mandel-type two-photon interference[23] instead of a phase-sensitive single photon interference. In this paper, we extend this idea to generate a GHZ-type multi-atomic-ensemble state, by which the shortcoming shown above could be overcome. We propose two schemes in this work, one is using quantum memory therefore the preparation can scale efficiently with the number of ensembles in contrast to the other. In our scheme, we use Hong–Ou–Mandel-type two-photon in-

*Project supported by National Natural Science Foundation of China (Grant Nos 10674126 and 10874171), National Fundamental Research Program of China (Grant Nos 2006CB921900 and 2009CB929601), the Innovation Fund from CAS, Program for NCET, and International Cooperation Program from CAS and Ministry of Science and Technology of China.

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terference instead of Mach–Zehnder-type single photon interference. As we know, this type of interference is insensitive to phase instability. The path difference should be kept on the length scale of a photon's coherence length, which is about meter for photon generated from an atomic ensemble, significantly improved in comparison with submicrometer of the scheme in Ref.[16]. Therefore both schemes are phase-insensitive and workable in practice. Besides, our schemes have other advantages over the scheme in Ref.[16], such as an inherent fault tolerance to noise and imperfections, which only lower the successful probability. Another advantage of our schemes is that our proposals are heralded in ideal case, this is conceptually different from the scheme in Ref.[16], where post-selection is inherent to the preparation process even in ideal case.

The basic element of our scheme is an ensemble which consists of many identical atoms, and can be a sample of cold trapped atoms or hot atomic vapor of alkali atoms, for example ^{85}Rb or ^{87}Rb. The atom has a Λ-type energy level structure, which is shown in Fig.1. The state $|g\rangle$ is the ground state, $|s\rangle$ is a metastable state, and $|e\rangle$ is an excited state. The atoms are initially prepared to be in the ground state $|g\rangle$ through optical pumping. In our scheme, we firstly prepare the entanglement between a collective atomic spin excitation and a Stokes photon emitted by spontaneous Raman scattering, this can be realized through a weak pulse applied to the transition from the ground state $|g\rangle$ to the excited state $|e\rangle$ of the atom, so that the forward-scattered Stokes light from transition $|e\rangle$ to $|s\rangle$ with the mean photon number much less than 1 generates. Two qubit states can consist of a spin excitation in, for example, two different parts of an atomic ensemble, like that in Ref.[18], or two different spatial modes of an atomic ensemble,

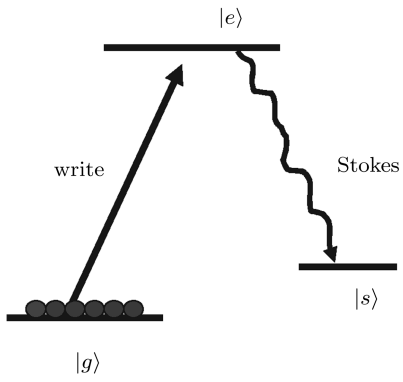


Fig.1. The energy level structure of the atom.

like that in Ref.[24]. By this spontaneous Raman process, the following atom-light field entangled state can be created if we omit the vacuum and high order terms

$$\Psi = \frac{1}{\sqrt{2}}[|H\rangle\,|L\rangle + |V\rangle\,|R\rangle], \tag{1}$$

where, H and V mean horizontal and vertical polarizations of the scattered Stokes photons respectively, L and R are atomic collective spin excitations, which could be in different parts[18] or in different spatial modes[24] of the atomic ensemble.

Now, let us show our scheme 1. We take the generation of a 3-atomic-ensemble GHZ-type state as an example, after that we will extend it to the preparation of a multi-atomic-ensemble entanglement. The schematic set-up is shown in Fig.2. First, we prepare

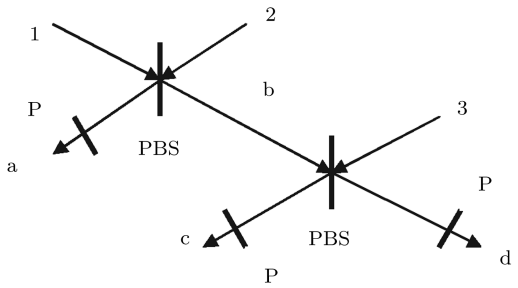


Fig.2. The schematic set-up for a 3-atomic-ensemble GHZ state preparation. PBS: polarization beamsplitter for H and V polarizations. P: polarizer.

