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Effects of bulk viscosity on hadron spectra and the Hanbury-Brown Twiss radius by causal viscous hydrodynamics*

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The particle spectra and Hanbury-Brown Twiss (HBT) radius of Au+Au collisions at RHIC energy are investigated by a hydrodynamical expanding source with both shear and bulk viscosities (ζ). With a large width of the ratio of ζ to entropy density s, both the particle transverse momentum spectra and the ratio $R_{\rm out}/R_{\rm side}$ of HBT radii in the direction of the total transverse momentum of detected two particles ($R_{\rm out}$) and perpendicular to both this direction and the beam direction ($R_{\rm side}$) become a little steeper.

Keywords: relativistic hydrodynamics, bulk viscosity, hadron spectra, HBT radius

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1. Introduction

Ideal hydrodynamics has successfully explained the experimental data of the elliptic flow coefficient $v_2(p_{\rm T})$ as well as the transverse momentum spectra of hadrons in central and semi-peripheral collisions up to $p_{\rm T} = 1.5 - 2$ GeV/c and also predicted hard photon production.[1,2] However, the almost perfect ideal fluid dynamical description of the experimental data gradually breaks down in regions of more peripheral collisions and/or high transverse momenta ($p_{\rm T}$), and it also fails to reproduce the Hanbury-Brown Twiss (HBT) radius.[3,4] Actually, ideal fluid is only an approximation;[5] the viscous hydrodynamics need to be investigated. The so-called first order relativistic viscous hydrodynamics was first formulated by Eckart with a variation by Landau and Lifshitz, and many calculations have been done so far.[6−8] But there are two problems in their approaches: one is the dissipative fluctuations which may propagate at a speed larger than the speed of light and thus lead to the causality problem; the other is solutions which may develop instabilities. The second order formalism developed by Israel and Stewart about thirty years ago avoided these difficulties.[9] But due to the complication of the evolution equations, numerical codes have not been developed to solve the Israel–Stewart equations until recent years.[10−16] For comparison with the experimental data, people only considered the shear viscosity and found that the particle spectra and elliptic flow became more consistent in a larger $p_{\rm T}$ range. But the HBT radius is still not consistent with the experimental data.[12]

Although one expects that the bulk viscosity is to be negligible for temperatures far away from the phase transition temperature $T_{\rm c}$, large effects are expected near $T_{\rm c}$.[17−21] As suggested in Ref.[22], the HBT results may come close to the experimental data if bulk viscosity is taken into account. Based on this argument, we want to check if this argument is valid in the present work. To this end, we use the algorithm which has been used in Ref.[12] to consider the effect of the bulk viscosity for central Au+Au collisions at RHIC energies.

2. Causal viscous hydrodynamics and results

General hydrodynamic equations arise from the local conservation of energy and momentum[1]

$$\partial_\mu T^{\mu\nu}(x) = 0, \tag{1}$$

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where the energy–momentum tensor without heat conduction is decomposed in the form[14]

$$T^{\mu\nu} = eu^{\mu}u^{\nu} - (p+\Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}. \tag{2}$$

Here e and p are the local energy density and thermal equilibrium pressure, and u^{μ} is the 4-velocity of the energy flow which obeys $u^{\mu}u_{\mu} = 1$. Π is the bulk viscous pressure, $\Delta^{\mu\nu} = g^{\mu\nu} - u^{\mu}u^{\nu}$ is transverse to the flow velocity, that is $\Delta^{\mu\nu}u_{\nu} = 0$. $\pi^{\mu\nu}$ is the traceless shear viscous pressure tensor. With Eqs.(1) and (2), we can obtain the evolution equations of the energy density and the 4-velocity of the energy flow

$$\begin{aligned}&(\epsilon + p + \Pi)Du^{\mu}\\ &= \nabla^{\mu}(p+\Pi) - \Delta^{\mu}_{\nu}\nabla_{\sigma}\pi^{\nu\sigma} + \pi^{\mu\nu}Du_{\nu}\,,\end{aligned} \tag{3}$$

$$D\epsilon = -(\epsilon + p + \Pi)\nabla_{\mu}u^{\mu} + \pi^{\mu\nu}\langle\nabla_{\nu}u_{\mu}\rangle\,, \tag{4}$$

