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Mode shift and stability control of a current mode controlled buck-boost converter operating in discontinuous conduction mode with ramp compensation*

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By establishing the discrete iterative mapping model of a current mode controlled buck-boost converter, this paper studies the mechanism of mode shift and stability control of the buck-boost converter operating in discontinuous conduction mode with a ramp compensation current. With the bifurcation diagram, Lyapunov exponent spectrum, time-domain waveform and parameter space map, the performance of the buck-boost converter circuit utilizing a compensating ramp current has been analysed. The obtained results indicate that the system trajectory is weakly chaotic and strongly intermittent under discontinuous conduction mode. By using ramp compensation, the buck-boost converter can shift from discontinuous conduction mode to continuous conduction mode, and effectively operates in the stable period-one region.

Keywords: buck-boost converter, iterative map model, mode shift, ramp compensation

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1. Introduction

Switching DC–DC converters are strongly nonlinear circuits with richly nonlinear phenomena like border-collision bifurcation, period-doubling bifurcation, Hopf bifurcation, time-bifurcation, chaos and quasi-periodicity etc.[1−14] Under the variation of circuit parameters, switching DC–DC converters will enter into chaos via bifurcation, which will deteriorate the converter's performance. Therefore, chaotic behaviour and nonlinear phenomena of switching DC–DC converters have attracted much attention and have been extensively studied recently.

A switching DC–DC converter may shift from CCM (Continuous Conduction Mode) to DCM (Discontinuous Conduction Mode) with variation of converter circuit parameters such as input voltage or reference current. It has been found that specific types of bifurcation phenomena like weak chaos and strong intermittence occur during CCM–DCM transition in current mode controlled converters.[15−18]

In Ref.[19], we have studied the mechanism of stability control of a switching DC–DC converter in CCM with ramp compensation. In this paper, we will study the dynamic behaviour of a current mode controlled buck-boost converter operating in DCM with ramp compensation and illustrate how the compensating ramp current will affect the dynamic behaviour of the switching DC–DC converter.

2. Discrete mapping model of the buck-boost converter with ramp compensation

A schematic diagram of the current mode controlled buck-boost converter utilizing a compensating ramp current is shown in Fig.1. A timer generates a free-running clock which controls the operation of the current mode control loop. The switch S is turned on at the beginning of each clock pulse. The converter is controlled by a feedback loop consisting of a comparator and a flip-flop. The inductor current is sensed and compared with a compensated reference current $i_{\rm ref}$, which is modified by a compensating ramp cur-

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rent given by

$$i_{\rm ref} = I_{\rm ref} - m_{\rm c} \bmod (t, T), \tag{1}$$

where $I_{\rm ref}$ is the reference current, $m_{\rm c}$ is the compensation ramp slope, T is the clock period and mod $(\cdot)$ is the modulus function. The switch S turns on at the beginning of each switching cycle and turns off when the inductor current is equal to the compensated reference current $i_{\rm ref}$. In CCM, the inductor current is always non-zero, while in DCM, the inductor current drops to zero during the switch off period and remains at zero until the end of the clock cycle.

Since the control logic has an external clock, we can obtain its discrete iterative model by sampling in synchronism with the clock. In Fig.1, the capacitor helps to smooth the output voltage ripple, and if the clock period is much smaller than the time constant of the RLC circuit, the output voltage can be assumed as constant. In this case, the system becomes one-dimensional and the inductor current waveform becomes piecewise linear.

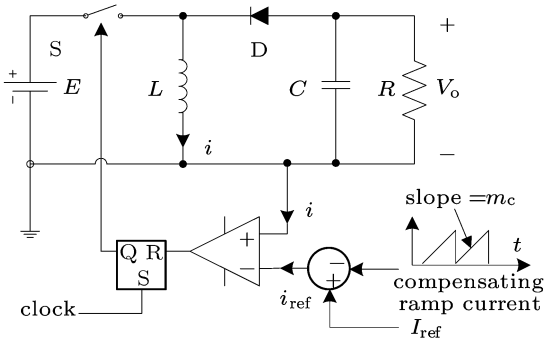


Fig.1. Current mode controlled buck-boost converter with ramp compensation.

