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Surface superlattice solitons in nonlocal nonlinear media*

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We analyse surface solitons at the interface between a one-dimensional photonic superlattice and a uniform medium with weak nonlocal nonlinearity. We demonstrate that in deep lattices there exist three kinds of surface solitons when the propagation constant exceeds a critical value, including two on-site solitons and one off-site soliton. These three kinds of surface solitons have unique dynamical properties. If the relative depth of the superlattice is low, there is only one kind of off-site soliton; however, the solitons of this kind can propagate stably, unlike their deep superlattice counterparts. Dipole surface solitons are also investigated, and the stable domain is given.

Keywords: surface solitons, superlattice, nonlocal nonlinearity

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1. Introduction

Surface solitons at the interface between a uniform medium and a periodic nonlinear medium have attracted a great deal of attention.[1−7] After theoretical prediction,[1,2] surface solitons have been observed in AlGaAs waveguide arrays[3] and $LiNbO_3$ waveguide arrays.[4] The combination of periodicity and nonlinearity can dramatically modify the property of a surface wave.[4] For example, it allows one to observe surface solitons with low power or in a self-defocusing medium, which otherwise is difficult or unavailable.[4] Moreover, the interface properties can be readily engineered by altering the depth, period, or structure of the lattice, enriching the physics and applications of surface solitons.[7−10] Spatiotemporal discrete surface solitons are also found in nonlinear optical waveguide arrays.[11−13] Recently, we investigated surface superlattice gap solitons and found some new dynamic effects.[5]

On the other hand, solitons in nonlocal media have also been extensively investigated, such as in liquid crystals,[14−20] and in thermal nonlinear media.[21−25] The nonlocal nonlinearity can lead to new families of waves that would otherwise have been impossible in local nonlinear media. Recently, nonlocality was introduced into surface solitons.[6,23−25] Stable fundamental and higher-order surface solitons were found to be stable in nonlocal nonlinear media.[6,23,24] Nonlocal surface solitons was also demonstrated experimentally, and did not exhibit a power threshold, distinctly different from their Kerr-like counterparts.[25]

To our knowledge, surface solitons at the interface between a photonic superlattice and a uniform medium with nonlocal nonlinearity has not been investigated yet. In the present paper, we investigate surface superlattice solitons (SSSs) in nonlocal nonlinear media. It is shown that in a deep enough superlattice, there exist three kinds of surface solitons when the propagation constant exceeds a critical value, including two on-site solitons and one off-site soliton. The two on-site solitons, located at the first and second lattices, have nearly the same energies and can propagate stably. The off-site solitons, located at the space between the first and second lattices, easily jump between the nearest lattice sites. If the relative depth of the superlattice is low, there occurs only one kind of off-site surface soliton in the middle of the first and second lattice sites; however, they can propagate sta-

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bly. We also investigate dipole surface solitons and find their stable domain.

2. The model

The normalized light field amplitude q and the nonlinear contribution to refractive index n obey the following system of equations:[6,26]

$$\begin{aligned} \mathrm{i}\frac{\partial q}{\partial \xi} &= -\frac{1}{2}\frac{\partial^2 q}{\partial \eta^2} - qn - I_{\mathrm{L}}q, \\ n - d\frac{\partial^2 n}{\partial \eta^2} &= |q|^2, \end{aligned} \tag{1}$$

where the transverse η and longitudinal ξ coordinates are normalized by the beam width and the diffraction length, respectively, the degree of nonlocality of the nonlinear response is described by parameter d, and I_{L} is the intensity profile of the superlattice, with $I_{\mathrm{L}} = 0$ at $\eta < 0$, and

$$I_{\mathrm{L}}(\eta) = I_0[\varepsilon \sin^2(\pi\eta/d_1) + (1-\varepsilon)\sin^2(\pi\eta/d_2)] \tag{2}$$

at $\eta \geq 0$. Such a superlattice potential given by expression (2) can be induced optically in a nonlocal nonlinear medium, by illuminating the sample with a broad laser beam passing through an appropriately designed amplitude mask. In the general case of incommensurate period d_1 and d_2, lattice (2) is quasi-periodic.[27] However, in the present paper we focus on the double-period lattice (e.g. see Refs.[5] and [28]), with $d_1 = 2d_2$. The superlattice can be engineered by parameter ε (which takes values between 0 and 1) and I_0, respectively. Figure 1(a) shows the intensity distributions of a superlattice with parameters $d_1 = 4$, $d_2 = 2$, $\varepsilon = 0.3$, and $I_0 = 2$. The regular lattice in Ref.[6] is a special case of the lattice given by Eq.(2) as $\varepsilon \to 0$ or $\varepsilon \to 1$.

