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Dispersion relations of lattice waves in three-dimensional strongly coupled complex plasma crystals*

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Dispersion relation matrices, with the screened Coulomb interaction between a charged dust particle and all other particles taken into account, are derived for waves in body centred cubic (bcc) and face centred cubic (fcc) lattices in three-dimensional strongly coupled complex plasma crystals separately. The matrices are then calculated in characteristic directions to obtain the longitudinal and transverse eigenmodes. The longitudinal and transverse waves for these cases are discussed separately.

Keywords: plasma crystals, dispersion relation matrices

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1. Introduction

Strongly coupled complex (dusty) plasma studies have been of substantial interest in the recent decade, particularly due to the fact that 'plasma crystals' were observed for the first time in experiment.[1,2] Dust particles are highly charged in plasmas and hence strongly coupled to each other through their mutual Coulomb interaction. Due to the Debye screening of a charged particle in plasmas, the interparticle potential, $\Phi(r)$, is considered to be of Debye–Yukawa type, i.e. $\Phi(r) = Q\,\mathrm{e}^{-r/\lambda_\mathrm{D}}/r$ (the CGS system is used in this paper), where λ_D is the Debye length and Q is the charge quantity of a dust particle. The intensity of the interaction is characterized by the coupling parameter $\Gamma = Q^2/\Delta T$, where Δ and T are the interparticle distance and dust temperature respectively. As Γ exceeds a critical value Γ_c, the dust component can be crystallized into a plasma crystal.

Experiments have shown that dust lattice waves (DLWs) can be excited in plasma crystals.[3−12] Most of them are for one-dimensional linear chains or two-dimensional hexagonal crystals. Theoretical modelling and analysis have also been carried out for waves propagating in those strongly coupled media.[5−8,13−28] The simplest model for the waves in plasma crystals is the one-dimensional linear approximation of dust chains.[3,13−16,27,28] For two-dimensional plasma crystals, Wang *et al* have developed a theoretical treatment of longitudinal and transverse modes in Debye–Yukawa crystals including the effects of damping.[17] The dispersion relation is derived and used for two-dimensional hexagonal crystals. It is also applied to three-dimensional bcc and fcc crystals in the long wavelength regime separately. The theoretical predictions are in agreement with the experimental measurements. Uchida *et al* have also used a molecular dynamics simulation to derive the wave dispersion relation in a two-dimensional plasma crystal in the presence of a constant magnetic field.[19] They have first reported two new wave modes named 'upper-hybrid and lower-hybrid dust lattice wave'. Duan *et al* have further investigated longitudinal and transverse particle vibrations in a two-dimensional hexagonal lattice, induced by the Debye–Yukawa interaction between microparticles with only its nearest neighbour interaction.[20] Vladimirov *et al* have derived a dispersion relation also in the nearest neighbour interaction approximation which explicitly depends on the main plasma parameters and the characteristic direction of the mode.[21] Farokhi *et al* have studied the dynamics of a two-dimensional hexagonal crystal, searching for a dispersion relation for longitudinal and transverse DLWs.[22,23] A ground state of an externally confined one-component Yukawa plasma has also been tested for three-dimensional

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spherical Coulomb crystals.[26] Meanwhile, although a few experiments in three-dimensional plasma crystals have been carried out, due to the limitations of ground-based experiments, it is necessary to derive the appropriate dispersion relation for three-dimensional plasma crystals theoretically.

In the present paper we generalize the theoretical treatment for DLWs in two-dimensional plasma crystals to the three-dimensional case. The general dispersion is obtained, with the screened Coulomb interaction between a charged dust particle and all other particles taken into account, in Section 2. In Section 3, we apply the general dispersion to different characteristic directions for bcc and fcc lattices to obtain various eigenmodes of DLWs, separately. Finally, a summary and a discussion are given in Section 4.

