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Measurement of a discontinuous object based on a dual-frequency grating*

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The dual-frequency grating measurement theory is proposed in order to carry out the measurement of a discontinuous object. Firstly, the reason why frequency spectra are produced by low frequency gratings and high frequency gratings in the field of frequency is analysed, and the relationship between the wrapped-phase and the unwrapping-phase is discussed. Secondly, a method to combine the advantages of the two kinds of gratings is proposed: one stripe is produced in the mutation part of the object measured by a suitable low frequency grating designed by MATLAB, then the phase produced by the low frequency grating need not be unfolded. The integer series of stripes is produced by a high frequency grating designed by MATLAB based on the frequency ratio of the two kinds of gratings and the high frequency wrapped-phase, and the high frequency unwrapping-phase is then obtained. In order to verify the correctness of the theoretical analysis, a steep discontinuous object of 600×600 pixels and 10.00 mm in height is simulated and a discontinuous object of ladder shape which is 32.00 mm in height is used in experiment. Both the simulation and the experiment can restore the discontinuous object height accurately by using the dual-frequency grating measurement theory.

Keywords: dual-frequency grating, Fourier transform profilometry, measurement of discontinuous object, phase unwrapping

PACC: 4230, 4230K, 4230V

1. Introduction

Measurement of 3-D object shape is very important in machine vision, solid modeling, industrial automeasuring, etc.[1−12] Fourier transform profilometry is a popular method for the automatic optical noncontact measurement of 3-D object shape and it has been applied extensively because of its advantages of high precision, automatism, implement facility, etc.[1−9] However, when we use such a method to measure the mutation part of a discontinuous object, it is difficult to unfold the phase because of the appearance of a wrapped-phase (the range of phase values is from $-\pi$ to π) in the measurement process, so it cannot restore object shape exactly.[2−4] To obtain actual phase values, phase unwrapping is necessary in this method. If the phase value exceeds the range of $-\pi$ to π, we can eliminate truncation by adding or subtracting $2n\pi$ (n is an integer). Accordingly, actual phase values can be obtained exactly.

Aiming at the complexity of phase unwrapping,[2−4] we need the following three steps to fulfill it. Firstly, we can use a projector to project a compound dual-frequency grating designed by MATLAB to the object to be measured, then carry out a Fourier transform[13−15] by using a CCD camera[16−18] to capture the deformed fringe pattern on the object to be measured. As a result, we can obtain Fourier spectra containing two components of low frequency and high frequency. Secondly, we can separate the components of the two spectra in order to extract them by designing a suitable filter. Finally, a reverse Fourier transform can be used to acquire the exact height distribution of the measured object by using one deformed fringe only.

2. Theoretical analysis

The optical configuration of the measurement system is shown in Fig.1, where the optical axis P_1P_2 of a projector lens crosses that of the CCD camera lens I_1I_2 at point O on a reference plane. D is a point on the object surface. A and C are two points on the reference plane. d is the distance between P_2 and I_2, and L_0 is the distance between I_2 and O. A sinusoidal grating has its lines normal to the plane of the figure, and its image is projected onto the object surface.

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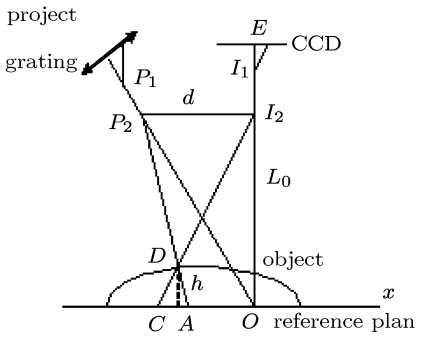


Fig.1. Optical configuration of the measurement system.

In Ref.[1], the measured object was located on the reference plan and a single-frequency grating was used to project to it by the projector. However, a compound dual-frequency grating is used in the present paper. According to Eqs.(7) and (11) in Ref.[1], the deformed fringe pattern observed through a CCD camera can be expressed as

$$\begin{aligned} g(x,y) &= r(x,y)\sum_{n=-\infty}^{\infty} a_{kn} \\ &\quad \times \exp\{\mathrm{j}[2\pi n f_k x + n\phi_k(x,y)]\} \\ &= \sum_{n=-\infty}^{\infty} q_{kn}(x,y)\exp(\mathrm{j}2\pi n f_k x), \\ &\quad k=1,2, \end{aligned} \tag{1}$$

