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Implementation of a many-qubit Grover search by cavity QED*

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We explore the possibility of an N-qubit ($N > 3$) Grover search in cavity QED, based on a fast operation of an N-qubit controlled phase-flip with atoms in resonance with the cavity mode. We demonstrate both analytically and numerically that our scheme can be achieved efficiently to find a marked state with high fidelity and high success probability. As an example, a ten-qubit Grover search is simulated specifically under the discussion of experimental feasibility and challenge. We argue that our scheme is applicable to the case involving an arbitrary number of qubits. As cavity decay is involved in our quantum trajectory treatment, we can analytically understand the implementation of a Grover search subject to dissipation, which will be very helpful for relevant experiments.

Keywords: cavity QED, Grover search, controlled phase flip gate
PACC: 4250, 0367

1. Introduction

Cavity quantum electrodynamics (QED) has seen much progress over the past several years.[1−3] People have achieved various optical and microwave cavities, with which some remarkable quantum phenomena have been demonstrated, such as strong coupling of a single atom with a single photon,[4], quantum logic gating between two atomic qubits,[5] photonic blocking,[6] efficient generation of single photons,[7] and even an observation of the birth and death of a single microwave photon.[8]

In the present work, we will investigate how to realize a Grover search with more than three qubits by cavity QED. The Grover search,[9] as one of the most frequently mentioned quantum algorithms, can effectively exemplify the potential speed-up offered by quantum computers.[10] Recently, many authors have dedicated themselves to the search algorithm by adiabatic evolution methods[11−13] or a non-adiabatic scheme.[14,15] Although achievement of these schemes needs stringent conditions and demanding techniques,[16] they are really wonderful ideas. On the other hand, some authors have addressed the effects of decoherence,[17,18] noise,[19] and gate imperfection or errors[20] on the efficiency of quantum algorithms. We also noticed that there has been intensive interest in achieving a Grover search algorithm theoretically and experimentally by using NMR systems,[21] trapped ions,[22−24] linear optical elements,[25,26] cavity QED,[27−31] and superconducting meso-circuits.[32]

Our interest is in the experimental feasibility of the Grover search by a microwave cavity. A recent work,[29] with which two of the present authors joined in, showed a three-qubit Grover search implementation in a microwave cavity. As it makes use of the resonant interaction of the atoms with the cavity mode and considers the detrimental influence of cavity decay, the implementation of Ref.[29] is close to the reach of the current technique. However, that scheme is hard to extend to the case with more than three qubits due to the limitation of the method[33] used there. Our present work will show that, after some modification of the method in Ref.[33], we could accomplish a high-fidelity conditional phase flip (CPF) for many atomic qubits flying through a microwave cavity, based on which an N-qubit ($N > 3$) Grover search might be

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achieved in a straightforward way. The main idea is that we make a smart qubit encoding on the atomic levels, and send the atoms through a microwave cavity for resonant interaction. As long as we can exactly control the interaction time and the trajectories of the flying atoms, we may accomplish the N-qubit CPF gate by a one-step implementation, based on which we could efficiently carry out an N-qubit Grover search with high fidelity. However, as the Grover search with more than two qubits works only probabilistically and also as our one-step implementation of a many-qubit CPF gate intrinsically has imperfections, the situation in our case is much more complicated than in the two-qubit case. To make our scheme more realistic, cavity decay is involved in our treatment, and as done in Ref.[29], we will try to present some analytical expressions for the state evolution during our implementation. As a result, we can explicitly assess how well a Grover search is made subject to these disadvantageous factors.

2. $N-$qubit CPF gate subject to cavity decay

As described in Refs.[29] and [33], we consider the resonant interaction of N three-level atoms with a single-mode cavity. As shown in Fig.1, the three levels can be described by $|i_j\rangle$, $|g_j\rangle$, and $|e_j\rangle$, in which the subscript j means the jth atom, the states $|e_j\rangle$ are excited states and $|i_j\rangle$, $|g_j\rangle$ are ground states. As $|g_j\rangle$ and $|e_j\rangle$ are coupled in resonance by the cavity mode, the states $|i_j\rangle$ are not involved in the interaction with the cavity mode throughout our treatment.

