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Theoretical investigation of synchronous totally asymmetric simple exclusion process on lattices with two consecutive junctions in multiple-input-multiple-output traffic system*

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In this paper, we study the dynamics of the synchronous totally asymmetric simple exclusion process (TASEP) on lattices with two consecutive junctions in a multiple-input-multiple-output (MIMO) traffic system, which consists of m sub-chains for the input and the output, respectively. In the middle of the system, there are n ($n < m$) sub-chains via two consecutive junctions linking m sub-chains of input and m sub-chains of output, respectively. This configuration is a type of complex geometry that is relevant to many biological processes as well as to vehicular traffic flow. We use a mean-field approach to calculate this typical geometry and obtain the theoretical results for stationary particle currents, density profiles, and a phase diagram. With the values of m and n synchronously increasing, the vertical phase boundary moves toward the right and the horizontal phase boundary moves toward the upside in the phase diagram. The boundary conditions of the system as well as the numbers of input and output determine the no-equilibrium stationary states, stationary-states phases, and phase boundaries. We use the results to compare with computer simulations and find that they are in very good agreement with each other.

Keywords: TASEPs, Monte Carlo simulations, traffic flow, junction
PACC: 0250

1. Introduction

Due to the rapid development of our economy and society, traffic jams have become one of the important subjects in urban traffic research. More and more people have come to study the subject in recent decades.[1,2]

The traffic bottleneck is one of the main factors which causes traffic congestion. These bottlenecks may be classified according to their formation mechanisms. Among them, the two typical types are the speed-bottleneck, and the road-bottleneck. Recently, people used cellular automaton (CA) traffic models to study traffic problems.[1,3−5] In the present paper, we use totally asymmetric simple exclusion processes (TASEPs) to study the road-bottleneck where a road changes from wide to narrow in the local location (i.e. synchronous TASEPs on lattices with two consecutive junctions in a multiple-input multiple-output (MIMO) traffic system). The TASEPs were used as a powerful tool in studying traffic widely and the two-lane TASEPs with variant lane-changing rules between two lanes are also well studied (e.g. in Refs.[6]–[11]).

Junctions can be seen as a road changing from wide to narrow, connected by junction points, which are one of the commonly used traffic facilities in nature. Recently, TASEPs updated with a one-input two-output junction in random were studied in Ref.[12]. Wang *et al*[13] further investigated TASEPs updated with a multiple-input single-output junction in parallel. These papers described either the merging process or diverging process of traffic flow. However, synchronous TASEPs on lattices with two consecutive junctions in the MIMO traffic system have not been studied so far.

The rest of the present paper is organized as follows. In Section 2 we introduce a model and discuss the synchronous TASEPs on lattices with two consecutive junctions in the MIMO traffic system (see Fig.1). We use the mean-field results to compare with Monte Carlo simulation in Section 3. The summary and conclusions are given in Section 4.

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2. Model and mean-field approach

The synchronous TASEP on lattices with two consecutive junctions in the MIMO traffic system is illustrated in Fig.1. The system consists of m sub-chains for the input and output, respectively. In the middle of the model, there are n ($n < m$) sub-chains via two consecutive junctions linking m sub-chains of input and m sub-chains of output, respectively. In this paper we study the case where $n = m - 1$, the first junction is between site L and site $L + 1$, the second junction is between site $2L$ and $2L+1$. In the system, each sub-chain includes L sites and the particles can move only from the left to the right.

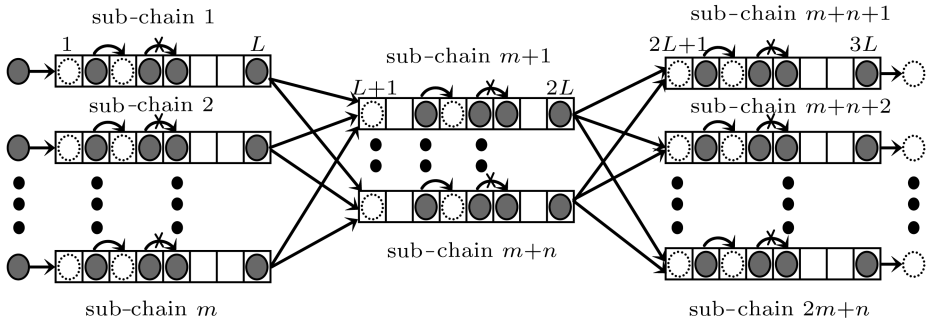


Fig.1. Synchronous TASEP on lattices with two consecutive junctions in the MIMO traffic system.