three maximal entangled pairs between the collective spin excitation and the Stokes photon in three atomic ensembles by the spontaneous Raman process, which are denoted as Ψ_1, Ψ_2 and Ψ_3 respectively. After that, we let the scattered Stokes photons from the ensembles 1 and 2 input simultaneously to a polarization beamsplitter PBS1. This PBS transmits the H polarization photon and reflects the V polarization photon. If we only consider the case in which there is one photon in each output ports a and b of PBS1, then we have the state

$$\begin{aligned}\Psi_1 \otimes \Psi_2 \;=\; & [|H\rangle_1\,|L\rangle_1 + |V\rangle_1\,|R\rangle_1] \\ & \otimes [|H\rangle_2\,|L\rangle_2 + |V\rangle_2\,|R\rangle_2] \\ \rightarrow\; & |H\rangle_a\,|H\rangle_b\,|L\rangle_1\,|L\rangle_2 \\ & + |V\rangle_a\,|V\rangle_b\,|R\rangle_1\,|R\rangle_2\,. \end{aligned} \tag{2}$$

Here, the normalization factor is omitted. After that, we let the photon in beam b and the scattered Stokes photon from the ensemble 3 input to the polarization beamsplitter PBS2 simultaneously. We still consider the case in which there is only one photon in each output port of a, c and d, then we get the state

$$[|H\rangle_a\,|H\rangle_b\,|L\rangle_1\,|L\rangle_2 + |V\rangle_a\,|V\rangle_b\,|R\rangle_1\,|R\rangle_2]$$

$$\otimes [|H\rangle_3 |L\rangle_3 + |V\rangle_3 |R\rangle_3] \rightarrow |H\rangle_a |H\rangle_c |H\rangle_d |L\rangle_1 |L\rangle_2 |L\rangle_3 + |V\rangle_a |V\rangle_c |V\rangle_d |R\rangle_1 |R\rangle_2 |R\rangle_3 . \quad (3)$$

If we measure these photons in beams a, c and d in $|+\rangle = (1/\sqrt{2})[|H\rangle + |V\rangle]$ polarization, then the state shown by Eq.(3) is projected into the state

$$\Phi = \frac{1}{\sqrt{2}}[|L\rangle_1 |L\rangle_2 |L\rangle_3 + |R\rangle_1 |R\rangle_2 |R\rangle_3]. \quad (4)$$

If we measure them in $|-\rangle = (1/\sqrt{2})[|H\rangle - |V\rangle]$, then the obtained state is

$$\Phi' = \frac{1}{\sqrt{2}}[|L\rangle_1 |L\rangle_2 |L\rangle_3 - |R\rangle_1 |R\rangle_2 |R\rangle_3]. \quad (5)$$

Obviously, both are the atomic ensemble GHZ-type entangled states and are the states we want to prepare. This scheme is heralded, because when we get the photons in each beam a, c, and d at the same time, we know that a GHZ state is created.

In previous discussions, we only consider the case in which only one Stokes photon is generated in spontaneous Raman scattering when we prepare the state Ψ, this means that only one collective spin excitation in an atomic ensemble is assumed to be generated, two and more collective spin excitation terms are omitted because of very lower probabilities in comparison with one excitation probability. In fact, we have to consider these terms, because the total probability of each ensemble generating one collective spin excitation is equal to the probability that one ensemble generates two collective spin excitations and any one of the other two ensembles generates one collective spin excitation, and is also equal to the probability that any one of the ensembles generates three collective spin excitations and the other two ensembles have no spin excitation at all (here the case in which the ensemble 3 generates three excitations has no contribution to successful detection is included. This case occurs with 1/3 probability.). All these cases can contribute to the successful detections, therefore we have to distinguish these cases from the case shown by Eq.(4) or Eq.(5). Fortunately, we could use the post-selection to solve this problem: we only consider the case in which one ensemble generates a photon when the spin excitations of the atomic ensembles are converted to photons for further applications. Therefore a post-selection technique is still needed in practice.