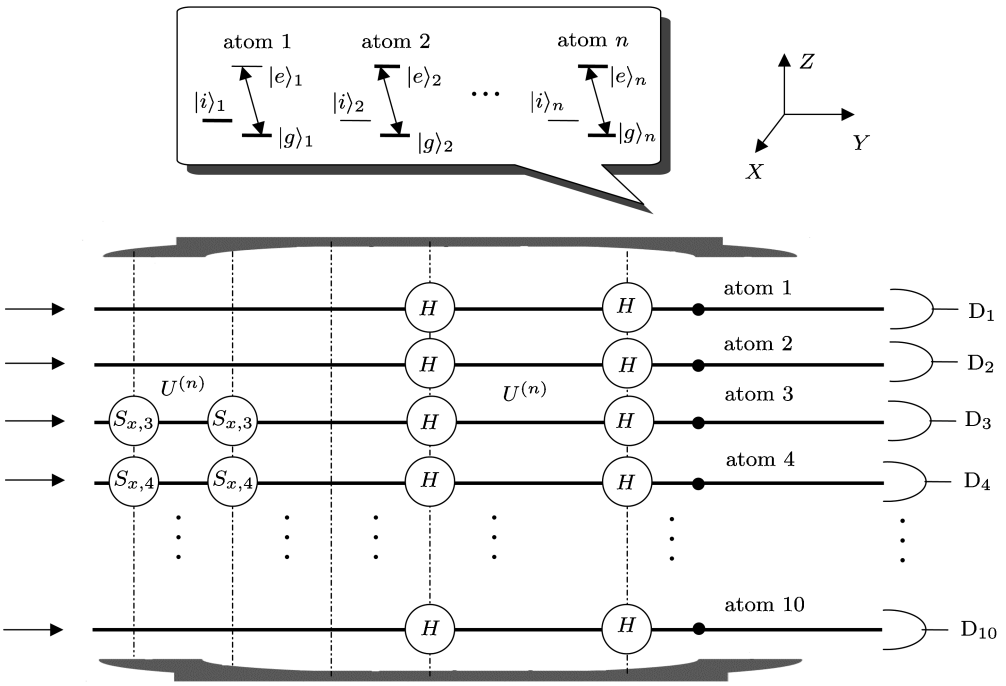


Fig.1. Schematic setup for finding the marked state $|e_1 i_2 g_3 g_4 i_5 i_6 i_7 i_8 i_9 i_{10}\rangle$ in a ten-qubit Grover search, where ten atoms are simultaneously sent through the cavity with proper speed. The ten state-selective field ionization detectors $D_1, \ldots, D_{10}$ are settled at the end of the passage for checking the states of the qubits. The operations $U^{(10)}$, $H^{\otimes 10}$, $S_{x,3}$, and $S_{x,4}$ are defined in the text. The inset shows the atomic level configuration, with bold lines for the states encoding qubits and arrows for the coupling between the atoms and the standing wave of the cavity.

We encode the first qubit $|0\rangle_1$ ($|1\rangle_1$) on $|e_1\rangle$ ($|g_1\rangle$), and other qubits $|0\rangle_k$ ($|1\rangle_k$) on $|i_k\rangle$ ($|g_k\rangle$) with $k = 2, 3, \ldots, N$. Considering a cavity with weak decay, which implies no photon actually leaking out of the cavity during our implementation, we may write the Hamiltonian in units of $\hbar = 1$ as,

$$H = \sum_{i=1}^{N} \Omega_i \left(a^\dagger \sigma_i^- + a\sigma_i^+\right) - \mathrm{i}\,\frac{\kappa}{2} a^\dagger a, \tag{1}$$

where Ω_i is the coupling constant of the ith atom to the cavity mode, and the atomic spin operators for raising and lowering are $\sigma_i^+ = |e_i\rangle\langle g_i|$ and $\sigma_i^- = |g_i\rangle\langle e_i|$, respectively. $a^\dagger$ (a) is the creation (annihilation) operator of the cavity mode. We will first consider an implementation of N-qubit CPF under weak cavity dissipation, based on which an N-qubit Grover search will be achieved in the next section. The weak dissipation condition $\kappa \ll \Omega_i$ makes sure that we can analytically obtain the time evolution of the state by the quantum trajectory notion. Figure 2 shows the variations of the fidelity F and probability P of success with respect to both η and μ.