Let the slopes of the inductor current during switch on-state and switch off-state be

$$m_1 = \frac{E}{L} \quad \text{and} \quad m_2 = \frac{V_{\rm o}}{L} \tag{2}$$

respectively, and the inductor current at the beginning of the n-th and $(n{+}1)$-th clock pulse be i_n and i_{n+1} respectively.

For operation in DCM, there are two borders in the discrete state-space. The first borderline value $I_{\rm b1}$ is defined as the value of the inductor current at the beginning of the clock pulse which reaches $i_{\rm ref}$ just at the end of the clock pulse. The second borderline value $I_{\rm b2}$ is defined as the value of the inductor current at the beginning of the clock pulse which touches zero just at the end of the clock pulse.

The inductor current waveforms of the current mode controlled buck-boost converter change if we consider the compensating ramp current as shown in Fig.2. Figure 2(a) shows the evolution of the inductor current if $i_n = I_{\rm b1}$. In this case, the current reaches $i_{\rm ref}$ at the end of the n-th clock pulse, the switch S remains on throughout the clock period. Figure 2(b) shows the evolution of the inductor current if $i_n = I_{\rm b2}$. In this case, at the end of the n-th clock pulse, the current decreases to zero.

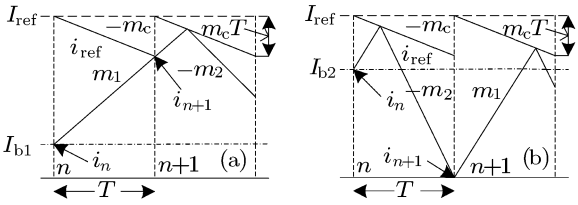


Fig.2. Illustration of the inductor current waveforms for a buck-boost converter with two borders: (a) $i_n = I_{\rm b1}$, $i_{n+1} = i_{\rm ref}$; (b) $i_n = I_{\rm b2}$, $i_{n+1} = 0$.

By the definitions of these two borderlines, from Fig.2(a), we can easily obtain the expression of borderline value $I_{\rm b1}$ as below:

$$I_{\rm b1} = I_{\rm ref} - (m_1 + m_{\rm c})T. \tag{3}$$

From Fig.2(b), the inductor current at the beginning of the n-th clock pulse equals the second borderline value, the switch S turns on at the beginning of the n-th clock pulse, the inductor current linearly rises, and then turns off when the inductor current equals the compensated reference current $i_{\rm ref}$. The turn-on time $t_{\rm on}$ can thus be obtained as

$$t_{\rm on} = \frac{I_{\rm ref} - i_n}{m_1 + m_{\rm c}}. \tag{4}$$

At the end of the clock pulse, i.e. at the beginning of the $(n{+}1)$-th clock pulse, $i_{n+1} = 0$, thus

$$I_{\rm ref} - m_{\rm c} t_{\rm on} - m_2(T - t_{\rm on}) = 0. \tag{5}$$

By putting $t_{\rm on}$ into Eq.(5), the borderline value $I_{\rm b2}$ can be obtained as

$$I_{\rm b2} = \frac{(m_1 + m_2) I_{\rm ref} - m_2(m_1 + m_{\rm c})T}{m_2 - m_{\rm c}}. \tag{6}$$

There can be three types of orbits between consecutive clock instants as below:

(1) If $i_n \le I_{\rm b1}$, the switch remains on throughout the clock period, and the map is easily given by

$$i_{n+1} = i_n + m_1 T; \tag{7}$$

(2) If $I_{\rm b2} > i_n > I_{\rm b1}$, the inductor current increases to $i_{\rm ref}$ and then decreases until the end of the

clock period. The increasing time, i.e. turn-on time $t_{\rm on}$ has been given in Eq.(4), thus the decreasing time, i.e. turn-off time, equals $(T - t_{\rm on})$. Therefore

$$I_{\rm ref} - m_{\rm c}t_{\rm on} - i_{n+1} = m_2(T - t_{\rm on}). \quad (8)$$

Substituting $t_{\rm on}$ into Eq.(8), we can obtain the following map:

$$i_{n+1} = -\frac{m_2 - m_{\rm c}}{m_1 + m_{\rm c}} i_n - m_2 T + \frac{m_1 + m_2}{m_1 + m_{\rm c}} I_{\rm ref}; \quad (9)$$