We have obtained the exactly solvable results of one-dimensional synchronous TASEP in Refs.[14] and [15]. We briefly review these conclusions, as the solution of our consecutive junctions can be derived from them. Three phases can be obtained, i.e. the low density (LD) phase, high density (HD) phase, and maximal current (MC) phase, and a transition line as $\alpha = \beta$. The MC phase (i.e., $J = 0.5$) can be achieved only at $\alpha = \beta = 1$.[15]

(i) If $\alpha < \beta \leq 1$, the system is found in a low density (LD) phase with

$$J = \rho, \quad \rho = \rho_1, \quad \rho_1 = \frac{\alpha}{1+\alpha}, \ \rho_n = \frac{\alpha}{\beta\,(1+\alpha)}, \quad (1)$$

where J is the system current, ρ is the bulk density, and ρ_1 (ρ_n) is the particle density at the first (last) site.

(ii) If $1 \geq \alpha > \beta$, the system is found in a high density (HD) phase with

$$J = 1 - \rho, \quad \rho = \frac{1}{1+\beta},$$

$$\rho_1 = 1 - \frac{\beta}{\alpha\,(1+\beta)}, \quad \text{and} \quad \rho_n = \rho. \qquad (2)$$

(iii) If $\alpha = \beta < 1$, we can obtain a transition line between LD and HD phases.

(iv) If $\alpha = \beta = 1$, the system is found to be in the maximal current (MC) phase with

$$J = 0.5, \quad \rho = 0.5. \qquad (3)$$

From the discussion above, we can draw a conclusion that the problems of TAESPs with consecutive junctions are exactly solvable. For the model, as the current which is at site $L+1$ ($2L+1$) is flow conservation through the system, we have

$$J_1 + J_2 + \cdots + J_m = J_{m+1} + J_{m+2} + \cdots + J_{m+n} = J_{m+n+1} + J_{m+n+2} + \cdots + J_{2m+n} \leq 0.5,$$
$$J_1 = J_2 = \cdots = J_m, \quad J_{m+1} = J_{m+2} = \cdots = J_{m+n}, \quad J_{m+n+1} = J_{m+n+2} = \cdots = J_{2m+n}. \qquad (4)$$

The system can be mapped onto $(2m + n)$ homogeneous TASEP sections (see Fig.2.) In section I, the injection rate and the ejection rate are α and β_{eff1}, respectively. In section II, the injection rate and the ejection rate are α_{eff1} and β_{eff2}, respectively. And in section III, the injection rate and the ejection rate are α_{eff2} and β, respectively. α_{eff1}, α_{eff2}, β_{eff1} and β_{eff2} can be given by[13]

$$\beta_{\text{eff1}} = 1 - \rho_{n+1}, \quad \alpha_{\text{eff1}} = 2\rho_n, \quad \beta_{\text{eff2}} = 1 - \rho_{2n+1}, \quad \text{and} \quad \alpha_{\text{eff2}} = \frac{1}{2}\rho_{2n}. \tag{5}$$

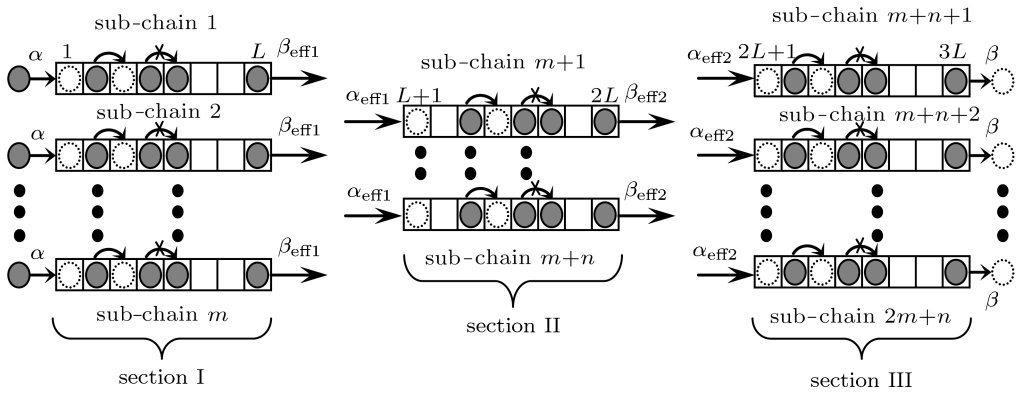


Fig.2. Synchronous TASEPs on lattices with two consecutive junctions in the MIMO traffic system, which can be mapped onto five homogeneous TASEP sections. The effective rates β_{eff1}, α_{eff1}, β_{eff2} and α_{eff2} can be seen in the schematic diagram.