Obviously, it is easy to extend this scheme to generate an N-ensemble GHZ-type entanglement. The schematic experimental set-up is shown in Fig.3. What we need is $N-1$ PBSs, and what we need to do is let the $(i+1)$-th scattered Stokes photon from the $(i+1)$-th ensemble and one photon at one output beam from the $(i-1)$-th PBS overlap at the inputs of the i-th PBS. Finally, only the case in which there is one and only one photon in each output of all $N-1$ PBSs is considered, the obtained state is

$$[|H\rangle_1 |L\rangle_1 + |V\rangle_1 |R\rangle_1] \otimes [|H\rangle_2 |L\rangle_2 + |V\rangle_2 |R\rangle_2] \otimes \cdots \otimes [|H\rangle_N |L\rangle_N + |V\rangle_N |R\rangle_N] \rightarrow |L\rangle_1 |L\rangle_2 |L\rangle_3 \cdots |L\rangle_N |H\rangle_a |H\rangle_c |H\rangle_e \cdots |H\rangle_n + |R\rangle_1 |R\rangle_2 |R\rangle_3 \cdots |R\rangle_N |V\rangle_a |V\rangle_c |V\rangle_e \cdots |V\rangle_n . \quad (6)$$

Here, the normalization factor is omitted for simplicity. If we measure these photons in output beams a, c, e, $\ldots, m, n$ in $|+\rangle$ polarization, then the state consisted of N ensembles will be projected into the state

$$\Theta = \frac{1}{\sqrt{2}}[|L\rangle_1 |L\rangle_2 |L\rangle_3 \cdots |L\rangle_N + |R\rangle_1 |R\rangle_2 |R\rangle_3 \cdots |R\rangle_N]. \quad (7)$$

If we measure these photons in $|-\rangle$ polarization, the state Θ or Θ' will be obtained, where, $\Theta' = (1/\sqrt{2})[|L\rangle_1 |L\rangle_2 |L\rangle_3 \cdots |L\rangle_N - |R\rangle_1 |R\rangle_2 |R\rangle_3 \cdots |R\rangle_N]$. Whether the state Θ or Θ' is obtained depends on the number of the ensembles N. If N is even, the state obtained is Θ. If N is odd, the state obtained is Θ'. Both are the N-ensemble GHZ-type states. Obviously, this scheme is heralded. Of course, here we only consider the ideal case, i.e. there is only one spin excitation in each ensemble. In practice, we have to consider the case in which more than one excitations occur in one ensemble, these cases also contribute to successful detections. This problem can

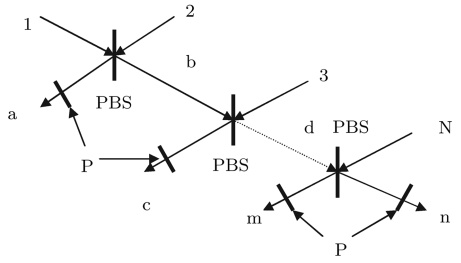


Fig.3. The schematic set-up for an N-atomic ensemble GHZ state preparation.

be solved by postselecting a case in which one ensemble generates a photon when the spin excitations of the atomic ensembles are converted to photons for