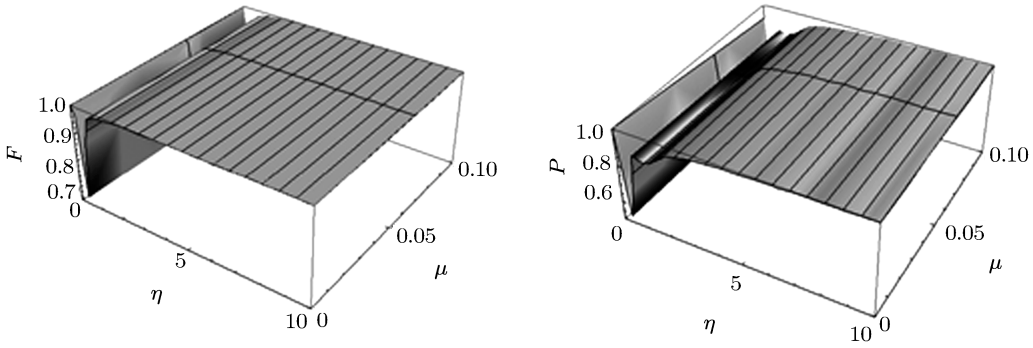


Fig.2. The variations of the fidelity F and probability P of success with respect to both η and μ.

Assuming the initial atomic state is in $|e_1\rangle\prod_{l,j=2,l\neq j}^{N}|g_l\rangle|i_j\rangle$ and the cavity is initially in the vacuum state $|0\rangle$, we can deduce the evolved state by directly solving the Schrődinger equation regarding Eq.(1), which yields

$$\begin{aligned}|e_1\rangle\prod_{l,j=2,l\neq j}^{N}|g_l\rangle|i_j\rangle|0\rangle \longrightarrow & \left[\frac{\Omega_1^2}{G_m^2}\exp(-\kappa t/4)\times\xi+\frac{G_m^2-\Omega_1^2}{G_m^2}\right]\\ & \times|e_1\rangle\prod_{l,j=2,l\neq j}^{N}|g_l\rangle|i_j\rangle|0\rangle+\frac{\Omega_1^2}{G_m^2}\left[\exp(-\kappa t/4)\times\xi-1\right]\\ & \times|g_1\rangle\sum_{k=2}^{m+1}\Omega_k|e_k\rangle\prod_{l,j=2,l\neq j\neq k}^{N}|g_l\rangle|i_j\rangle|0\rangle\\ & -(\mathrm{i}\,\Omega_1/A_{m+1,\kappa})\sin(A_{m+1,\kappa}t)|g_1\rangle\prod_{l,j=2,l\neq j}^{N}|g_l\rangle|i_j\rangle|1\rangle, \end{aligned}\tag{2}$$

where $\xi=\left(\cos(A_{m+1,\kappa}t)+\dfrac{\kappa}{4A_{m+1,\kappa}}\sin(A_{m+1,\kappa}t)\right)$,

$$G_m=\sqrt{\sum_{j=1}^{m+1}\Omega_j^2}$$

and

$$A_{m+1,\kappa}=\sqrt{\sum_{j=1}^{m+1}\Omega_j^2-\kappa^2/16}$$

with m the number of the atoms initially in the ground state $|g\rangle$. We emphasize that no strict normalization is made here and hereafter, like in Ref.[33], due to involvement of the dissipative factor. But as Eq.(1) is non-Hermitian, we have obtained Eq.(2) by using the time evolution of the norm of the state over an effective normalization factor $\sqrt{\langle|(\mathrm{e}^{-\mathrm{i}Ht})^\dagger\mathrm{e}^{-\mathrm{i}Ht}|\rangle}$ with $|\rangle$ the initial state of the system. Equation (2) could be justified either by putting $N=3$ to return to the expressions in Ref.[33] or by putting other values of N to check the corresponding Schrődinger equation by Mathematica software.