2. General dispersion relation

We consider three-dimensional infinite extended plasma crystals with periodic boundary conditions and assume that the dust population is cold. The three-dimensional dispersion relation of such a plasma crystal can then be derived from the equation of motion for the charged dust particle at the origin,[17]

$$\frac{\mathrm{d}^2\tilde{\boldsymbol{r}}(X,Y,Z)}{\mathrm{d}t^2}+\nu\frac{\mathrm{d}\tilde{\boldsymbol{r}}(X,Y,Z)}{\mathrm{d}t} = \frac{Q}{M}\boldsymbol{E}(\tilde{\boldsymbol{r}}(X,Y,Z),t), \tag{1}$$

where the electrical field is generated by the screened Coulomb interaction between the charge at the origin and all other particles, ν is the frictional drag coefficient. Let Δ be the interparticle spacing, $\kappa=\Delta/\lambda_{\mathrm{D}}$ the screening parameter, and $\omega_0=(Q^2/M\Delta^3)^{1/2}$ the dust plasma frequency,[17] and then the space and the time will be rescaled by Δ and ω_0 respectively, therefore $(X,Y,Z,R)/\Delta\rightarrow(X,Y,Z,R)$, $\lambda_{\mathrm{D}}/\Delta\rightarrow\lambda_{\mathrm{D}}$, $\boldsymbol{k}\Delta\rightarrow\boldsymbol{k}$, $(\omega,\nu)/\omega_0\rightarrow(\omega,\nu)$, and $(\xi,\eta,\varsigma)=(\xi_0,\eta_0,\varsigma_0)\exp\left[\mathrm{i}(k_xX+k_yY+k_zZ)-\mathrm{i}\omega t\right]$. The general dispersion relation of a plasma crystal can then be derived as

$$\omega(\omega+\mathrm{i}\nu)\xi_0=F_{11}\xi_0+F_{12}\eta_0+F_{13}\zeta_0, \tag{2a}$$
$$\omega(\omega+\mathrm{i}\nu)\eta_0=F_{21}\xi_0+F_{22}\eta_0+F_{23}\zeta_0, \tag{2b}$$
$$\omega(\omega+\mathrm{i}\nu)\zeta_0=F_{31}\xi_0+F_{32}\eta_0+F_{33}\zeta_0, \tag{2c}$$

where (due to the symmetry of the system)

$$F_{11}=2\sum_{X,Y,Z}F(X,Y,Z)\sin^2\left(\frac{k_xX+k_yY+k_zZ}{2}\right),$$
$$F_{22}=2\sum_{X,Y,Z}F(Y,Z,X)\sin^2\left(\frac{k_xX+k_yY+k_zZ}{2}\right),$$
$$F_{33}=2\sum_{X,Y,Z}F(Z,X,Y)\sin^2\left(\frac{k_xX+k_yY+k_zZ}{2}\right),$$
$$F_{12}=F_{21}=2\sum_{X,Y,Z}G(X,Y,Z)\times\sin^2\left(\frac{k_xX+k_yY+k_zZ}{2}\right),$$
$$F_{13}=F_{31}=2\sum_{X,Y,Z}G(Z,X,Y)\times\sin^2\left(\frac{k_xX+k_yY+k_zZ}{2}\right),$$
$$F_{23}=F_{32}=2\sum_{X,Y,Z}G(Y,Z,X)\times\sin^2\left(\frac{k_xX+k_yY+k_zZ}{2}\right)$$

with

$$F(X,Y,Z)=\frac{1}{R^5}\mathrm{e}^{-\kappa R}[X^2\left(3+3\kappa R+\kappa^2R^2\right)-R^2\left(1+\kappa R\right)],$$
$$G(X,Y,Z)=\frac{XY}{R^5}\mathrm{e}^{-\kappa R}\left(3+3\kappa R+\kappa^2R^2\right),$$

and $\omega(\omega+\mathrm{i}\nu)$ and $(\xi_0,\eta_0,\varsigma_0)$ are eigenvalues and eigenvectors of the symmetric matrix

$$\boldsymbol{\Psi}=\begin{pmatrix}F_{11}&F_{12}&F_{13}\\F_{21}&F_{22}&F_{23}\\F_{31}&F_{32}&F_{33}\end{pmatrix}. \tag{3}$$

We call matrix $\boldsymbol{\Psi}$ the general dispersion relation matrix. We can obtain the dispersion relation for any symmetric lattice and any wave by calculating the eigenvalue of matrix $\boldsymbol{\Psi}$.