where $r(x,y)$ is a nonuniform distribution of reflectivity on the object surface, a_{kn} are the weighting factors of the Fourier series, $\phi_k(x,y)$ is the phase modulation resulting from the object height variation, n is the grade of the frequency, and f_k is the fundamental frequency of the observed low frequency grating image when $k=1$ and that of the observed high frequency grating image when $k=2$ (thereinafter the same),

$$q_{kn}(x,y) = a_{kn} r(x,y)\exp[\mathrm{j}n\phi_k(x,y)], \tag{2}$$

$$\phi_k(x,y) = 2\pi f_k \overline{CA} \tag{3}$$

with $\overline{CA}$ being the distance between point C and point A as shown in Fig.1.

Supposing that the y axis is fixed, a one-dimensional Fourier transform is applied to expression (1) in the x axis direction; its spectra can then be given by the following formula:

$$G(f_x,y) = \sum_{n=-\infty}^{\infty} Q_{kn}(f_x - nf_k, y), \quad k=1,2, \tag{4}$$

where $Q_{kn}(f_x,y)$ are spectra after Fourier transform of $q_{kn}(x,y)$. It can be seen from expression (4) that higher order spectra and lower order spectra are produced in the field of spectra, and the spectra are separated on condition that they have no overlap with each other. If we take no account of the nonlinearity of CCD,[5,6] and use the weighted method to get rid of the zero order frequency component,[7] by filtering, we can obtain only fundamental frequency components produced by the low frequency grating and the high frequency grating in the field of spectra. Then the reversed Fourier transform is applied to the fundamental frequency components to obtain the low and high frequency phase distribution, because the height distribution contains the phase distribution, and the height distribution can be obtained too.

In real conditions, when $L_0 \gg h(x,y)$, we assume that $\phi_k(x,y) \approx \Delta\phi_k(x,y)$. Then the relationship between the phase distribution $\phi_k(x,y)$ and the height distribution $h(x,y)$ can be described as follows:

$$h(x,y) = \lambda_k \phi_k(x,y), \quad k=1,2, \tag{5}$$

where $\lambda_k = -L_0/2\pi f_k d$ is a constant related to the system structure. Because the object measured is discontinuous, the phase is wrapped in the range from $-\pi$ to π and the phase distribution obtained is discontinuous when we calculate the phase by using a reverse trigonometric function. In order to obtain the height distribution of the object calculated from the phase function, the wrapped-phase produced during the calculation of the reverse trigonometric function must be restored to the actual continuous phase distribution. The practical process is called phase unwrapping, through which the phase unwrapped is close to the actual phase. The wrapped-phase is denoted by $\varphi_k(x,y)$, and the continuous phase obtained from phase unwrapping is denoted by $\phi_k(x,y)$.

When we consider only the fundamental frequency components from the low frequency grating and the high frequency grating, we can select a suitable compound dual-frequency grating to obtain suitable low frequency f_1 and high frequency f_2, thus $Q_{11}(f_x,y)$ and $Q_{21}(f_x,y)$ from expression (4) can be separated, and $\varphi_k(x,y)$ can be obtained. The relationship between $\phi_k(x,y)$ and $\varphi_k(x,y)$ can be expressed as

$$\phi_k(x,y) = \varphi_k(x,y) + 2\pi n_k(x,y), \quad k=1,2, \tag{6}$$

where $n_k(x,y)$ is an integer series of stripes produced by the grating. The relationship between $n_1(x,y)$ and

$n_2(x,y)$ at the same point of the object measured can be given by the following formula:

$$n_2(x,y) = \frac{f_2}{f_1} n_1(x,y). \tag{7}$$

Obviously, when one fringe produced by the low frequency grating is situated in the mutation part of the object measured, the wrapped-phase is equal to the continuous phase, so the problem of phase unwrapping does not exist. But the high frequency grating can produce many fringes in the mutation part of the object measured, thus phase wrap will emerge, which makes the phase difficult to unwrap. We can obtain continuous phase by selecting a suitable low frequency grating to measure the object, and what is obtained by the high frequency grating is a wrapped-phase, so we must get rid of its wrap. We can unwrap the wrapped-phase $\varphi_2(x,y)$ from expression (6) to obtain the continuous phase $\phi_2(x,y)$. It is obvious that the process of phase unwrapping is actually the process of determining $n_2(x,y)$ from expression (6).