In order to accomplish an N-qubit CPF gate, we should also pay attention to the evolution with the initial state $|e_1\rangle\prod_{j=2}^{N}|i_j\rangle|0\rangle$, which is

$$
\begin{aligned}
&|e_1\rangle \prod_{j=2}^{N} |i_j\rangle\,|0\rangle \\
\longrightarrow\ &[\exp(-\kappa t/4)\times\xi]\times|e_1\rangle \prod_{j=2}^{N}|i_j\rangle\,|0\rangle \\
&-(\mathrm{i}\,\Omega_1/A_{1,\kappa})\sin(A_{1,\kappa}t)\,|g_1\rangle \prod_{j=2}^{n}|i_j\rangle\,|1\rangle
\end{aligned}
\tag{3}
$$

with $A_{1,\kappa}=\sqrt{\Omega_1^2-\kappa^2/16}$. Equation (3) implies the time evolution due to interaction only with the first qubit, and is thereby easily understood. Assuming that the coupling constants satisfy the condition $\Omega=\Omega_2=\Omega_3=\ldots=\Omega_N\gg\Omega_1$ making $\sin(A_{1,\kappa}t)\ll\cos(A_{1,\kappa}t)$ and $\sin(A_{m+1,\kappa}t)\ll\cos(A_{m+1,\kappa}t)$ $(m=1,2,3\ldots N-1)$, we may effectively delete the terms regarding sin functions in Eqs.(2) and (3). In order to achieve a CPF gating, we try to set the evolution time $t_0=\pi/\sqrt{\Omega_1^2-\kappa^2/16}$, which yields $\sin(A_{1,\kappa}t_0)=0$ and $\cos(A_{1,\kappa}t_0)=-1$. Meanwhile, if the condition $A_{m+1,\kappa}t_0=A_{1,\kappa}t_0+(2x-1)\pi$ (x is an arbitrary integer) is met, we may have $\sin(A_{m+1,\kappa}t_0)=0$ and $\cos(A_{m+1,\kappa}t_0)=1$. As a result, the conditions given above are completely satisfied and thereby Eqs.(2) and (3) are reduced to

$$
|e_1\rangle \prod_{\substack{j,k=2,\\ j\neq k}}^{N}|g_k\rangle\,|i_j\rangle\,|0\rangle\longrightarrow\beta_m\,|e_1\rangle \prod_{\substack{j,k=2,\\ j\neq k}}^{N}|g_k\rangle\,|i_j\rangle\,|0\rangle\,, \tag{4}
$$

$$
|e_1\rangle \prod_{j=2}^{N}|i_j\rangle\,|0\rangle\longrightarrow\alpha\,|e_1\rangle \prod_{j=2}^{N}|i_j\rangle\,|0\rangle \tag{5}
$$

with

$$
\alpha=\exp(-\kappa t_0/4)\cos(A_{1\kappa}t_0)
$$

and

$$
\beta_m=\left[\frac{\Omega_1^2}{G_m^2}\exp(-\kappa t_0/4)\cos\left(A_{(m+1)\kappa}t_0\right)+\frac{G_m^2-\Omega_1^2}{G_m^2}\right].
$$

To obtain Eq.(4), we have also used an approximation $\exp(-\kappa t_0/4)\approx 1$ in the case of $\kappa\ll\Omega_1$. Putting the value of t_0 in Eqs.(4) and (5), we have

$$
\alpha=-\exp\left(-\pi\mu/\sqrt{16-\mu^2}\right), \tag{6}
$$

$$
\begin{aligned}
\beta_m=&\frac{-\alpha}{1+m\eta^2}(\cos\vartheta+\mu\sin\vartheta/\sqrt{16+16m\eta^2-\mu^2}) \\
&+\frac{m\eta^2}{1+m\eta^2}
\end{aligned}
\tag{7}
$$

with $\mu=\kappa/\Omega_1$, $\vartheta=\pi\sqrt{1+16m\eta^2/(16-\mu^2)}$ and $\eta=\Omega/\Omega_1$.