future applications, like that in the 3-ensemble case shown above. Therefore the postselection is still used in practice. The main advantage of this scheme is its robustness to the phase fluctuation in comparison with the scheme in Ref.[16]. In Ref.[16], N phase-sensitive single-photon interferences are needed in order to generate an N-ensemble GHZ-type entanglement, this is not easy to be realized in practice because this needs to keep the relative length difference between two interfering beams in each interferometer to be submicrometer order, and needs to control N interferometers. This is very hard to be realized, especially when two atomic ensembles are very far away from each other.[21,22] In Ref.[16], one method is proposed to cancel the phase fluctuations of the interferometers by preparing two pairs of ensembles using the same channel, but this needs to prepare two pairs of ensembles in the same trial, because the phase fluctuation is not only dependent on the channel, but also dependent on time. If the two pairs of ensembles use the channel in different times, the phase fluctuations are different even if they use the same channel. If these two pairs have the same phase fluctuation, it requires that the fluctuation time of the channel should be much larger than the time difference between two preparation processes. In our scheme, we use Hong–Ou–Mandel-type two-photon interference instead. The path difference should be kept on the length scale of a photon's coherence length, which is about meter for a photon generated from an atomic ensemble, significantly improved in comparison with submicrometer of the scheme in Ref.[16]. This makes our scheme easier in comparison with the scheme in Ref.[16], because what we need to do is to let two photon wavepackets overlap at a PBS. Therefore it is possible to generate an N-ensemble GHZ-type entanglement in practice. Besides, our scheme also has some similar advantages with the scheme shown in Ref.[16]: it is inherent fault tolerant to noise and imperfections. The dominant noises are the photon detector inefficiency, the transferring loss of the excitation from the atomic spin state to photon. These only lower the successful probability. One disadvantage of this scheme is the lower efficiency in comparison with the scheme in Ref.[16]. In Ref.[16], the quantum memory is used, the preparation efficiency is sub-polynimical decreased with the increased number of the ensembles. In our scheme, N atomic ensembles are needed to be excited at the same time in order to prepare an N-atomic ensemble GHZ state, this will significantly reduce the successful probability, therefore this scheme cannot be scaled efficiently. We calculate that the successful probability is $P \sim \chi^n(1-\eta)^n/2^{n-1}$, where, χ is the spin excitation probability of the atomic ensemble with $\chi \ll 1$, which can be realized by using a weak pulses applied to the ensemble, η is the overall loss probability, including the detector inefficiency, propagating loss of the photon from the atomic ensemble to detector. Therefore, the total time for preparing an N-ensemble GHZ state is $T \sim 2^{n-1}/(\chi^n(1-\eta)^n)$, which increases exponentially with the number of the ensembles. So we have to make the revision on this scheme. Fortunately, this shortcoming could be overcome by our scheme 2.

In scheme 2, the entanglement swapping technique is used to generate a multi-ensemble GHZ-type state. In Ref.[25], a proposal to generate a more particles entanglement from a less particles entanglement based on entanglement swapping is presented. Inspired by this idea, we design a scheme to generate a more-atomic ensemble GHZ-type state from two less-atomic ensemble GHZ states by using entanglement swapping. In this method, we firstly prepare many 3-atomic-ensemble GHZ states independently by using the way shown in scheme 1, and each 3-ensemble GHZ-type state could be stored because of the built-in quantum memory. When we need to entangle two neighbouring 3-atomic-ensemble states, readable pulses applied to the transition of the atom from the state $|s\rangle$ to the state $|e\rangle$ are used to transfer one spin excitation in for example ensemble 1 of the first 3-atomic-ensemble GHZ state to an anti-Stokes photon, and at the same time one spin excitation in ensemble $1'$ of the second 3-atomic-ensemble GHZ state to another anti-Stokes photon, then let these two photons from two 3-atomic-ensemble GHZ states input to a PBS and make their wavepackets overlap at this PBS. In more detail, we let the first 3-atomic-ensemble GHZ state be $\Phi_{123} = (1/\sqrt{2})[|L\rangle_1|L\rangle_2|L\rangle_3 + |R\rangle_1|R\rangle_2|R\rangle_3]$, and the second GHZ state be $\Phi_{1'2'3'} = (1/\sqrt{2})[|L\rangle_{1'}|L\rangle_{2'}|L\rangle_{3'} + |R\rangle_{1'}|R\rangle_{2'}|R\rangle_{3'}]$. We convert the spin excitation in the ensemble 1 of the first 3-atomic-ensemble GHZ state to an anti-Stokes photon, and at the same time the spin excitation in the ensemble $1'$ of the second 3-atomic-ensemble GHZ state to another anti-Stokes photon, then the obtained state is