We search for the stationary surface superlattice soliton profiles in the form of $q(\eta, \xi) = w(\eta)\exp(\mathrm{i}b\xi)$, where b is the real propagation constant and $w(\eta)$ is a real function satisfying the following equations:

$$\begin{aligned} \frac{\mathrm{d}^2 w}{\mathrm{d}\eta^2} + 2wn + 2I_{\mathrm{L}}w - 2bw = 0, \\ d\frac{\mathrm{d}^2 n}{\mathrm{d}\eta^2} - n + w^2 = 0, \end{aligned} \tag{3}$$

then by using the standard relaxation method, we solve these equations and obtain the stationary soliton solutions with boundary conditions $w, n \to 0$ at $\eta \to \pm\infty$. This method converges to a stationary solution quickly provided that appropriate initial guesses for w and n are given.

3. Numerical results

Figure 1(b) presents a numerically found family of the SSS solutions, in terms of the soliton power versus the propagation constant, corresponding to Fig.1(a) with nonolocality d=3 and d=5. A notable phenomenon is that as the propagation constant exceeds a critical value b_{c}, there appear three kinds of surface solitons (Fig.1(b)). Figure 1(c) shows that the critical value b_{c} increases monotonically with the growth of nonlocality.

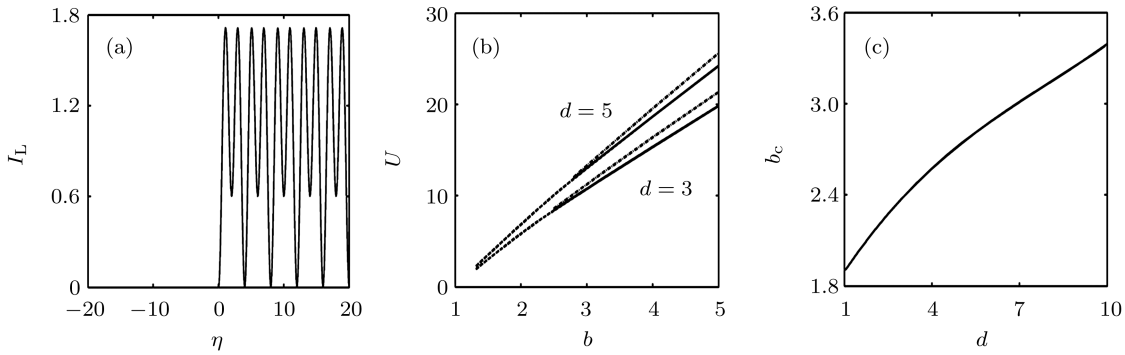


Fig.1. (a) Intensity distribution of optical supperlattice with $d_1 = 4$, $d_2 = 2$, $\varepsilon = 0.3$, and $I_0 = 2$. (b) Curves for soliton total power versus b for $d = 3$ and 5, with solid, dash, and dash-dot lines corresponding to on-site solitons at the first and second lattices, and the off-site solitons, respectively. Note that part of the dash lines nearly overlap the solid lines. (c) Values of critical propagation constant b_{c} for different degrees of nonlocality.

Taking the $d = 5$ case in Fig.1(b) as example, when the propagation constant is below the critical value $b_c = 2.7$, there is only one kind of surface soliton as shown in Fig.2(a), for $b = 1.4$. As the propagation constant increases, the profiles of surface solitons become narrower and locate at the second lattice site (see Fig.2(b)) (dash line in Fig.1(b)). When the propagation constant exceeds b_c, in addition to these second lattice on-site solitons, there appear two more kinds of surface solitons: one locates at the first lattice site (Fig.2(e)), the other locates at the space between the first and second lattices (Fig.2(f)) (solid and dash-dot line in Fig.1(b), respectively).

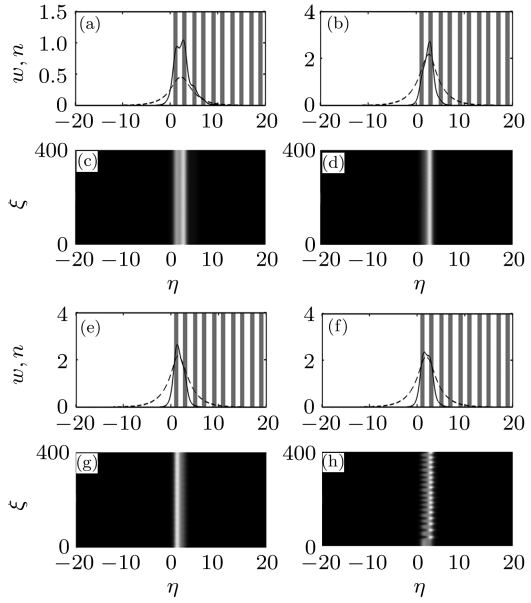


Fig.2. Profiles of the SSSs corresponding to $d = 5$ in Fig.1(b), for (a) $b = 1.4$, below b_c, and (b, e, f) $b = 2.9$, exceeds b_c. Solid curves show the field distribution, dash curves indicate the refractive index profiles (this is also the same for those in other figures in this paper). Panels c, d, g and h show the propagation dynamics of SSSs.