We may take $N=4$ as an example to show more explicitly the CPF gate we designed. Following the deduction above, we have the four-qubit CPF written as $U^{(4)}=J_{0000}=\mathrm{diag}\{\alpha,\beta_1,\beta_1,\beta_1,\beta_2,\beta_2,\beta_2,\beta_3,1,1,1,1,1,1,1,1\}$ with respect to the bases $|e_1i_2i_3i_4\rangle$, $|e_1i_2i_3g_4\rangle$, $|e_1i_2g_3i_4\rangle$, $|e_1g_2i_3i_4\rangle$, $|e_1i_2g_3g_4\rangle$, $|e_1g_2i_3g_4\rangle$, $|e_1g_2g_3i_4\rangle$, $|e_1g_2g_3g_4\rangle$, $|g_1i_2i_3i_4\rangle$, $|g_1i_2i_3g_4\rangle$, $|g_1i_2g_3i_4\rangle$, $|g_1g_2i_3i_4\rangle$, $|g_1i_2g_3g_4\rangle$, $|g_1g_2i_3g_4\rangle$, $|g_1g_2g_3i_4\rangle$, and $|g_1g_2g_3g_4\rangle$, respectively, where the latter eight states do not evolve under the Hamiltonian of Eq.(1) in the case of the vacuum cavity state. To assess the CPF gating, we assume the atoms to be initially in $|\Psi_0\rangle=1/\sqrt{2}\,(|g_1\rangle+|e_1\rangle)\,(|g_2\rangle+|i_2\rangle)\ldots(|g_N\rangle+|i_N\rangle)$. We may calculate the fidelity and the success probability by employing the relations $F=\overline{\langle\Psi_f|\,U^{(n)\dagger}\,|\Psi_0\rangle\,\langle\Psi_0|\,U^{(n)}\,|\Psi_f\rangle}$[34] and $P=\overline{\langle\Psi_f|\Psi_f\rangle}$.[29,33] As shown in Fig.2, we can reach considerably high fidelity F and high probability of success P as long as η approaches 10, even in the case of a relatively large decay rate. In what follows, to show the potential application of our CPF gating, we will carry out a Grover search under weak dissipation.

3. $N-$qubit Grover search

Generally speaking, provided that the initial state of the system has been prepared in an average superposition state $|\Psi_0\rangle=(1/\sqrt{2^N})\sum_{i=0}^{2^N}|i\rangle$, a Grover search can be depicted as the iterative operation $\widehat{D}^{(N)}J_\tau$ by at least $\pi\sqrt{2^N}/4$ times for finding the marked state $|\tau\rangle$ with an optimal probability, where the quantum phase gate $J_\tau=\boldsymbol{I}-2\,|\tau\rangle\,\langle\tau|$ (with $\boldsymbol{I}$ being the identity matrix) plays an important role to invert the amplitude of the marked state, and the diffusion transform $\widehat{D}^{(N)}$ is defined as $\widehat{D}_{ij}=2/K-\delta_{ij}$ $(i,j=1,2,\ldots,K,\ K=2^N)$. Considering the qubit subspace spanned by $\{|e_1\rangle,\ |g_1\rangle,\ |i_2\rangle,\ |g_2\rangle\ \ldots\ \ |i_{N-1}\rangle,\ |g_{N-1}\rangle,\ |i_N\rangle,\ |g_N\rangle\}$, to make a Grover search, we could construct the following transformation,

$$
\begin{aligned}
Q^{(N)}&=W^{\otimes N}J^{(N)}_{00\ldots0}W^{\otimes N}J_\rho \\
&=W^{\otimes N}J^{(N)}_{e_1i_2\ldots i_n}W^{\otimes N}J_\rho \\
&=-\hat{D}^{(N)}J_\rho,
\end{aligned}
\tag{8}
$$

where

$$
\begin{aligned}
J^{(N)}_{00\ldots0}&=\mathrm{diag}\{-1,1,\ldots,1\} \\
&=I^{(N)}-2\,|00\ldots0\rangle\,\langle00\ldots0|
\end{aligned}
$$

(with superscript (N) denoting the N-qubit case and $\ldots$ denoting N elements) and the Hadamard gate in our computational subspace is given by

$$W^{\otimes N}=\prod_{i=1}^{N}W_i=\left(\frac{1}{\sqrt{2}}\right)^N\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\otimes\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\otimes\ldots\otimes\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}. \tag{9}$$