In each of the chains, there are three possible phases (LD, HD, and MC) as they are in a stationary state. So the number of possible stationary phases of the system is equal to 3^3=27. Namely, a stationary state can be one of the following 27 phases, According to expression (4), it is impossible that the maximal current phase exists in section I and section III, since the maximal current in the system is no more than 0.5. Hence, the number of possible phases in the system is reduced to 12, i.e. (LD, LD, LD), (LD, LD, HD), (LD, HD, LD), (LD, HD, HD), (LD, MC, LD), (LD, MC, HD), (HD, LD, LD), (HD, LD, HD), (HD, HD, LD), (HD, HD, HD), (HD, MC, LD), and (HD, MC, HD) phases. Now we can use the same means like that in case 1 to calculate each phase. The results are listed as follows:

i) The LD/LD/LD phase. The conditions for this phase are

$$\alpha < \beta_{\text{eff1}} \le 1, \quad \alpha_{\text{eff1}} < \beta_{\text{eff2}} \le 1, \quad \text{and} \quad \alpha_{\text{eff2}} < \beta \le 1. \tag{6}$$

According to expression (1), the stationary properties of this phase are given by

$$\begin{aligned}
&J_1 = \cdots = J_m = \frac{\alpha}{1+\alpha},\\
&J_{m+1} = \cdots = J_{m+n} = \frac{\alpha_{\text{eff1}}}{1+\alpha_{\text{eff1}}}, \quad \text{and}\\
&J_{m+n+1} = \cdots = J_{2m+n} = \frac{\alpha_{\text{eff2}}}{1+\alpha_{\text{eff2}}},\\
&\rho_n = \frac{\alpha}{\beta_{\text{eff1}}(1+\alpha)}, \quad \rho_{n+1} = \frac{\alpha_{\text{eff1}}}{1+\alpha_{\text{eff1}}},\\
&\rho_{2n} = \frac{\alpha_{\text{eff1}}}{\beta_{\text{eff2}}(1+\alpha_{\text{eff2}})}, \quad \text{and} \quad \rho_{2n+1} = \frac{\alpha_{\text{eff2}}}{1+\alpha_{\text{eff2}}}.
\end{aligned} \tag{7}$$

According to expressions (4) and (5), we can obtain

$$\begin{aligned}
J_1 + \cdots + J_m &= \frac{m\alpha}{1+\alpha} = J_{m+1} + \cdots + J_{m+n}\\
&= \frac{n\alpha_{\text{eff1}}}{1+\alpha_{\text{eff1}}} = J_{m+n+1} + \cdots + J_{2m+n}\\
&= \frac{m\alpha_{\text{eff2}}}{1+\alpha_{\text{eff2}}},\\
\beta_{\text{eff1}} &= 1 - \rho_{n+1} = 1 - \frac{\alpha_{\text{eff1}}}{1+\alpha_{\text{eff1}}}\\
&= \frac{1}{1-\alpha_{\text{eff1}}}.
\end{aligned} \tag{8}$$

Using expression (8), the effective rates α_{eff1}, α_{eff2}, and β_{eff1} can be calculated in terms of inject rate α, respectively to be

$$\begin{aligned}
&\alpha_{\text{eff1}} = \frac{m\alpha}{n + (n-m)\,\alpha},\\
&\alpha_{\text{eff2}} = \alpha, \quad \text{and} \quad \beta_{\text{eff1}} = \frac{n + (n-m)\,\alpha}{n\,(1+\alpha)}.
\end{aligned} \tag{9}$$

Therefore, α_{eff1}, α_{eff2} and β_{eff1} always satisfy conditions (6). Thus, when the system is in the LD/LD/LD phase, the conditions of α and β can be obtained, respectively

$$\alpha < \frac{n}{2m-n}, \quad \text{and} \quad \alpha < \beta. \tag{10}$$

ii) The LD/LD/HD phase. This phase corresponds to the following conditions

$$\begin{aligned}
&\alpha < \beta_{\text{eff1}} \le 1, \quad \alpha_{\text{eff1}} < \beta_{\text{eff2}} \le 1, \quad \text{and}\\
&\beta < \alpha_{\text{eff2}} \le 1.
\end{aligned} \tag{11}$$

According to expressions (1) and (2), we can obtain

$$J_1 = \cdots = J_m = \frac{\alpha}{1+\alpha}, \quad J_{m+1} = \cdots = J_{m+n} = \frac{\alpha_{\text{eff1}}}{1+\alpha_{\text{eff1}}}, \quad \text{and} \quad J_{m+n+1} = \cdots = J_{2m+n} = \frac{\beta}{1+\beta},$$
$$\rho_n = \frac{\alpha}{\beta_{\text{eff1}}(1+\alpha)}, \quad \rho_{n+1} = \frac{\alpha_{\text{eff1}}}{1+\alpha_{\text{eff1}}}, \quad \rho_{2n} = \frac{\alpha_{\text{eff1}}}{\beta_{\text{eff2}}(1+\alpha_{\text{eff2}})}, \quad \text{and} \quad \rho_{2n+1} = \frac{\alpha_{\text{eff2}}(1+\beta)-\beta}{\alpha_{\text{eff2}}(1+\beta)}. \tag{12}$$