To further study the stability of these three kinds of SSS solutions, we numerically simulate their propagations by using the split-step Fourier method. In all numerical simulations, we add a noise to the input SSSs by multiplying them with $[1 + \rho_{1.2}(\eta)]$, where $\rho_{1.2}(\eta)$ is a Gaussian random function with $\langle \rho_{1,2} \rangle = 0$ and $\langle \rho_{1,2}^2 \rangle = \sigma_{1,2}^2$ (the value of $\sigma_{1,2}$ is chosen to be 10% of the input solution amplitude). The on-site surface solitons whose peak positions locate in the first and second lattices can propagate stably up to $\xi = 400$, which is tantamount to 400 diffraction lengths (Figs.2(c), 2(d) and 2(g)). However, the off-site surface solitons locating at the space between the first and second lattices begin to jump back and forth between these two lattice sites, when propagating a distance up to $z \approx 40$.

We tune the lattice depth by varying the parameter ε and find that there appear three kinds of surface solitons only when the lattice is deep enough, otherwise there exists only one kind. Figure 3 shows typical examples with parameters $d = 5$ and $b = 2.9$. For deep lattices with $\varepsilon = 0.1$ (Fig.3(b)), three kinds of surface solitons exist (Fig.3(d)) (only the off-site solitons are shown in Fig.3), similar to the scenario in Fig.2 with $\varepsilon = 0.3$, besides, the off-site solitons jump to the second lattice site when propagating (Fig.3(f)), because the on-site solitons at the second lattice have lower potentials. For low lattices with $\varepsilon = 0.5$ (Fig.3(a)), only one kind of off-site surface soliton exists (Fig.3(c)), which also locates at the space between the first and second lattices. However, they can propagate stably (Fig.3(e)), unlike their deep superlattice counterparts.

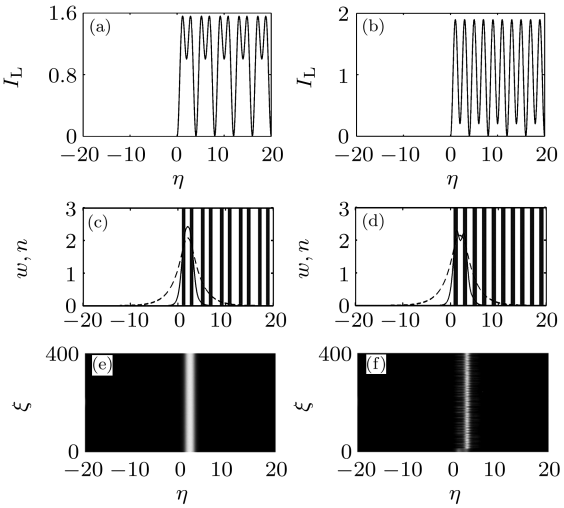


Fig.3. Intensity distribution of (a) low superlattices $\varepsilon = 0.5$, (b) deep superlattices $\varepsilon = 0.1$, with $d_1 = 4$, $d_2 = 2$, and $I_0 = 2$. (c), (e) Stable and (d), (f) unstable solitons in low and deep superlattices, respectively, for $d = 5$ and $b = 2.9$.

We find, by additional simulations, that the stable off-site surface solitons mentioned above do not exist in regular periodic lattices ($\varepsilon = 0$), no matter what values parameters d and I_0 take. Figures 4(a) and 4(b) show typical examples in which off-site surface solitons in regular lattices propagate unstably by penetrating into lattice sites or jumping to other lattice sites (Figs.4(c) and 4(d)). In other words, the structure of the superlattice is a necessary ingredient

for such stable off-site surface solitons. Figures 4(e) and 4(f) show the off-site surface solitons, repectively, localized in the weakly nonlocal and the local nonlinear media in the presence of the superlattices. They propagate stably and unstably, respectively, shown in Figs.4(g) and 4(h).