3. Wave dispersion relations in three characteristic directions

The eigenmodes of DLWs can be found in characteristic directions. In the coordinates with the three edges of the lattice cell as the x-, y-, and z-axes, the characteristic vectors of the lattice cell are (1, 0, 0), (1, 1, 0)/$\sqrt{2}$, and (1, 1, 1)/$\sqrt{3}$, shown in Fig.1(a). We consider a crystal lattice with dust particles arranged in the vertexes and the centre of a bcc cell (Fig.1(b)), or in the vertexes and each face centre of an fcc cell (Fig.1(c)), and then derive the dispersion relation for the eigenmodes propagating along these characteristic directions.

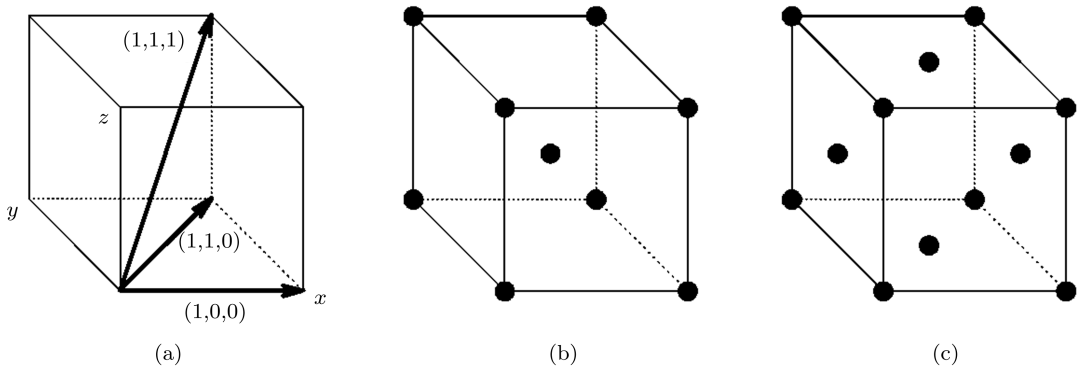


Fig.1. Dust lattices: (a) lattice cell and characteristic vectors, (b) bcc lattice, (c) fcc lattice.

3.1. Waves in the (1, 0, 0) direction

For the eigenmode in the (1, 0, 0) direction,

$$\boldsymbol{k} = (k_x, k_y, k_z) = k(1, 0, 0)\,,$$

the general dispersion relation matrix then is

$$\boldsymbol{\Psi} = \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_2 \end{pmatrix}. \tag{4}$$

The dispersion relations can then be obtained as follows:

$$\omega_l(\omega_l + \mathrm{i}\nu) = f_1, \tag{5a}$$

$$\omega_t(\omega_t + \mathrm{i}\nu) = f_2. \tag{5b}$$

Expression (5a) represents the dispersion relation of the longitudinal waves oscillating parallel to (1, 0, 0) and expression (5b) represents the dispersion relation of the transverse waves oscillating perpendicularly to (1, 0, 0). The two dispersions for the bcc and fcc lattices are then shown in Figs.2(a) and 2(b) respectively.

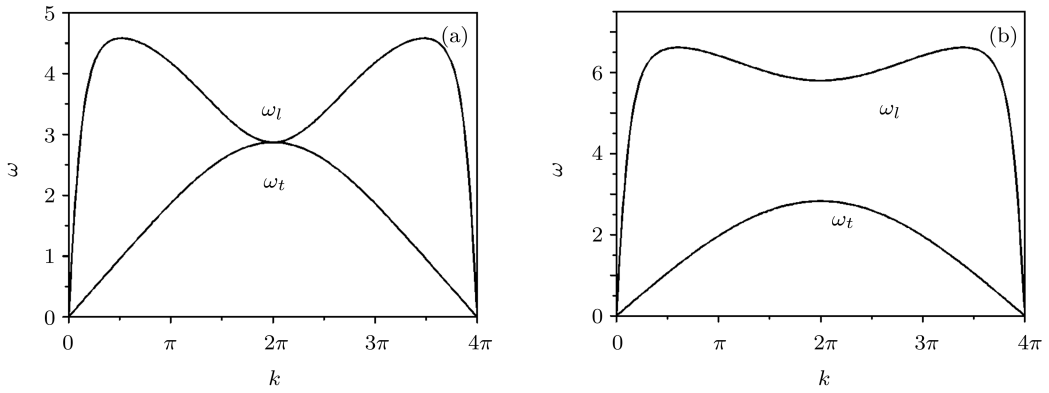


Fig.2. Dispersion relations of the longitudinal mode ω_l and transverse mode ω_t propagating in the direction (1, 0, 0), in (a) bcc lattice; and (b) fcc lattice separately.