where D is the time derivative in the local fluid rest frame and fulfils $D = u^{\mu}\partial_{\mu}$. The angular bracket notation is defined by $\langle\nabla^{\mu}u^{\nu}\rangle = 1/2(\nabla^{\mu}u^{\nu}+\nabla^{\nu}u^{\mu}) - 1/3(\nabla\cdot u)\Delta^{\mu\nu}$. In the Israel–Stewart approach the kinetic evolution equations of the bulk pressure Π and the traceless shear viscous tensor $\pi^{\mu\nu}$ are[14]

$$D\Pi = -\frac{1}{\tau_{\Pi}}\big(\Pi + \zeta\nabla\cdot u\big), \tag{5}$$

$$\begin{aligned}D\pi^{\mu\nu} =\ & -\frac{1}{\tau_{\pi}}\big(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\big)\\ & -\big(u^{\mu}\pi^{\nu\alpha} + u^{\nu}\pi^{\mu\alpha}\big)Du_{\alpha},\end{aligned} \tag{6}$$

where η and ζ denote the bulk and shear viscous coefficients, respectively. τ_{π} and τ_{Π} are the relaxation times for the bulk pressure and the shear viscous pressure tensor, respectively. They can be related to η and ζ as follows[10]

$$\tau_{\pi} = 2\eta\beta_2\ , \quad \tau_{\Pi} = \zeta\beta_0. \tag{7}$$

For a massless Boltzmann gas $\beta_2 \approx 3/(4p)$, so we can get $\tau_{\pi} \approx 3\eta/2p$ which translates to $\tau_{\pi} \approx 6\eta/sT$,[10,11] where s is the entropy density and this is the value we use in this paper. In this case the relaxation and coupling coefficients for bulk pressure vanish, $\beta_0 \to \infty$, and thus there are no effects of the bulk viscosity.[10] However, as we already know, the bulk viscous coefficient ζ becomes large near the quark-hadron phase transition temperature $T_{\rm c}$.[23] So the behaviour of the bulk viscous pressure must change and β_0 does not tend to be infinite any more. However, as far as we know, there are no quantitative results for β_0 which are relevant to the applications in nuclear collisions (an attempt has been made in Ref.[10]). In this paper we will use two different β_0 in our calculations which correspond to two different τ_{Π}, namely, $\tau_{\Pi} = 8\zeta/sT$ and $\tau_{\Pi} = 30\zeta/sT$, respectively. Of course people could try smaller β_0. But with a smaller β_0, there will be casuality problems and this may cause instability.[24]

As in ideal hydrodynamics we need initial conditions at τ_0, the equation of state of bulk matter and the freeze-out condition to obtain the results of these evolution equations. Generally there are two models for the initialization of the hydrodynamics: the Glauber-type model and the colour glass condensate-type model. Here we use the Glauber model and the energy density at the initial time τ_0 is parameterized by the number density of wounded nucleons $\epsilon \sim n_{\rm WN}$,[12,25] where $n_{\rm WN} = 2T_A\left[1 - \left(1 - \frac{\sigma T_{\rm A}}{A}\right)^{A}\right]$ and $T_A = \int_{-\infty}^{\infty} {\rm d}z\rho_0/\{1 + \exp[(\sqrt{r^2+z^2} - R_0)/\chi]\}$. For gold nuclei, $A = 197$, $R_0 = 6.4$ fm, $\chi = 0.54$ fm, the nucleon–nucleon cross section σ at $\sqrt{s_{\rm NN}} = 200$ GeV is assumed to be 40 mb and ρ_0 is chosen to fulfil $2\pi\int r{\rm d}rT_A = A$. The EOS we use here is calculated by Laine and Schroder which corresponds to a phase transition temperature $T_{\rm c} \approx 190 \pm 20$ MeV.[26] In their calculation they considered a hadron resonance gas at low temperatures and used a high order weak-coupling QCD result at high temperatures. The freeze-out mechanism is the Cooper–Frye prescription in which particles freeze out at a single temperature $T_{\rm f}$.[27]