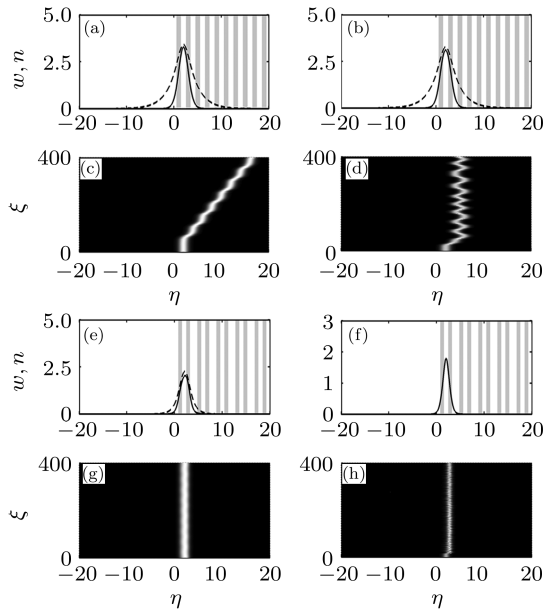


Fig.4. Profiles of the SSSs in regular lattice ($\varepsilon = 0$) with $d = 5$ for (a) $I_0 = 0.05$, (b) $I_0 = 0.3$, and in superlattice ($\varepsilon = 0.5$) with $I_0 = 2$ for (e) $d = 1$, (f) $d = 0$. Panels c, d, g and h show the propagation dynamics of SSSs.

For other superlattice configurations with a relatively low depth, there exist similar results. Thus we can see that the stability of these off-site surface solitons is due to the influence of the structure of the superlattice and the nonlocality together. This may be explained as follows. The basis of the first and second lattices serves as a potential to trap the off-site solitons, while the nonlocal nonlinear refraction index fills the groove between the two lattice sites, resulting in the stability of the off-site solitons. Based on these results, we find that the ground-state surface solitons at the nonlocal nonlinear superlattice have multiformity as mentioned above. These phenomena appear only in the superlattice rather than in regular periodic lattices.

Finally, we investigate dipole surface solitons in these superlattice systems. Such dipole surface solitons are composed of two poles that reside at different sides of the interface (Figs.5(c) and 5(d)).[6] Figure 5(a) shows the curves for soliton power versus propagation constant, corresponding to a superlattice with $d_1 = 4$, $d_2 = 2$, $\varepsilon = 0.3$, and $I_0 = 2$. The stable domain is given in Fig.5(b). Inside this domain, solitons can propagate stably over a long distance ($\xi = 1600$) (Fig.5(e)), while outside this domain, solitons unstably propagate (Fig.5(f)).

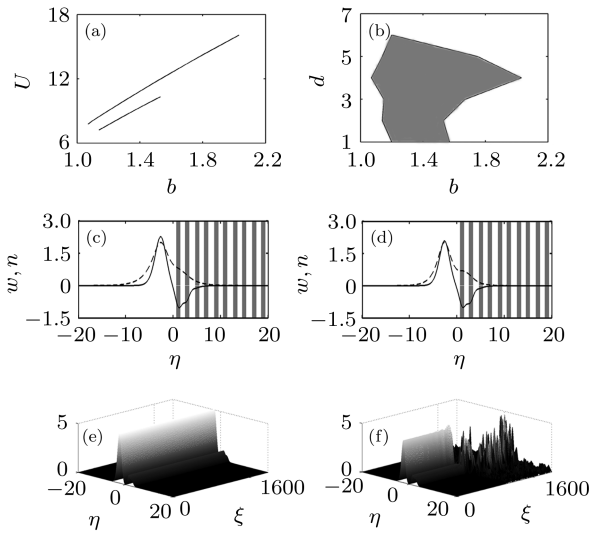


Fig.5. (a) Power versus propagation constant for dipole surface solitons for the superlattice in Fig.1(a). (b) Stable domain (grey area) for dipole surface solitons in (b, d) plane. (c), (e) $d = 4$, $b = 1.6$, stable and (d), (f) $d = 2$, $b = 1.54$, unstable dipole surface solitons.

4. Summary

We demonstrate that solitons exist at the interface between a one-dimensional photonic superlattice and a uniform medium with nonlocal nonlinearity. It is shown that the ground-state surface solitons in such a system have multiformity. In a deep enough superlattice, there exist two on-site solitons and one off-site soliton when the propagation constant exceeds a critical value. These two on-site solitons locate at the first and second lattice sites, respectively, have nearly the same energies and can propagate stably. The off-site solitons, located at the space between the first and second lattice sites, will jump between these two adjacent lattice sites. If the relative depth of the superlattice is low, there is only one kind of surface soliton in the middle of the first and second lattice sites; however, the off-site solitons of this kind can propagate stably, unlike their deep superlattice counterparts. Moreover, we find that these stable off-site surface solitons do not exist in regular periodic lattices. Dipole surface solitons are also investigated, and the stable domain is given. The stable dipole surface solitons can propagate a very long distance.

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