The spring constants f_j $(j = 1,\ 2)$ are given as follows: for the bcc lattice,

$$f_j = 4\sum_{m=1}^{+\infty}\sum_{n,l=-\infty}^{+\infty}\left[g_j\,(m,n,l)\sin^2\frac{km}{2} + g_j\left(m-\frac{1}{2}, n-\frac{1}{2}, l-\frac{1}{2}\right)\sin^2\frac{k(2m-1)}{4}\right],$$

and for the fcc lattice,

$$f_j = 4\sum_{m=1}^{+\infty}\sum_{n,l=-\infty}^{+\infty}\left[g_j\left(m,n,l\right)\sin^2\frac{km}{2} + g_j\left(m,n-\frac{1}{2},l-\frac{1}{2}\right)\sin^2\frac{km}{2} + g_j\left(m-\frac{1}{2},n,l-\frac{1}{2}\right)\sin^2\frac{k(2m-1)}{4} + g_j\left(m-\frac{1}{2},n-\frac{1}{2},l\right)\sin^2\frac{k(2m-1)}{4}\right],$$

with $g_1(m,n,l) \equiv F(m,n,l)$, and $g_2(m,n,l) \equiv F(n,l,m)$.

3.2. Waves in the (1, 1, 0) direction

For the eigenmode in the (1, 1, 0) direction, $\boldsymbol{k} = (k_x, k_y, k_z) = k(1,1,0)/\sqrt{2}$, the general dispersion relation matrix then is

$$\boldsymbol{\Psi} = \begin{pmatrix} f_1 & f_2 & 0 \\ f_2 & f_1 & 0 \\ 0 & 0 & f_3 \end{pmatrix}. \tag{6}$$

The dispersion relations can then be obtained as

$$\omega_l(\omega_l + \mathrm{i}\nu) = f_1 + f_2, \tag{7a}$$

$$\omega_{t1}(\omega_{t1} + \mathrm{i}\nu) = f_1 - f_2, \tag{7b}$$

$$\omega_{t2}(\omega_{t2} + \mathrm{i}\nu) = f_3. \tag{7c}$$

Equation (7a) represents the dispersion relation of the longitudinal waves oscillating parallel to (1, 1, 0), Eq.(7b) represents the dispersion relation of the transverse waves oscillating in the (1, –1, 0) direction and Eq.(7c) represents the dispersion relation of the transverse waves oscillating in the (0, 0, 1) direction. The three dispersions for the bcc and the fcc lattices are then shown in Figs.3(a) and 3(b) respectively.

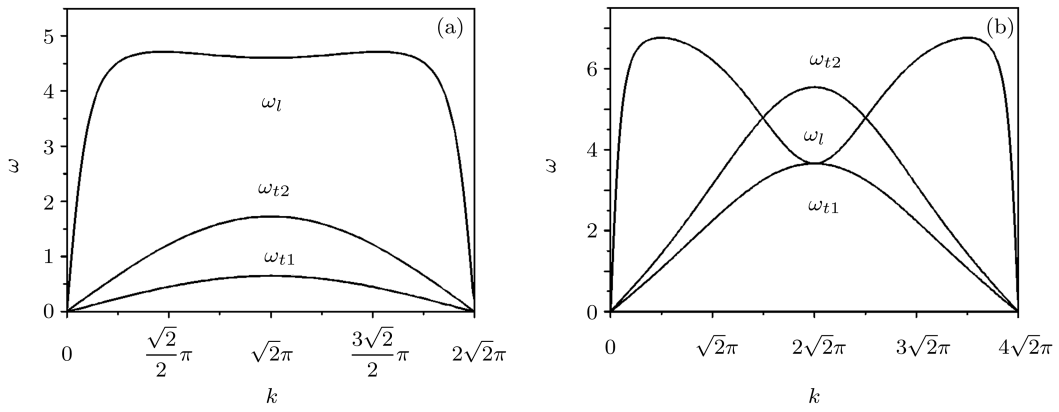


Fig.3. Dispersion relations of the longitudinal mode ω_l and transverse modes ω_{t1}, ω_{t2} propagating in the direction (1, 1, 0), in (a) bcc lattice and (b) fcc lattice separately.