Because the heights at the same point of the object, measured by different grating projections, are equal, expression (5) can be written as

$$\lambda_1 \phi_1(x,y) = \lambda_2 \phi_2(x,y). \tag{8}$$

We can design a suitable low frequency grating to produce one fringe in the mutation part of the object measured, so that the phase produced by the low frequency grating does not need phase unwrapping, and it satisfies $\varphi_1(x,y) = \phi_1(x,y)$. Then, combining expressions (6) and (8) yields

$$n_2(x,y) = \mathrm{NT}\left\{\frac{1}{2\pi}[k_f \phi_1(x,y) - \varphi_2(x,y)]\right\}, \quad k = 1, 2, \tag{9}$$

where NT is an integer operator, and $k_f = f_2/f_1$. We can obtain $n_2(x,y)$ through k_f, $\phi_1(x,y)$ and $\varphi_2(x,y)$, and determine the value of the high frequency unwrapped phase $\phi_2(x,y)$ from expression (6).

The measurement accuracy is determined by the structure parameters d and L_0 of the measurement system and the fundamental frequency components f_k of the grating,[1] and their relationship can be described as follows:

$$\left|\frac{\partial h(x,y)}{\partial \phi_k(x,y)}\right| = \frac{L_0}{2\pi f_k d}, \quad k = 1, 2. \tag{10}$$

The higher the measurement accuracy is, the smaller the value of expression (10) will be. On the condition that the measurement system is fixed, the measurement accuracy is reduced by using the low frequency grating only, so it will lose some part of the height distribution in the mutation part of the discontinuous object, but the phase obtained does not need phase unwrapping. It is difficult to unfold the phase when we use a high frequency grating to measure the discontinuous object, but we can obtain the height distribution in the mutation part of the discontinuous object. What is more, the measurement accuracy is higher than that of the low frequency grating. Combining their advantages, we can measure the height distribution in the mutation part of the discontinuous object by using the low frequency grating, and obtain the object shape with high precision by using the high frequency grating.

3. Computer simulation and experiment

3.1. Computer simulation

In order to verify the correctness of the theoretical analysis, we numerically simulate a steep discontinuous object of 600×600 pixels and 10.00 mm in height. The structure parameters in the measurement system are d = 200.00 mm and L_0 = 900.00 mm. We use two kinds of gratings (f_1 = 1/16, f_2 = 1/4, so k_f = 4) designed by MATLAB in the computer to produce normalized spectra in different ranges as shown in Fig.2(a). There are two spectra produced by the low frequency grating, situated in the middle of the figure, and the other two spectra situated on both sides of the figure are produced by the high frequency grating.

We measure the object three times. First, only the low frequency grating is used, second, only the high frequency grating is used, and finally, a compound dual-frequency grating is used. The restored object shapes are shown in Figs.2(b)–2(d), separately.