According to expresssions (4) and (5), we can have

$$J_1 + \cdots + J_m = \frac{m\alpha}{1+\alpha} = J_{m+1} + \cdots + J_{m+n} = \frac{n\alpha_{\text{eff1}}}{1+\alpha_{\text{eff1}}} = J_{m+n+1} + \cdots + J_{2m+n} = \frac{m\beta}{1+\beta},$$
$$\beta_{\text{eff1}} = 1 - \rho_{n+1} = 1 - \frac{\alpha_{\text{eff1}}}{1+\alpha_{\text{eff1}}} = \frac{1}{1-\alpha_{\text{eff1}}}. \tag{13}$$

From expression (13), it can be easily shown that

$$\alpha = \beta, \quad \alpha_{\text{eff1}} = \frac{m\alpha}{n+(n-m)\alpha}, \quad \text{and} \quad \beta_{\text{eff1}} = \frac{n+(n-m)\alpha}{n(1+\alpha)}. \tag{14}$$

Thus, α_{eff1} and β_{eff1} must satisfy inequality (11). When the system is in the LD/LD/HD phase, we obtain

$$\alpha < \frac{n}{2m-n}, \quad \text{and} \quad \alpha = \beta. \tag{15}$$

iii) The LD/HD/LD phase. The following conditions should be satisfied:

$$\alpha < \beta_{\text{eff1}} \leq 1, \quad \beta < \alpha_{\text{eff1}} \leq 1, \quad \text{and} \quad \alpha_{\text{eff2}} < \beta \leq 1. \tag{16}$$

According to expressions (1) and (2), the stationary currents are given by

$$J_1 = \cdots = J_m = \frac{\alpha}{1+\alpha}, \quad J_{m+1} = \cdots = J_{m+n} = \frac{\beta_{\text{eff1}}}{1+\beta_{\text{eff1}}}, \quad \text{and} \quad J_{m+n+1} = \cdots = J_{2m+n} = \frac{\alpha_{\text{eff2}}}{1+\alpha_{\text{eff2}}},$$
$$\rho_n = \frac{\alpha}{\beta_{\text{eff1}}(1+\alpha)}, \quad \rho_{n+1} = 1 - \frac{\beta_{\text{eff2}}}{\alpha_{\text{eff1}}(1+\beta_{\text{eff2}})}, \quad \rho_{2n} = \frac{1}{1+\beta_{\text{eff2}}}, \quad \text{and} \quad \rho_{2n+1} = \frac{\alpha_{\text{eff2}}}{1+\alpha_{\text{eff2}}}. \tag{17}$$

According to expressions (4) and (5), we can have

$$J_1 + \cdots + J_m = \frac{m\alpha}{1+\alpha} = J_{m+1} + \cdots + J_{m+n} = \frac{n\beta_{\text{eff2}}}{1+\beta_{\text{eff2}}} = J_{m+n+1} + \cdots + J_{2m+n} = \frac{m\alpha_{\text{eff2}}}{1+\alpha_{\text{eff2}}},$$
$$\beta_{\text{eff1}} = 1 - \rho_{n+1} = 1 - \frac{\alpha_{\text{eff1}}}{1+\alpha_{\text{eff1}}} = \frac{1}{1-\alpha_{\text{eff1}}}, \quad \text{and} \quad \beta_{\text{eff2}} = 1 - \rho_{2n+1} = \frac{1}{1+\alpha_{\text{eff2}}}. \tag{18}$$

Using expression (18), α_{eff1}, α_{eff2}, β_{eff1} and β_{eff2} are expressed as

$$\alpha_{\text{eff1}} = \frac{m\alpha}{n\beta_{\text{eff1}}(1+\alpha)}, \quad \alpha_{\text{eff2}} = \alpha, \quad \beta_{\text{eff1}} = \frac{m\alpha}{n\alpha_{\text{eff1}}(1+\alpha)}, \quad \text{and} \quad \beta_{\text{eff2}} = \frac{m\alpha}{n+(n-m)\alpha}. \tag{19}$$

However, we cannot use expressions (16) and (19) to obtain the conditions of α and β, for the LD/HD/LD phase does not exist.