In the case of N qubits, Eq.(8) implies that the diffusion transform $\hat{D}^{(N)}=-W^{\otimes N}J^{(N)}_{e_1i_2\ldots i_n}W^{\otimes N}$ is always unchanged no matter which state is to be searched. The only change is the operator J_ρ for different marked states. Based on the CPF gate constructed in last section, we have $J^{(N)}_{00\ldots0}\approx \mathrm{diag}\{-1,1,\ldots,1\}=U^{(N)}_{\mathrm{CPF}}$, from which we could achieve the gate $J^{(N)}_{11\ldots1}$ as,

$$J^{(N)}_{11\ldots1}=S_{x,N}S_{x,N-1}\ldots\sigma_{x,1}J^{(N)}_{00\ldots0}\sigma_{x,1}\ldots S_{x,n-1}S_{x,n}, \tag{10}$$

where $S_{x,j}=|i_j\rangle\langle g_j|+|g_j\rangle\langle i_j|$ with $j\neq1$, and $\sigma_{x,1}=|e_1\rangle\langle g_1|+|g_1\rangle\langle e_1|$. To achieve $Q^{(N)}$, we will construct the CPF gate $J_\rho=I-2\,|\rho\rangle\,\langle\rho|$.

As an example, we may consider the qubit number $N=10$. In this case, the number of the involved quantum states is 2^{10}, and the operation is to label a marked state by J_ρ with ρ one of the states from $\{|0000000000\rangle, |0000000001\rangle, |0000000010\rangle, \ldots, |1111111111\rangle\}$. To carry out the ten-qubit Grover search, we need two ten-qubit Hadamard gates $W^{\otimes10}$. Based on the CPF gate marking the state $|g_1g_2g_3g_4g_5g_6g_7g_8g_9g_{10}\rangle$, we could construct other $2^{10}-1$ gates for the marking job by single-qubit rotations. For example,

$$\begin{aligned}&J_{g_1g_2g_3g_4g_5g_6g_7g_8g_9g_{10}}\\ &=J_{1111111111}\\ &=S_{x,10}S_{x,9}S_{x,8}S_{x,7}S_{x,6}S_{x,5}S_{x,4}S_{x,3}S_{x,2}\sigma_{x,1}\\ &\quad J_{0000000000}\sigma_{x,1}S_{x,2}S_{x,3}S_{x,4}S_{x,5}S_{x,6}S_{x,7}S_{x,8}S_{x,9}S_{x,10}.\end{aligned} \tag{11}$$

So with a state marked, and with the ten-qubit diffusion transform $\hat{D}^{(10)}$ which is generated by combining two Hadamard gates $W^{\otimes10}$ with $J_{0000000000}$, a full Grover search for ten qubits is available. Specifically, we may consider the marked state $|e_1i_2g_3g_4i_5i_6i_7i_8i_9i_{10}\rangle$. The Rydberg atoms from the 1st to the 10th are input, as shown in Fig.1, with principal quantum numbers 49, 50 and 51 denoting $|g_j\rangle$, $|i_j\rangle$ and $|e_j\rangle$, respectively. Suppose the atoms initially prepared in a superposition $|\Psi_0\rangle=2^{-5}\times(|e_1\rangle+|g_1\rangle)(|i_2\rangle+|g_2\rangle)(|i_3\rangle+|g_3\rangle)\ldots(|i_{10}\rangle+|g_{10}\rangle)$. We plot in Fig.3 the success rate of the Grover search in different cases. As the Grover search involving more than two qubits is intrinsically probabilistic, how to obtain an optimal search is a problem of interest. In contrast to the previous study for an ideal Grover search,[35] our analytical expressions along with some numerics could identify the optimal search under cavity decay and other imperfections due to gating. We know from Fig.3 that, because of the cavity decay, the more iteration steps, the more detrimental the effect from the cavity decay is. As a result, we prefer to have fewer iteration steps for an optimal search. The detrimental effect from the cavity decay is also reflected in the figure that the maximum value of P is slightly lower in the case of a larger decay rate.