(3) If $i_n \geq I_{\rm b2}$, the inductor current reaches zero, i.e., the converter enters into DCM. Thus, at the end of the n-th clock period, we have

$$i_{n+1} = 0. \quad (10)$$

This piecewise-smooth system can exhibit bifurcation to chaotic behaviour with the variation of circuit parameters. When without ramp compensation, if we take the input voltage as the bifurcation parameter, we can obtain its bifurcation diagram and its corresponding Lyapunov exponent as shown in Figs.3(a) and 3(b) respectively. The numerical simulations are performed with the following parameters: $V_{\rm o} = 10$ V, $L = 0.9$ mH, $I_{\rm ref} = 1$ A, $T = 100$ μs and $m_{\rm c} = 0$.

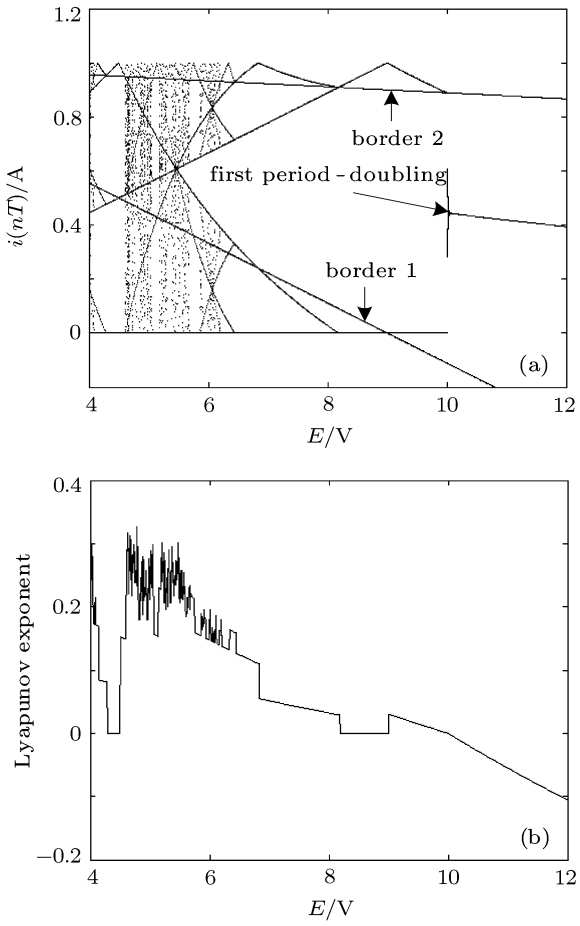


Fig.3. (a) Bifurcation diagram with E as parameter. (b) Corresponding Lyapunov spectrum.

From Fig.3(a), it is found that the system has complex dynamical behaviours. The system has a reverse period-doubling route to chaos with the increase of E, its first period-doubling bifurcation occurs when $E = 10$ V. Moreover, the period-doubling bifurcation and the border collision bifurcation occur at the same parameter value. Some special dynamical phenomena appear in this system: (i) after the first period-doubling bifurcation occurs, the system operates in DCM, i.e., the sample inductor current touches zero; (ii) at each period-doubling bifurcation point, the system orbit collides with the second borderline $I_{\rm b2}$, resulting in the border collision bifurcation; (iii) there are lots of periodic windows in the chaotic region, the system shows weak chaos and strong intermittence, which means that the chaotic behaviour weakens in DCM. Where the behaviour is chaotic in DCM, the bifurcation diagrams show a high density of points in the neighbourhood of unstable periodic orbits, which implies that there is intermittent periodic behaviour within chaos.