For the same reason, the LD/HD/HD phase, HD/LD/LD phase, HD/LD/HD phase and HD/HD/LD phase do not exist for any conditions of the inject rate α and eject rate β.

iv) The LD/MC/LD phase. The conditions for this case are given by

$$\alpha < \beta_{\text{eff1}} \leq 1, \quad \alpha_{\text{eff1}} = \beta_{\text{eff2}} = 1, \quad \text{and} \quad \alpha_{\text{eff2}} < \beta \leq 1. \tag{20}$$

The currents of this phase in a stationary state are

$$J_1 = \cdots = J_m = \frac{\alpha}{1+\alpha}, \quad J_{m+1} = \cdots = J_{m+n} = \frac{1}{2}, \quad \text{and} \quad J_{m+n+1} = \cdots = J_{2m+n} = \frac{\alpha_{\text{eff2}}}{1+\alpha_{\text{eff2}}},$$
$$\rho_n = \frac{\alpha}{\beta_{\text{eff1}}(1+\alpha)}, \quad \rho_{n+1} = \rho_{2n} = \frac{1}{2}, \quad \text{and} \quad \rho_{2n+1} = \frac{\alpha_{\text{eff2}}}{1+\alpha_{\text{eff2}}}. \tag{21}$$

From expression (4), we obtain

$$J_1 + \cdots + J_m = \frac{m\alpha}{1+\alpha} = J_{m+1} + \cdots + J_{m+n} = \frac{n}{2} = J_{m+n+1} + \cdots + J_{2m+n} = \frac{m\alpha_{\text{eff2}}}{1+\alpha_{\text{eff2}}}. \tag{22}$$

Using expression (22), α and α_{eff2} are expressed as

$$\alpha_{\text{eff2}} = \alpha, \quad \text{and} \quad \alpha = \frac{n}{2m-n}. \tag{23}$$

Thus, the model is in the LD/MC/LD phase when

$$\beta > \alpha, \quad \text{and} \quad \alpha = \frac{n}{2m-n}. \tag{24}$$

v) The LD/MC/HD phase. The corresponding conditions for this phase are

$$\begin{aligned} &\alpha < \beta_{\text{eff1}} \leq 1, \quad \alpha_{\text{eff1}} = \beta_{\text{eff2}} = 1, \quad \text{and} \\ &\beta < \alpha_{\text{eff2}} \leq 1. \end{aligned} \tag{25}$$

According to expressions (1), (2) and (3), we obtain

$$\begin{aligned} &J_1 = \cdots = J_m = \frac{\alpha}{1+\alpha}, \\ &J_{m+1} = \cdots = J_{m+n} = \frac{1}{2}, \quad \text{and} \\ &J_{m+n+1} = \cdots = J_{2m+n} = \frac{\beta}{1+\beta}, \\ &\rho_n = \frac{\alpha}{\beta_{\text{eff1}}(1+\alpha)}, \quad \rho_{n+1} = \rho_{2n} = \frac{1}{2}, \quad \text{and} \\ &\rho_{2n+1} = 1 - \frac{\beta}{\alpha_{\text{eff2}}(1+\beta)}. \end{aligned} \tag{26}$$

From expression (4), we obtain

$$\begin{aligned} J_1 + \cdots + J_m &= \frac{m\alpha}{1+\alpha} = J_{m+1} + \cdots + J_{m+n} \\ &= \frac{n}{2} = J_{m+n+1} + \cdots + J_{2m+n} \\ &= \frac{m\beta}{1+\beta}. \end{aligned} \tag{27}$$

Using expression(27), α and α_{eff2} are expressed as

$$\alpha_{\text{eff2}} = \alpha, \quad \text{and} \quad \alpha = \frac{n}{2m-n}. \tag{28}$$

Hence, the LD/MC/HD phase can exist in the model when

$$\beta = \alpha, \quad \text{and} \quad \alpha = \frac{n}{2m-n}. \tag{29}$$

vi) The HD/HD/HD phase. The conditions for this phase to exist can be written as

$$\begin{aligned} &\beta_{\text{eff1}} < \alpha \leq 1, \quad \beta_{\text{eff2}} < \alpha_{\text{eff1}} \leq 1, \quad \text{and} \\ &\beta < \alpha_{\text{eff2}} \leq 1. \end{aligned} \tag{30}$$

The corresponding expressions for the stationary currents are

$$\begin{aligned} &J_1 = \cdots = J_m = \frac{\beta_{\text{eff1}}}{1+\beta_{\text{eff1}}}, \\ &J_{m+1} = \cdots = J_{m+n} = \frac{\beta_{\text{eff2}}}{1+\beta_{\text{eff2}}}, \quad \text{and} \\ &J_{m+n+1} = \cdots = J_{2m+n} = \frac{\beta}{1+\beta}, \\ &\rho_n = \frac{1}{1+\beta_{\text{eff1}}}, \quad \rho_{n+1} = 1 - \frac{\beta_{\text{eff2}}}{\alpha_{\text{eff1}}(1+\beta_{\text{eff2}})}, \\ &\rho_{2n} = \frac{1}{1+\beta_{\text{eff2}}}, \quad \text{and} \quad \rho_{2n+1} = \frac{\alpha_{\text{eff2}}(1+\beta) - \beta}{\alpha_{\text{eff2}}(1+\beta)}. \end{aligned} \tag{31}$$