$$\begin{aligned} & [|L\rangle_1|L\rangle_2|L\rangle_3 + |R\rangle_1|R\rangle_2|R\rangle_3] \\ & \otimes [|L\rangle_{1'}|L\rangle_{2'}|L\rangle_{3'} + |R\rangle_{1'}|R\rangle_{2'}|R\rangle_{3'}] \\ \rightarrow\ & [|H\rangle_1|L\rangle_2|L\rangle_3 + |V\rangle_1|R\rangle_2|R\rangle_3] \\ & \otimes [|H\rangle_{1'}|L\rangle_{2'}|L\rangle_{3'} + |V\rangle_{1'}|R\rangle_{2'}|R\rangle_{3'}], \end{aligned} \quad (8)$$

where, the normalization factor is omitted. We let the anti-Stokes photons from both GHZ states input to a PBS and make their wavepackets overlap at this PBS. If we only consider the case in which there is only one photon at each output of the PBS, this happens with a half probability, the obtained state is

$$|L\rangle_2 |L\rangle_3 |L\rangle_{2'} |L\rangle_{3'} |H\rangle_1 |H\rangle_{1'} + |R\rangle_2 |R\rangle_3 |R\rangle_{2'} |R\rangle_{3'} |V\rangle_1 |V\rangle_{1'} . \quad (9)$$

If we measure photons in output beams 1 and $1'$ of the PBS in $|+\rangle = (1/\sqrt{2})[|H\rangle + |V\rangle]$ or $|-\rangle = (1/\sqrt{2})[|H\rangle - |V\rangle]$ polarization, the obtained state is

$$|L\rangle_2 |L\rangle_3 |L\rangle_{2'} |L\rangle_{3'} + |R\rangle_2 |R\rangle_3 |R\rangle_{2'} |R\rangle_{3'} . \quad (10)$$

By this way, a 4-atomic-ensemble GHZ state is obtained. The schematic set-up is shown in Fig.4. In this process, a two-photon interference is used, therefore this scheme is still phase-insensitive. We could generate a 6-atomic-ensemble GHZ state from two 4-atomic-ensemble GHZ states by using the same process. Of course, a 5-atomic-ensemble GHZ state can be obtained from a 4-atomic-ensemble GHZ state and a 3-atomic-ensemble GHZ state. There are two ways to prepare an N-ensemble GHZ state. One way is to connect many 3-ensemble GHZ states one by one by using entanglement swapping shown above. In this case, the successful probability is $P \sim [\chi^3(1-\eta)^3/4][(1-\eta_r)^2(1-\eta)^2]^{n-3}$, where, η_r is the retrieving inefficiency from the spin excitation to the anti-Stokes photon in an ensemble and $n > 3$. Although the successful probability is more efficient in comparison with the scheme 1, it still decreases exponentially with the number of the ensembles. We can improve this way to get a much more efficient scaling of the efficiency by using another way. We take the preparation of a 10-ensemble GHZ state as an example. In order for that, eight 3-ensemble GHZ states are prepared firstly, we denote them as Φ_1, Φ_2, ..., Φ_8 respectively. Then we get four 4-ensemble GHZ states $\Phi_{12}, \Phi_{34}, \Phi_{56}$ and Φ_{78} by connecting Φ_1 and Φ_2, Φ_3 and Φ_4, Φ_5 and Φ_6, Φ_7 and Φ_8 respectively using the way shown above. We can further prepare two 6-ensemble GHZ states Φ_{1234} and Φ_{5678} by connecting Φ_{12} and Φ_{34}, Φ_{56} and Φ_{78}. Finally, we get the 10-ensemble GHZ state Φ_{1-8} by connecting Φ_{1234} and Φ_{5678}. In this way, only 3 steps are needed to be done in order to get a wanted state, in contrast to 7 steps in the first way. The calculated successful probability for preparing an N-ensemble GHZ state is $P \sim [\chi^3(1-\eta)^3/4]\Pi_i[(1-\eta_r)^2(1-\eta)^2]^{i+1} \sim [\chi^3(1-\eta)^3/4][(1-\eta_r)^2(1-\eta)^2]^{(1/2)[\log_2(n-2)][\log_2(n-2)+1]}$, where, $n = 2^{i+1}+2$ and $i \geq 0$. This probability decreases subexponentially with n, therefore it can be scaled efficiently with the number of ensembles. Of course, this scheme has the same advantages with the scheme 1 besides the efficient scaling of the efficiency.