3. Mechanism of mode shift by utilizing ramp compensation

Based on the above discrete iterative mapping model of the current mode controlled buck-boost converter, we can investigate its performance by utilizing a compensating ramp current. Figures 4(a) and 4(b) show bifurcation diagrams with two different compensation slopes. It can be observed that with the increase of compensation slope, the buck-boost converter shifts from DCM to CCM, which makes its weakly chaotic orbit become strongly chaotic and its stable period-1 operation region become wider. It should be noted that when the system enters into CCM, the border collision bifurcation occurs at the first period-doubling bifurcation point caused by the unstable period-2 orbit hitting the first borderline $I_{\rm b1}$, while the system orbits cannot touch the second borderline $I_{\rm b2}$ in the parameter range. Following the border collision, a chaotic orbit develops. With further reduction of the input voltage, the pieces of the attractor join pair-wise to give a one-piece attractor at

about $E = 4$ V.

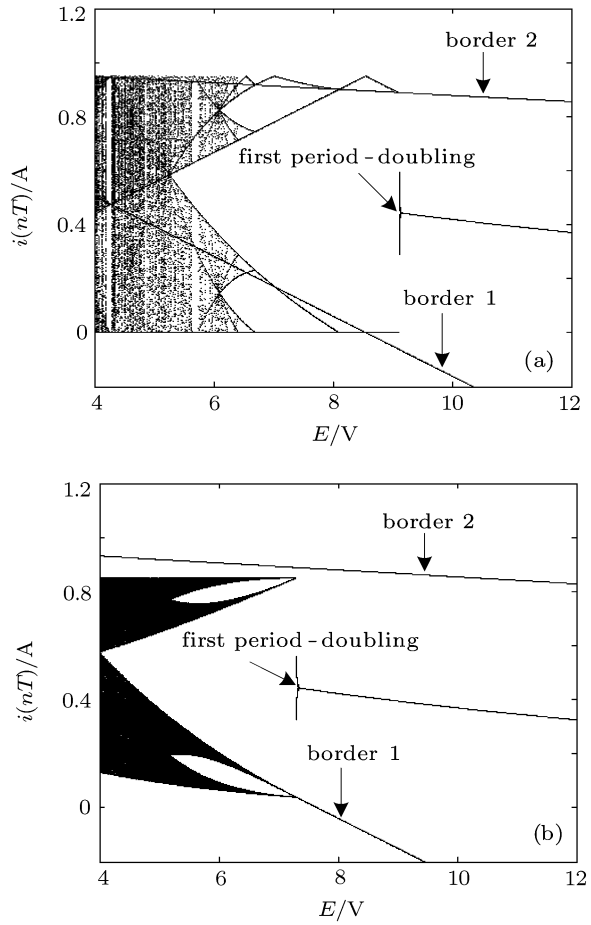


Fig.4. Bifurcation diagram with two different compensation slopes: (a) $m_c = 500$; (b) $m_c = 1500$.

Clearly, during stable period-one operation, the case of $i_n \leq I_{b1}$ cannot occur. Thus, the stability of the converter is only affected in the case of $I_{b2} > i_n > I_{b1}$. Considering Eq.(9), we obtain the eigenvalue λ of the characteristic equation for the inner compensated loop,

$$\lambda = -\frac{m_2 - m_c}{m_1 + m_c}. \tag{11}$$

To ensure stable operation, λ must fall between –1 and 1. In particular, the first period-doubling occurs when $\lambda = -1$. Hence, by putting $\lambda = -1$, the critical compensation slope M_c can be obtained as

$$M_c = \frac{m_2 - m_1}{2} = \frac{V_o - E}{2L}. \tag{12}$$

Note that the critical compensation slope decreases linearly with the increase of the input voltage. Let the compensation slope of buck-boost converter be m_c, the critical input voltage E_c can then be yielded as

$$E_c = V_o - 2m_c L. \tag{13}$$

It is found that the maximum value of inductor current at the end of the n-th clock cycle is

$$i_{n+1,\max} = I_{ref} - m_c T. \tag{14}$$

The chaotic orbit collides with the second borderline I_{b2}, resulting in the border collision bifurcation and CCM–DCM shift. Under this condition, there exists $I_{b2} = i_{n+1,\max}$, i.e.,

$$I_{ref} - m_c T = \frac{(m_1 + m_2) I_{ref} - m_2 (m_1 + m_c) T}{m_2 - m_c}. \tag{15}$$