According to expressions (4) and (5), we can obtain

$$\begin{aligned} J_1 + \cdots + J_m &= \frac{m\beta_{\text{eff1}}}{1+\beta_{\text{eff1}}} = J_{m+1} + \cdots + J_{m+n} \\ &= \frac{n\beta_{\text{eff2}}}{1+\beta_{\text{eff2}}} = J_{m+n+1} + \cdots + J_{2m+n} \\ &= \frac{m\beta}{1+\beta}, \\ \beta_{\text{eff2}} &= 1 - \rho_{2n+1} \\ &= 1 - \frac{\alpha_{\text{eff2}}(1+\beta) - \beta}{\alpha_{\text{eff2}}(1+\beta)}, \quad \text{and} \\ \alpha_{\text{eff2}} &= \frac{1}{2}\rho_{2n} = \frac{1}{2+2\beta_{\text{eff2}}}. \end{aligned} \tag{32}$$

According to expression (32), we can obtain

$$\beta_{\text{eff1}} = \beta, \quad \text{and} \quad \beta_{\text{eff2}} = \frac{m\beta}{n+(n-m)\beta}. \tag{33}$$

When expression (33) satisfies the condition (26), we can obtain the corresponding conclusions about α and β, i.e.

$$\alpha > \beta, \quad \text{and} \quad \beta < \frac{n}{2m-n}. \tag{34}$$

vii) The HD/MC/LD phase. The following conditions can be determined:

$$\begin{aligned} &\beta_{\text{eff1}} < \alpha \leq 1, \quad \alpha_{\text{eff1}} = \beta_{\text{eff2}} = 1, \quad \text{and} \\ &\alpha_{\text{eff2}} < \beta \leq 1. \end{aligned} \tag{35}$$

The stationary properties of this phase are obtained from expressions (1), (2) and (3) as follows:

$$\begin{aligned} &J_1 = \cdots = J_m = \frac{\beta_{\text{eff1}}}{1+\beta_{\text{eff1}}}, \\ &J_{m+1} = \cdots = J_{m+n} = \frac{1}{2}, \quad \text{and} \\ &J_{m+n+1} = \cdots = J_{2m+n} = \frac{\alpha_{\text{eff2}}}{1+\alpha_{\text{eff2}}}, \\ &\rho_n = \frac{1}{1+\beta_{\text{eff1}}}, \quad \rho_{n+1} = \rho_{2n} = tfrac12, \quad \text{and} \\ &\rho_{2n+1} = \frac{\alpha_{\text{eff2}}}{1+\alpha_{\text{eff2}}}. \end{aligned} \tag{36}$$

From expression (4), we obtain

$$\begin{aligned} J_1 + \cdots + J_m &= \frac{m\beta_{\text{eff1}}}{1+\beta_{\text{eff1}}} = J_{m+1} + \cdots + J_{m+n} \\ &= \frac{n}{2} = J_{m+n+1} + \cdots + J_{2m+n} \\ &= m\alpha_{\text{eff2}}/(1+\alpha_{\text{eff2}}). \end{aligned} \tag{37}$$

The stationary condition for the particle currents helps to determine the α_{eff2} and β_{eff1}

$$\alpha_{\text{eff2}} = \beta_{\text{eff1}}, \quad \text{and} \quad \beta_{\text{eff1}} = n/(2m-n). \tag{38}$$

According to phase existence requirements (35), we can obtain the final conditions of α and β

$$\alpha > n/(2m-n), \quad \text{and} \quad \beta > n/(2m-n). \tag{39}$$

viii) The HD/MC/HD phase, which is specified by the following conditions:

$$\beta_{\text{eff1}} < \alpha \leq 1, \quad \alpha_{\text{eff1}} = \beta_{\text{eff2}} = 1, \ \beta < \alpha_{\text{eff2}} \leq 1. \tag{40}$$

The stationary currents are given by

$$\begin{aligned} &J_1 = \cdots = J_m = \frac{\beta_{\text{eff1}}}{1+\beta_{\text{eff1}}}, \\ &J_{m+1} = \cdots = J_{m+n} = \frac{1}{2}, \quad \text{and} \\ &J_{m+n+1} = \cdots = J_{2m+n} = \frac{\beta}{1+\beta}, \\ &\rho_n = \frac{1}{1+\beta_{\text{eff1}}}, \quad \rho_{n+1} = \rho_{2n} = \frac{1}{2}, \quad \text{and} \\ &\rho_{2n+1} = 1 - \frac{\beta}{\alpha_{\text{eff2}}\,(1+\beta)}. \end{aligned} \tag{41}$$