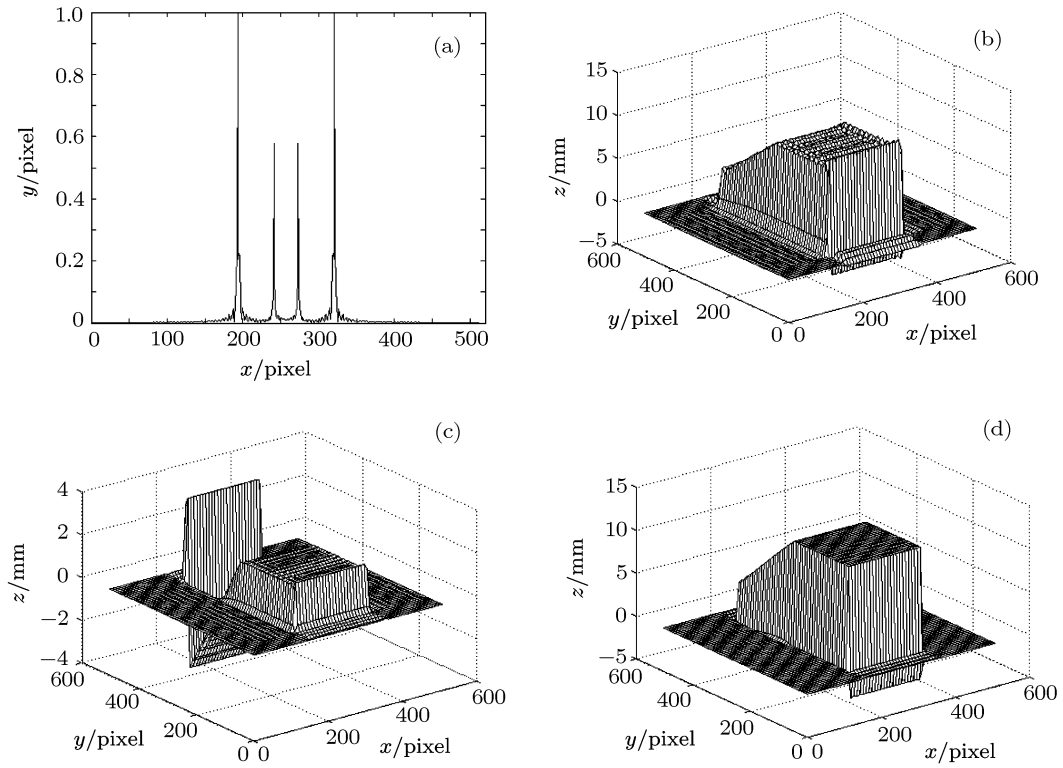


Fig.2. Results of simulation. They are (a) normalized spectra of different ranges produced by two kinds of gratings ($k_f = 4$), (b) restored object shape measured by the low frequency grating, (c) restored object shape measured by the high frequency grating, and (d) restored object shape measured by the compound frequency grating.

We can draw a conclusion that the object shape can be restored by using a low frequency grating only, but its accuracy is not high and the profile of the mutation part of the object measured is not clear. When we use a high frequency grating only, the profile details of the section can be obtained clearly, but the phase is difficult to unfold, and it cannot restore the object shape. When we use a compound dual-frequency grating, we can restore the object shape because of the combination of advantages from the two kinds of gratings. What is more, the section has clear profile details.

3.2. Experiment

In order to verify the correctness of the theoretical analysis further, we use a discontinuous object of ladder shape which is 32.00 mm in height to make an actual experiment. A schematic diagram of the simple experimental equipment is shown in Fig.3, and the structure parameters in the measurement system are $d = 250.00$ mm and $L_0 = 1000.00$ mm. We use a DMD

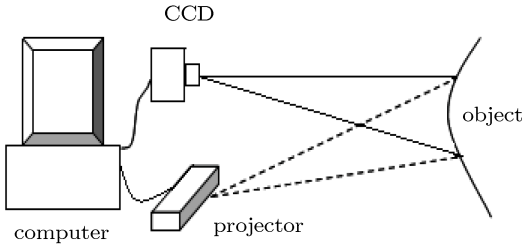


Fig.3. Schematic diagram of experimental equipment.

figure projector, whose resolving power is 800×600, to project low frequency grating fringes, high frequency grating fringes and compound dual-frequency grating fringes (all of them are designed by MATLAB, likewise, $f_1 = 1/16$, $f_2 = 1/4$, so $k_f = 4$) produced by a computer to the object measured in the experiment, separately. Then we use a low aberrance CCD camera to capture these fringes.

In the experiment, the Fourier transform is applied to deformed dual-frequency fringes of the object, and the normalized spectra obtained are shown in Fig.4(a). There are two spectra produced by the low frequency grating situated in the middle of the figure, and the other two spectra produced by the high frequency grating are situated on both sides of the figure. We measure the object by using the same method as the simulation. The sections with the same restored object height are shown as a solid line, a dotted curve and an asterisk curve in Fig.4(b) (the sample interval in the figure has 8 pixels), separately. We can draw the same conclusion from Fig.4(b) as from the simulation.

When we use a compound dual-frequency to measure the discontinuous ladder object, we can obtain its restored shape as shown in Fig.4(c).