Unlike ideal hydrodynamics, in causal viscous hydrodynamics we need to give the initial condition for $\pi^{\mu\nu}$ and Π as well as the values of η/s and ζ/s.[10−12,14] Here we choose $\pi^{\mu\nu} = 0$ and $\Pi = 0$ at the initial time which minimize the effects of viscosity. There have been many calculations for η/s in which there is a conjectured minimal bound for η/s in AdS/CFT with $\eta/s \geq 1/4\pi \approx 0.08$,[28] and we can see in Ref.[29] that η/s does not change a lot for different temperatures. In this paper we want to see the influence of the bulk viscosity, so we use the lower bound of η/s during the whole evolution of the fluid. In Ref.[21] the value of ζ/s reaches the maximum at $T_{\rm c}$ and changes linearly to a very small value when the temperature goes away from $T_{\rm c}$ and the width is small. However, in Refs.[19] and [20] the width is large. In this paper, we will take two linearly changing ζ/s with different widths and heights as shown in Fig.1 in our calculations.

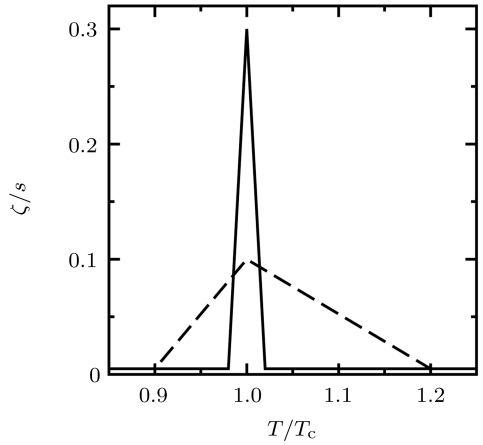


Fig.1. Two linearly changed ζ/s with different widths and heights. See text for details.

For central Au+Au collisions at RHIC, the geometries which are longitudinally expanding are space–time rapidity independent and have radial symmetry. For these geometries it is convenient to work in the co-moving and radial coordinates τ, r, ϕ, η with the relations $\tau = \sqrt{t^2 - z^2}$, $r^2 = x^2 + y^2$, $\tan\phi = y/x$ and $\eta = \text{artanh}(z/t)$.

The only non-vanishing fluid velocity components are then u^τ and u^r with the relation $u^\tau = \sqrt{1 + (u^r)^2}$. To solve Israel–Stewart hydrodynamic equations which have this kind of geometry we use the one dimensional algorithm used in Ref.[12].

As in the procedure adopted in ideal hydrodynamics,[30] the initial central temperature T_0 and the freeze-out temperature T_f are chosen to make both the normalization and the slope of the resulting pion spectrum be in reasonable agreement with the experimental data and we get $T_0 = 0.34$ MeV and $T_\text{f} = 0.17$ MeV. The freeze-out temperature is larger than that typically used in ideal hydrodynamics. In ideal hydrodynamics interactions of the system can keep the system in thermal equilibrium, but the interactions are not so efficient in viscous hydrodynamics which allows departures from equilibrium. So an earlier freeze-out and thus a higher T_f is to be expected in viscous hydrodynamics. In Ref.[13] there are other parameters that can fit both the pion spectrum and the HBT radii, but either the η/s is too large or the freeze-out temperature is unreasonable large.

Figure 2 shows the velocity and temperature profiles for different times of the evolution of the fluid using the parameters after fitting the pion spectra. The left and right panels are for relaxation time values $\tau_\Pi = 8\zeta/sT$ and $\tau_\Pi = 30\zeta/sT$, respectively. In each panel, we calculate three cases: (a) $\eta/s = 0.08$ and without bulk viscosity, (b) $\eta/s = 0.08$, $(\zeta/s)_\text{max} = 0.3$ at first but then later changes linearly to $(\zeta/s)_\text{min} = 0.005$ at $0.98\,T_\text{c}$ and $1.02\,T_\text{c}$ respectively, corresponding to the solid line in Fig.1, and (c) $\eta/s = 0.08$, $(\zeta/s)_\text{max} = 0.1$ at first but later changes linearly to $(\zeta/s)_\text{min} = 0.005$ at $0.9\,T_\text{c}$ and $1.2\,T_\text{c}$ respectively, corresponding to the dashed line in Fig.1. We can see that if bulk viscosity is taken into account, at earlier times the velocity changes near T_c, and the velocity in case (c) changes more rapidly than case (b). This is due to the narrower width and larger height of ζ/s. Also, with a much larger peak, there may be non-periodic oscillation in velocity and temperature. So the method of hydrodynamics may not be suitable and the source may clusterize to small fragments.[31] Later the effect of bulk viscosity extends to other places where the temperature is far away from T_c. We can see that the evolution of the fluid is sensitive to the width of ζ/s rather than its height. In both cases (b) and (c), the fluid tends to be retarded and the effect is stronger with larger widths. We can also see that the effect of bulk viscosity is not much stronger with a smaller relaxation time.