From expression (4), we can have the results

$$\begin{aligned} J_1 + \cdots + J_m &= \frac{m\beta_{\text{eff1}}}{1+\beta_{\text{eff1}}} = J_{m+1} + \cdots + J_{m+n} \\ &= \frac{n}{2} = J_{m+n+1} + \cdots + J_{2m+n} \\ &= \frac{m\beta}{1+\beta}. \end{aligned} \tag{42}$$

According to expression (42), we can obtain

$$\beta_{\text{eff1}} = \beta = \frac{n}{2m-n}. \tag{43}$$

According to phase existence requirement (36), we can have the final conditions of α and β as follows:

$$\alpha > \beta, \quad \text{and} \quad \beta = \frac{n}{2m-n}. \tag{44}$$

From the analysis above, one can see that there are seven possible phases, i.e. (LD/LD/LD), (LD/LD/HD), (HD/HD/HD), (LD/MC/HD), (HD/MC/HD), (LD/MC/LD), and (HD/MC/LD)] in the system. Figure 3(a) shows the possible phase boundaries for $m = 2$, 3, and 4, corresponding to $n = 1$, 2, and 3. With the values of m and n increasing, we can predict that the vertical phase boundary moves toward the right and the horizontal phase boundary moves toward the upside in the phase diagram, which means that when the values of m and n go to infinity, this model will change to TASEPs with one lane model. The phase diagram for $m = 2$ and $n = 1$ is also shown in Fig.3(b), and one can see the following points: 1) the (LD/LD/HD) phase corresponds to the transition line between the (LD/LD/LD) and the (HD/HD/HD) phases specified by $\alpha = \beta$ and $\alpha < n/(2m-n)$; 2) the (LD/MC/LD) phase is the transition phase between the (LD/LD/LD) and the (HD/MC/LD) phases specified by $\alpha = n/(2m-n)$ and $\alpha < \beta$; 3) the (HD/MC/HD) phase is

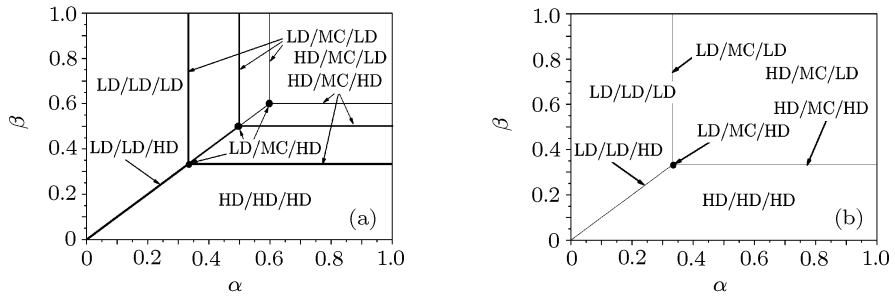


Fig.3. (a) Phase boundaries (or transition lines) for $m = 2$, 3, and 4, corresponding to $n = 1$, 2, and 3 in a synchronous TASEP with two consecutive junctions, and (b) phase diagram for $m = 2$ and $n = 1$ in a synchronous TASEP with two consecutive junctions. The vertical solid line and horizontal solid line represent the (LD/LD/HD) and (HD/MC/HD) phases specified by $\alpha = n/(2m-n)$, $\beta > \alpha$ and $\beta = n/(2m-n)$, $\alpha > \beta$, respectively; the black oval corresponds to the (LD/MC/HD) phase specified by $\alpha = n/(2m-n)$ and $\beta = \alpha$; and the diagonal solid line corresponds to the (LD/LD/HD) phase specified by $\alpha < n/(2m-n)$ and $\beta = \alpha$.

the transition line between the (HD/HD/HD) and (HD/MC/LD) phases specified by $\beta = n/(2m-n)$ and $\alpha > \beta$. The values of α and β are determined by the values of m and n.

We can see that the non-equilibrium stationary states, stationary-state, and the phase boundaries are determined by the boundary conditions of the system as well as by the number of sub-chains. In other words, the dynamics of the system is determined by its environment and its own structure.

3. Monte Carlo simulation result and discussion

We use computer simulation to validate our theoretical analysis. Here, we present only a synchronous TASEP on lattices with two consecutive junctions in the MIMO traffic system, that is, $m = 2$ and $n = 1$. The numbers of sites and the sub-chains are all equal to 500. In simulations, one can obtain the stationary density profiles by averaging 10^8 sampling at each site. The first 10^5 N time steps are discarded as transients.