4. Discussion

Our CPF gating plays an essential role in our Grover search scheme, as it significantly reduces the implementation time compared to that by a series of two-qubit conditional gates and single-qubit gates.[36] Section 2 has shown that the CPF gating time only depends on the weakest coupling Ω_1 and the cavity decay rate κ, but is irrelevant to the qubit number. So the CPF gating in our scheme could keep a constant implementation time no matter how many atoms are involved, which favours a large-scale QIP. As a result, if we suppose the single-qubit rotation takes a negligible time compared to the CPF gating, then one iteration of our Grover search could also be carried out by a constant time irrelevant to the qubit number. The lifetime of the photon in the superconducting cavity could reach half a second with the current technique.[8] Here what we consider is a cavity with photon lifetime 0.2 s, which should be easily achievable. So, assuming $\Omega=2\pi\times51$ kHz,[8] $\Omega_1=2\pi\times5.1$ kHz, and $\kappa=10^{-3}\Omega_1$, we have $t_0=\pi/\sqrt{\Omega_1^2-\kappa^2/16}\approx98$ μs, which is much shorter than either the cavity decay time, i.e., $2\pi/\kappa\approx0.2$ s, or the Rydberg atomic lifetime 30 ms.[1] Therefore, our assumption of no photon actually leaking out of the cavity during our implementation is reasonable and our quantum trajectory treatment is valid.

For a full consideration of a Grover search, however, the operation $Q^{(N)}$ has to be made for several times depending on the specific search task. In the case of $N=10$, the total required time for a Grover search could be simply calculated as $2[\pi\sqrt{2^N}/4]\times t_0\approx5$ ms, which is still much shorter than the cavity

decay time. Consequently, in view of the cavity decay rate, we may consider that our scheme is in the reach of current super-conducting microwave cavity technology.

For a realistic implementation with unavoidable dissipation, we prefer fewer iteration steps in order to suppress the detrimental effect from the cavity decay to the best. Our numerics in Fig.3 show that the iteration steps for an optimal search are 25, 24, 18, in the case of $\mu = 0$, 0.001 and 0.1 respectively. Since these cavity decay rates are very small compared to the coupling constant, the operation time $t_0 = \pi/\sqrt{\Omega_1^2 - \kappa^2/16}$ only varies very little in different cases of cavity dissipation. Considering the search steps, the photon lifetime and the operation time altogether, we certainly prefer the case of $\mu = 0.001$ with 24 iteration steps.

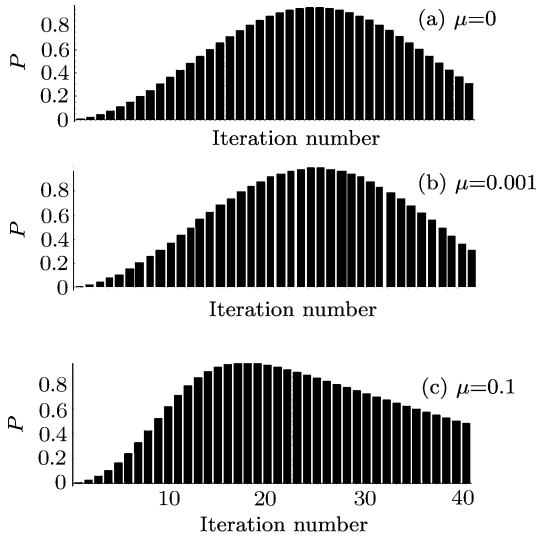


Fig.3. The search probability of ten-qubit with respect to the iteration number under the cavity decay rate (a) $\mu = 0$, (b) $\mu = 0.001 - 0.01$, and (c) $\mu = 0.1$, respectively.