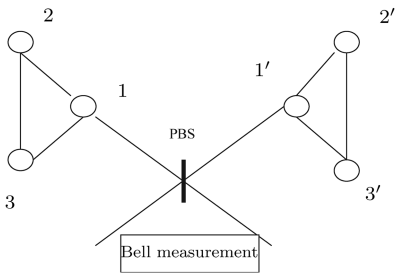


Fig.4. The schematic set-up for generating a 4-atomic-ensemble GHZ state by using entanglement swapping. P: polarizer.

Besides their robustness to phase instability and inherent fault tolerance to noise, another advantage of our schemes is that both schemes are heralded in ideal case, this is conceptually different from the scheme of Ref.[16], where post-selection is inherent to the process even in ideal case.

In the following, we briefly discuss the decoherence in our schemes. The main decoherence effect is the decay of the state $|s\rangle$ with rate of $\sim \Gamma_s$, corresponding to the dephase effect. Here, we assume that all ensembles have the same Γ_s. In our scheme 1, this effect is not needed to be considered because quantum memory is not used for preparing an N-ensemble GHZ state. What we want to point out is that if this GHZ state will be used after a while, then this effect should be considered. It will make the obtained state become a mixed state with the time t, $\rho_1 = F_1 \Theta\Theta^* + (1-F_1)\rho_1'$, where, $F_1 \sim \mathrm{e}^{-n\Gamma_s t}$ and $\rho_1' = |L\rangle_1 |L\rangle_2 |L\rangle_3 \cdots |L\rangle_N \langle L|_1 \langle L|_2 \langle L|_3 \cdots \langle L|_N + |R\rangle_1 |R\rangle_2 |R\rangle_3 \cdots |R\rangle_N \langle R|_1 \langle R|_2 \langle R|_3 \cdots \langle R|_N$. In our scheme 2, this dephase effect makes the 3-ensemble state Φ become a mixed state $\rho = F\ \Phi\Phi^* + (1-F)[|L\rangle_1 |L\rangle_2 |L\rangle_3 \langle L|_1 \langle L|_2 \langle L|_3 + |R\rangle_1 |R\rangle_2 |R\rangle_3 \langle R|_1 \langle R|_2 \langle R|_3]$, where $F \sim \mathrm{e}^{-3\Gamma_s t}$, t is the time keeping the spin excitation in an ensemble. Both terms of ρ will contribute to successful detections in preparing a more ensembles GHZ state, therefore the state we get is not a pure GHZ state, but a mixed state. In N-ensemble case, the state we get is $\rho' = F'\Theta\Theta^* + (1-F')\rho''$, where, ρ'' includes the other terms which contribute to successful detections beside

the term Θ, $F' \sim \mathrm{e}^{-3\Gamma_s \sum_{j=1}^{n-2} t_j}$, the successful probability of geting an N-ensemble GHZ state Θ is $P \sim [\chi^3(1-\eta)^3 \mathrm{e}^{-3\Gamma_s \sum_{j=1}^{n-2} t_j}/4][(1-\eta_r)^2(1-\eta)^2]^{n-3}$ in the way 1, and $P \sim [\chi^3(1-\eta)^3 \mathrm{e}^{-3\Gamma_s \sum_{j=1}^{n-2} t_j}/4]\Pi_i[(1-\eta_r)^2(1-\eta)^2]^{i+1}$ in the way 2, where, $n = 2^{i+1}+2$ and $i \geq 0$, t_j stands for the stored time of the spin excitation in the j-th 3-ensemble GHZ state. One possible method to solve the decoherence is to use the multi-system entanglement purification protocol.[26,27]

In summary, we proposed two schemes to generate an N-ensemble GHZ-type entanglement based on two-photon interference. The main advantage of these schemes is their robustness to phase instability, therefore they are able to be realized in practice. Both schemes are heralded in ideal case. Besides, both schemes have inherent fault tolerance to noise and imperfections. One scheme can scale efficiently with the number of ensembles because quantum memory is used. Both schemes are also robust to the noise and within the reach of current technology.

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