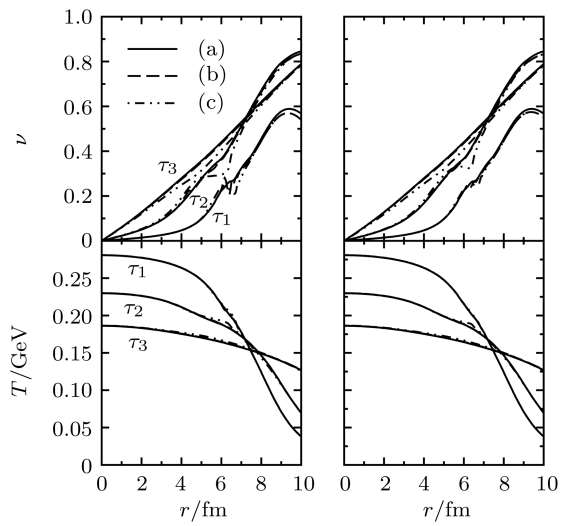


Fig.2. Temperature and velocity profiles for different conditions (a), (b) and (c) with $\tau_\Pi = 8\zeta/sT$ (left panels) and $\tau_\Pi = 30\zeta/sT$ (right panels) of the relaxation time values at different times $\tau_n = 2n$ fm/c. See text for details.

We plot the pion, kaon and proton spectra together with the RHIC STAR and PHENIX experimental data for the most central 5% of Au + Au collisions at $\sqrt{s_\text{NN}} = 200$ GeV in Fig.3.[32,33] It shows that cases (a) and (b) are almost the same and case (c) become steeper which indicates smaller transverse flow. From this we can conclude that the effect of

the width is more important than the height of ζ/s in hydrodynamics.

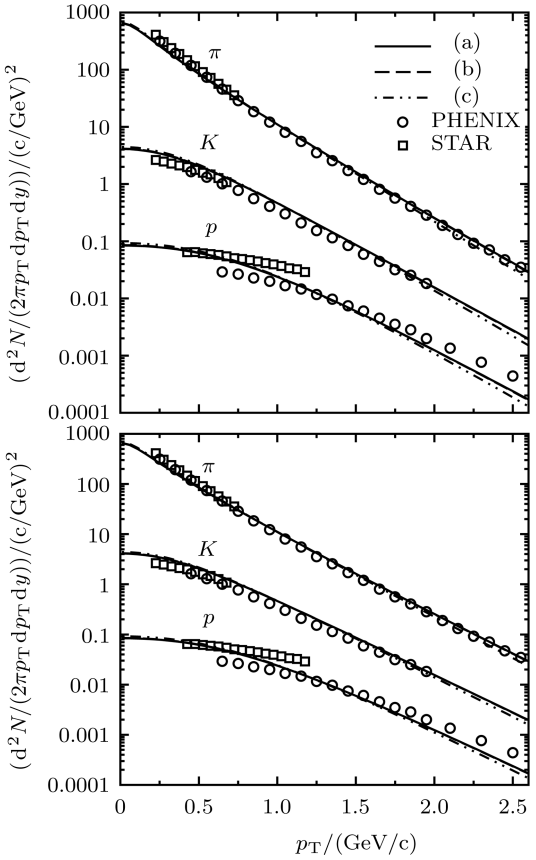


Fig.3. Comparison of the calculated spectra of π, K (scaled by 0.1) and P (scaled by 0.01) for different conditions (a), (b) and (c) with the STAR and PHENIX data. The upper and bottom panels correspond to $\tau_\Pi = 8\zeta/sT$ and $\tau_\Pi = 30\zeta/sT$ of the relaxation time values, respectively.