The spring constants f_j ($j = 1,\ 2,\ 3$) are given as follows: for the bcc lattice,

$$f_j = 2\sum_{m,n=-\infty}^{+\infty} g_j\left(m,n,0\right)\sin^2\frac{k(m+n)}{2\sqrt{2}} + 4\sum_{l=1}^{+\infty}\sum_{m,n=-\infty}^{+\infty}\left[g_j\left(m,n,l\right)\sin^2\frac{k(m+n)}{2\sqrt{2}} + g_j\left(m-\frac{1}{2},n-\frac{1}{2},l-\frac{1}{2}\right)\sin^2\frac{k(m+n-1)}{2\sqrt{2}}\right],$$

for the fcc lattice,

$$f_j = 2\sum_{m,n=-\infty}^{+\infty}\left[g_j\left(m,n,0\right)\sin^2\frac{k(m+n)}{2\sqrt{2}} + g_j\left(m-\frac{1}{2},n-\frac{1}{2},0\right)\sin^2\frac{k(m+n-1)}{2\sqrt{2}}\right]$$

$$+4\sum_{l=1}^{+\infty}\sum_{m,n=-\infty}^{+\infty}\left[g_j\left(m,n,l\right)\sin^2\frac{k(m+n)}{2\sqrt{2}}+g_j\left(m-\frac{1}{2},n-\frac{1}{2},l\right)\sin^2\frac{k(m+n-1)}{2\sqrt{2}}\right.$$
$$\left.+g_j\left(m-\frac{1}{2},n,l-\frac{1}{2}\right)\sin^2\frac{k(2m+2n-1)}{4\sqrt{2}}+g_j\left(m,n-\frac{1}{2},l-\frac{1}{2}\right)\sin^2\frac{k(2m+2n-1)}{4\sqrt{2}}\right],$$

where $g_1(m,n,l)\equiv F(m,n,l)$, $g_2(m,n,l)\equiv G(m,n,l)$, and $g_3(m,n,l)\equiv F(l,m,n)$.

3.3. Waves in the (1, 1, 1) direction

For the eigenmode in the (1, 1, 1) direction,

$$\boldsymbol{k}=(k_x,k_y,k_z)=k(1,1,1)/\sqrt{3},$$

the general dispersion relation matrix then is

$$\boldsymbol{\Psi}=\begin{pmatrix}f_1 & f_2 & f_2\\ f_2 & f_1 & f_2\\ f_2 & f_2 & f_1\end{pmatrix}. \tag{8}$$

The dispersion relations can then be obtained as

$$\omega_l(\omega_l+\mathrm{i}\nu)=f_1+2f_2, \tag{9a}$$
$$\omega_t(\omega_t+\mathrm{i}\nu)=f_1-f_2. \tag{9b}$$

Equation (9a) represents the dispersion relation of the longitudinal waves oscillating parallel to the (1, 1, 1) direction, and Eq.(9b) represents the dispersion relation of the transverse waves oscillating perpendicular to the (1, 1, 1) direction. The two dispersions for the bcc and fcc lattices are then shown in Figs.4(a) and 4(b), respectively.

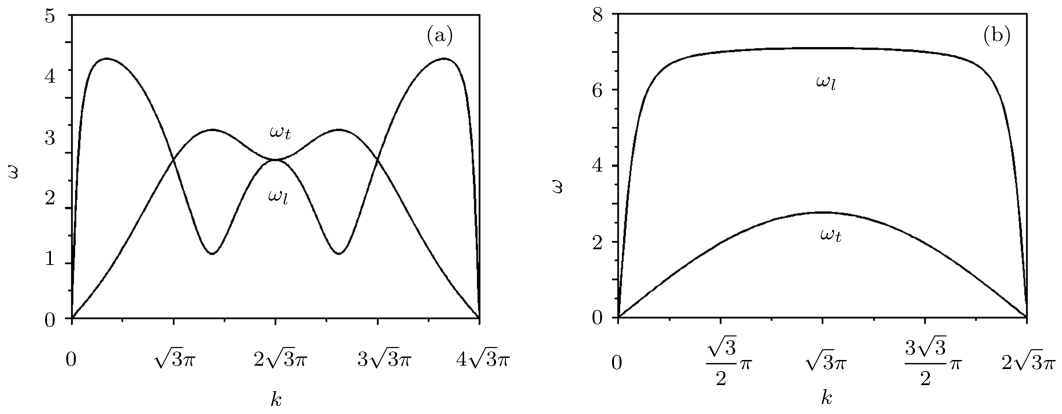


Fig.4. Dispersion relations of longitudinal mode ω_l and transverse mode ω_t propagating in the direction (1, 1, 1), in (a) bcc lattice and (b) fcc lattice.