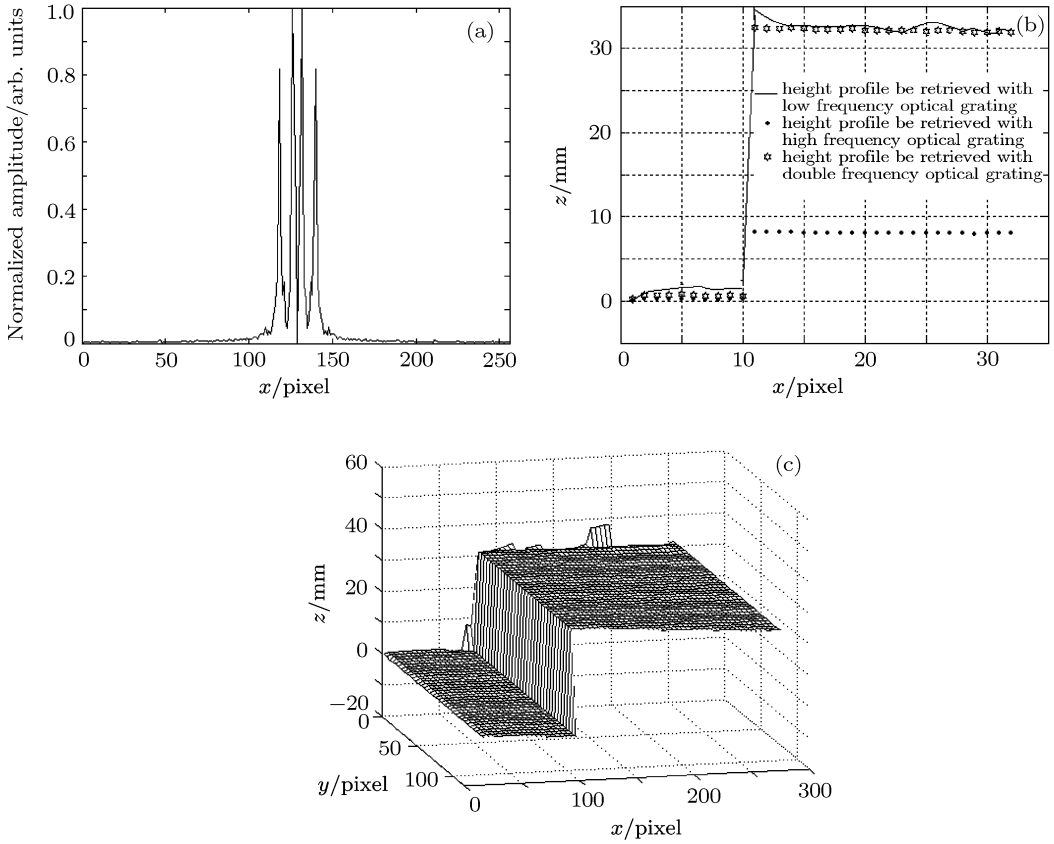


Fig.4. Results of experiment. They are (a) normalized spectra of different ranges produced by two kinds of gratings, (b) the same height section of the object shape restored by three different measurement methods, and (c) object shape restored by the compound dual-frequency grating measurement method.

3.3. Errors and their causes in computer simulation and experiment

According to expressions (3) and (5), we can obtain the mean height of the measured value of the discontinuous object. It is 10.13 mm in the computer simulation and 32.46 mm in the experiment.

Therefore, in the simulation, the height error between the measured value and the actual value is 0.13 mm, and the percentage error of relative height is 1.30%; in the experiment, the height error is 0.46 mm, and the percentage error of relative height is 1.44%.

The discrepancy between the simulations, measurements and actual values are mainly due to the following factors:

1) in the simulation and the experiment, when $L_0 \gg h(x,y)$, we assume that $\phi_k(x,y) \approx \Delta\phi_k(x,y)$, so expression (3) is approximative.

2) it is difficult to obtain a real sinusoidal grating because of the nonlinearity of CCD.[5,6]

4. Conclusions

Although the shape of a discontinuous object can be restored easily by using a low frequency grating, its accuracy is not high and the profile details of the mutation part are not clear. Opposite results are seen when a high frequency grating is used; however, it has a wrapped-phase because of the existence of the phase wrapping phenomenon. So, it is difficult to unfold the phase and the object shape cannot exactly be restored. Therefore, we combine the advantages of the two kinds of gratings. Firstly, we design a suitable low frequency grating to produce one fringe in the mutation part of the object measured, so that the phase produced by the low frequency grating does not need phase unwrapping. Secondly, we use the frequency ratio of the two kinds of gratings and the wrapped-phase of the high frequency one to determine an integer series of the fringes produced by the high frequency grating. Finally, the phase unwrapped by high frequency is obtained, and the height distribution of the measured discontinuous object is restored with high accuracy.

We use computer simulation and experiment to prove the theory proposed in this paper. The results verify the correctness of the theoretical analysis.

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