Finally we also show the viscous hydrodynamic results for the HBT radii together with experimental data. The results are shown in Fig.4 where the ratios $R_\mathrm{out}/R_\mathrm{side}$ are for different relaxation time values and different cases.[34] The radius R_out represents the radius in the direction of the total transverse momentum of the two detected particles and R_side corresponds to the radius perpendicular to both the total transverse momentum direction and the beam direction. There are some quantitative effects for the HBT radius' ratio in different p_T ranges. Essentially, $R_\mathrm{out}/R_\mathrm{side}$ becomes steeper versus the transverse momentum when the bulk viscosity is taken into account, which is consistent with the steeper transverse momentum spectra observed in Fig.3. Nevertheless we still did not see that the bulk viscosity can change the ratio substantially. This can also be seen in Ref.[35]. The reason could be that the thermodynamical quantities change only a little at T_f for different cases, and the freeze-out surface is almost the same.

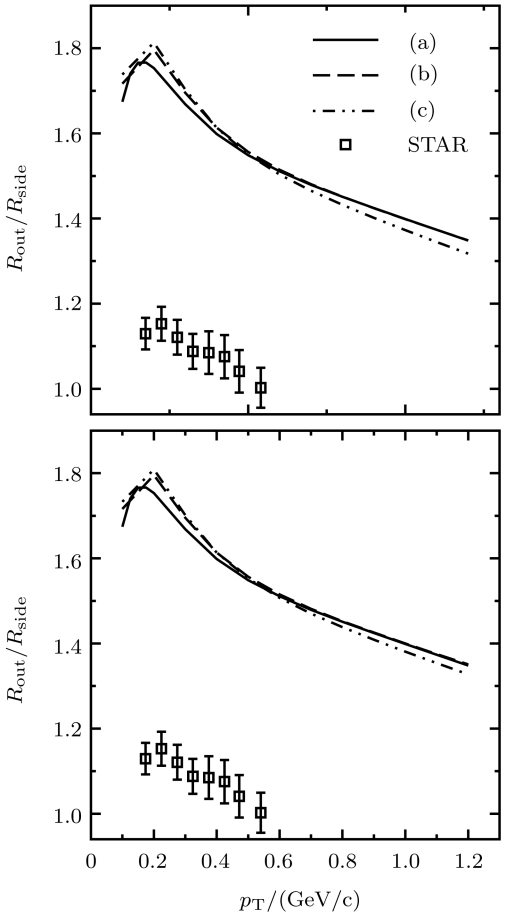


Fig.4. Calculated ratio $R_\mathrm{out}/R_\mathrm{side}$ of HBT radii for different conditions (a), (b) and (c) together with STAR data. The upper and bottom panel correspond to $\tau_\Pi = 8\zeta/sT$ and $\tau_\Pi = 30\zeta/sT$ of the relaxation time values, respectively.

3. Summary and discussions

In summary, we have solved the causal viscous hydrodynamic equations for central Au+Au collisions at $\sqrt{s_\mathrm{NN}} = 200$ GeV using a simple numerical code. Using the parameters that fitted the pion spectra, we calculated the temperature and velocity profiles, the pion, kaon and proton spectra as well as the HBT results for different relaxation time values and different bulk viscosity cases. We find that the fluid evolves slowly when the bulk viscosity is taken into account. The particle spectra become steeper which indicates smaller transverse flow when the width of ζ/s becomes larger. We can also see that the HBT results are still far away from the experimental data when the bulk viscosity is taken into account in our approach.

There are other problems that have to be considered to obtain a proper description of the experimental HBT radii. Firstly, we should investigate the influence of different EOS. In Ref.[22] the author uses different EOS to calculate the causal viscous hydrodynamics and finds that the behaviour of $R_{\rm out}/R_{\rm side}$ for different ζ/s changes substantially. Secondly, larger ζ/s may develop instabilities in the mixed phase, and the source may be clusterized to droplets; this freeze-out mechanism is different from Cooper–Frye formalism.[31,36] It has been claimed that a granular source method can explain the HBT puzzle.[37]

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