The spring constants f_j $(j=1,\ 2)$ are given as follows: for the bcc lattice,

$$f_j=2\sum_{n,l=-\infty}^{+\infty}g_j\left(0,n,l\right)\sin^2\frac{k(n+l)}{2\sqrt{3}}+4\sum_{m=1}^{+\infty}\sum_{n,l=-\infty}^{+\infty}\left[g_j\left(m,n,l\right)\sin^2\frac{k(m+n+l)}{2\sqrt{3}}\right.$$
$$\left.+g_j\left(m-\frac{1}{2},n-\frac{1}{2},l-\frac{1}{2}\right)\sin^2\frac{k(2m+2n+2l-3)}{4\sqrt{3}}\right];$$

for the fcc lattice,

$$f_j=2\sum_{n,l=-\infty}^{+\infty}\left[g_j\left(0,n,l\right)\sin^2\frac{k(n+l)}{2\sqrt{3}}+g_j\left(0,n-\frac{1}{2},l-\frac{1}{2}\right)\sin^2\frac{k(n+l-1)}{2\sqrt{3}}\right]$$
$$+4\sum_{m=1}^{+\infty}\sum_{n,l=-\infty}^{+\infty}\left[g_j\left(m,n,l\right)\sin^2\frac{k(m+n+l)}{2\sqrt{3}}+g_j\left(m,n-\frac{1}{2},l-\frac{1}{2}\right)\sin^2\frac{k(m+n+l-1)}{2\sqrt{3}}\right.$$

$$+g_j\left(m-\frac{1}{2},n,l-\frac{1}{2}\right)\sin^2\frac{k(m+n+l-1)}{2\sqrt{3}}+g_j\left(m-\frac{1}{2},n-\frac{1}{2},l\right)\sin^2\frac{k(m+n+l-1)}{2\sqrt{3}}\Bigg],$$

where $g_1(m,n,l)\equiv F(m,n,l)$, and $g_2(m,n,l)\equiv G(m,n,l)$.

4. Summary and discussion

We have in this paper derived general dispersion relation matrices for dust lattice waves (DLWs) in bcc and fcc lattices of three-dimensional strongly coupled complex plasma crystals separately. The matrices are calculated in characteristic directions to obtain the longitudinal and transverse eigenmodes, shown in Figs.2–4 for the (1, 0, 0), (1, 1, 0) and (1, 1, 1) directions respectively, with the screening parameter $\kappa = 0.5$.

It is found that for waves propagating in the (1, 0, 0) direction, shown in Fig.2, the longitudinal mode is strongly dispersive at a very fast velocity in the long wavelength regime, while the transverse mode has almost no dispersion and is relatively slow in the regime. For waves propagating in the (1, 1, 0) direction, shown in Fig.3, the behaviours of the modes in the fcc lattice are similar to those in the (1, 0, 0) case, though there is another transverse mode due to the anisotropy in the plane perpendicular to the (1, 0, 0) direction. Nevertheless, the negative dispersion of the longitudinal mode is weakened for the bcc case, due to the fact that the particle spacing in the (1, 1, 0) direction for the bcc lattice is longer than that in the (1, 0, 0) direction. For waves propagating in the (1, 1, 1) direction, shown in Fig.4, the behaviours of the modes in the bcc lattice are similar to those in the (1, 0, 0) case, though both longitudinal and transverse modes are more dispersive due to the closer particle spacing in this direction than that in the (1, 0, 0) direction. However, for the fcc lattice, the modes behave similarly to that in the (1, 1, 0) direction for the bcc case. Furthermore, the negative dispersion disappears in the regime for the wavelength $2\pi/k < 2/\sqrt{3}$, since for an fcc lattice, the particle spacing is the longest in this direction. These characteristics of the DLW dispersion relations can be applied to particle simulation benchmark and experimental studies.

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