The density profiles for (LD/LD/LD), (HD/HD/HD), (HD/MC/LD), (LD/MC/LD) and (HD/MC/HD) phases are shown in Fig.4. We illustrate only the density properties of sub-chain 1, sub-chain $(m+1)$ and sub-chain $(m+n+1)$, for the density properties of the other sub-chains are essentially the same as those of sub-chains above, respectively. It is found that Monte Carlo simulations and mean-field analysis are well consistent with each other (see Figs.4(a)–4(g)), thereby verifying our theoretical results.

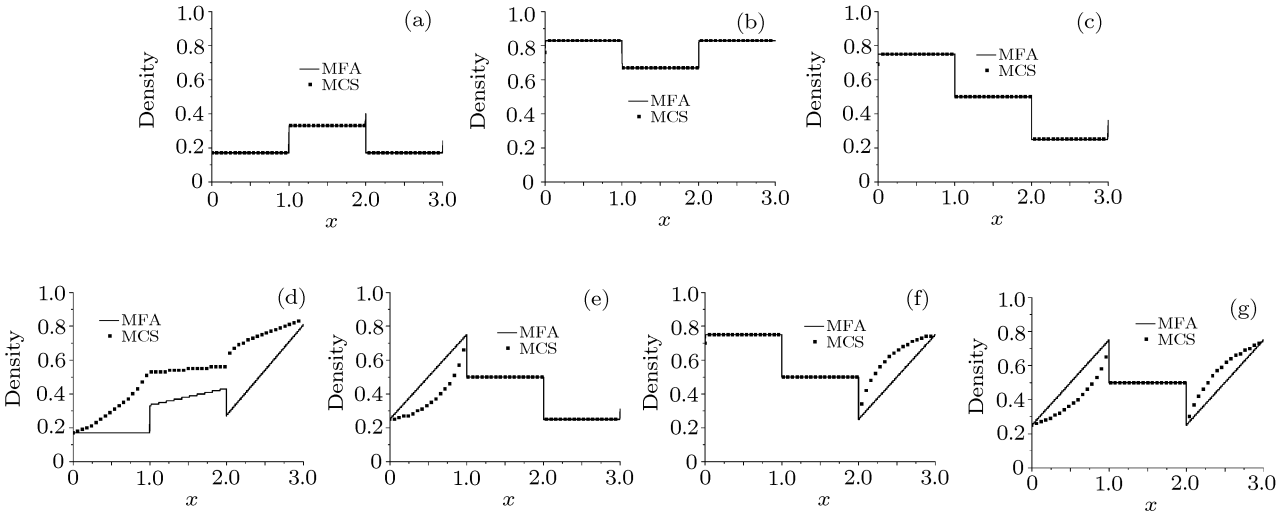


Fig.4. Density profiles obtained from our theoretical calculations and Monte Carlo simulations when $m = 2$ and $n = 1$: (a) (LD/LD/LD), (b) (HD/HD/HD), (c) (HD/MC/LD), (d) (LD/LD/HD), (e) (LD/MC/LD), (f) (HD/MC/HD) and (g) (LD/MC/HD) phases. The parameters are (a) α=0.2, β=0.7, (b) α=0.7, β=0.2, (c) α=0.7, β=0.7, (d) α=0.2, β=0.2, (e) α=0.333, β=0.333, (f) α=0.8, β=0.333, and (g) α=0.333, β=0.333.

4. Summary and conclusions

Two consecutive junctions in the MIMO models are relevant to vehicular and pedestrian traffic flow. Synchronous TASEPs with two consecutive junctions in the MIMO models are studied in this paper. Our theoretical analysis suggests that there are seven possible stationary phases, i.e. (LD/LD/LD), (HD/HD/HD), (HD/MC/LD), (LD/LD/HD), (LD/MC/LD), (HD/MC/HD) and (LD/MC/HD). In the model, the non-equilibrium stationary state, the stationary-state phases, and the phase boundaries are determined by the boundary conditions of the system as well as by the values of m and n. The density profiles are simulated and shown to be in good agreement with the theoretical analysis.

In the model, section II is the bottleneck of the system. The vertical boundary shifts toward the right and the horizontal boundary shifts toward the upside in the phase diagram when the values of m and n increase.

When the values of m and n go to infinity infinite, the system will change to TASEPs with a one lane system. It implies that the bottleneck effect will decrease when the numbers of section I and section III increase.

We can use the approach directly to study another case, i.e. the synchronous TASEPs on lattices with two consecutive junctions in the MIMO traffic system with $m < n$.

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