In current microwave cavity experiments,[1] we may consider the interaction between atoms and the cavity mode by $\Omega_0 \cos(2\pi z/\lambda_0) \exp(-r^2/w^2) \sim \Omega_0 \cos(2\pi z/\lambda_0)$,[1] where Ω_0 is the coupling strength at the cavity centre, r is the distance of the atom away from the cavity centre, and λ_0 and w are the wavelength and the waist of the cavity mode, respectively. To meet the condition of $\Omega = 10\Omega_1$, we may send the first atom going through the cavity along y axis deviating from the nodes by $(\lambda_0/2\pi)\arccos(1/10)$, but other atoms through the antinodes. Specifically, for the N input atoms (suppose N is an even number for simplicity), we should have the tracks of the atomic movement as $z_1 = -(N/2)\lambda_0 + (\lambda_0/2\pi)\arccos(1/10)$, $z_2 = -(N/2-1)\lambda_0$, ..., $z_N = (N/2)\lambda_0$. This is of course highly challenging with current experimental technology. Moreover, with current cavity QED techniques, the design in Fig.1 should be realized by 50 separate microwave cavities with each Ramsey zone in between the neighbouring cavities. Since each microwave cavity is employed to carry out a ten-qubit phase gate $J^{(10)}_{00\ldots0}$, the cavities should be identical. This is also a challenge at present. Fortunately, if N is smaller, the repetition for Grover searching would be significantly decreased. For example, for $N = 4$, we will only need six separate microwave cavities. Furthermore, for searching different states, we employ different single-qubit operations. So the Ramsey zones should be long enough to finish the necessary single-qubit operations.

Now we make a simple estimate for experimental imperfection in simultaneously sending many Rydberg atoms through a cavity with precise tracks and velocities. We know that two Rydberg atoms going through a microwave cavity simultaneously have been achieved,[27] but no experiment has been reported so far for more than two atoms going through a microwave cavity simultaneously due to the difficulty in the exact control of tracks and velocities of the atoms. Here we assess the infidelity due to the imperfection in atomic velocity by a simple example below. Consider that the first atom moves a little bit more slowly in the cavity than other atoms, that is, $t_1 = t_0 + \delta t$, $t_2 = t_3 = \ldots = t_n = t_0$, with $t_0 = \pi/\sqrt{\Omega_1^2 - \kappa^2/16}$. As shown in Fig.4 with $\eta = 10$, we can see that the infidelity increases with both δt and μ. In actual experiments, the situation is much more complicated. Besides the diversity of atomic velocity, there will be other imperfections such as classical pulse imperfection, differences of the cavities, deflected atomic trajectories and so on. Anyway, our simple estimate has shown explicitly the importance of highly controllable movement of the atoms in implementing our scheme.

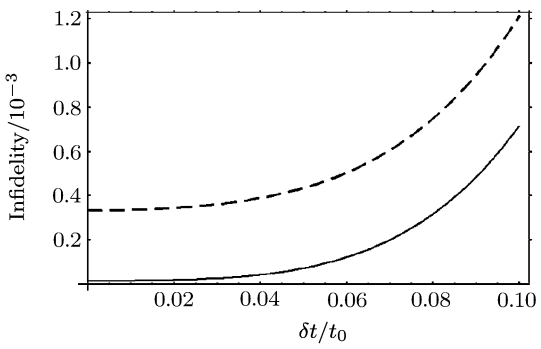


Fig.4. Infidelity of ten-qubit phase gate versus time delay in the case of $\eta = 10$, where $\mu = 0.001$ and $\mu = 0.1$ are plotted by solid and dashed curves, respectively.

5. Conclusion

In conclusion, we have studied a many-qubit CPF gate in cavity QED under weak cavity dissipation, and explored the possibility of a many-qubit Grover search in a dissipative cavity QED system. As a theoretical work, like previous publications in cavity QED, e.g.,[37] we have simplified the complexity in the realistic system, and tried to present some analytical expressions for the state evolution subject to cavity decay. Although we took the ten-qubit case as an example, our approach could actually be used for arbitrary numbers of atomic qubits in cavity QED. On the other hand, we have noticed experimentally achieved entanglement of six photons,[38] eight trapped ions[39] and twelve NMR ensemble qubits.[40] So we also hope that our idea could be extended to other systems, such as trapped ions. In particular, we have just found a very recent experimental report to realize a Toffoli gate with two trapped-ion qubits plus a qubit by the quantized vibration mode.[41] With slight modification, our scheme could be applied to a line of ions as qubits using the vibrational mode as a quantum data bus. Furthermore, from the above discussion, we have shown the experimental difficulty for ten atomic qubits to be entangled and used for a Grover search by the current cavity QED technique. As the cavity Q factor could high enough with current techniques, we argue that our study could be useful for future QIP experiments by microwave cavity.

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