By substituting Eq.(2) into Eq.(15), the critical input voltage E_{Mode} for the operation mode shifting from CCM to DCM will be

$$E_{Mode} = \frac{m_c^2 L^2 T - m_c L^2 I_{ref}}{L I_{ref} - V_o T}, \tag{16}$$

or the critical compensation slope m_{Mode} for the operation mode shifting from CCM to DCM will be

$$m_{Mode} = \frac{I_{ref}}{2T} - \sqrt{\left(\frac{I_{ref}}{2T}\right)^2 + E\left(\frac{I_{ref}}{TL} - \frac{V_o}{L^2}\right)}. \tag{17}$$

Figure 5 shows the bifurcation diagram of the buck-boost converter under the variation of ramp compensation slope m_c, with $V_o = 10$ V, $L = 0.9$ mH, $I_{ref} = 1$ A, $T = 100$ μs and $E = 6$ V. It can be observed that the first period-doubling bifurcation occurs when $m_c = 2222$, which coincides well with $M_c = 2222$ calculated from Eq.(12). After the first period-doubling bifurcation occurs, the border collision bifurcation occurs since a fixed point of the system collides with the borderline I_{b1} separating two

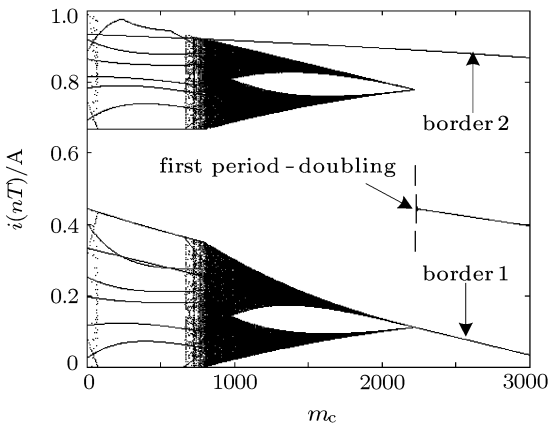


Fig.5. Bifurcation diagram with m_c as parameter.

smooth regions. Around $m_c = 806$, which coincides well with $m_{Mode} = 805.6475$ calculated from Eq.(17), the chaotic orbit collides with the second borderline

I_{b2} separating three smooth regions, the system enters into DCM from CCM.

Figure 6 shows the time-domain simulation waveforms of inductor current. During time 0.06 s$\leq t <$0.08 s, the system operates in DCM without ramp compensation, i.e., the inductor current touches zero in some switching cycle intervals, and shows high-periodic behaviour and intermittent locking into the periodic orbits. At time $t = 0.08$ s, by using a compensating ramp current with $m_c = 1500$, the system enters into CCM and shows chaotic behaviour. At time $t = 0.09$ s, with $m_c = 2500$, the system soon comes out of chaos, which means that compensating ramp current control can be used to control the chaotic behaviour of the buck-boost converter and thus realize stabilization control of the current mode controlled buck-boost converter.

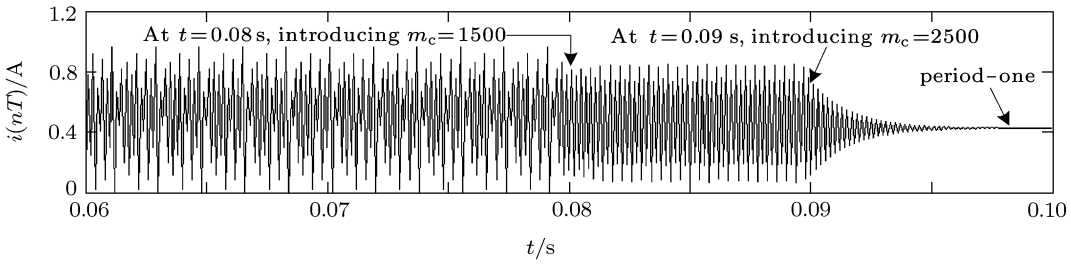


Fig.6. Waveform of inductor current with different values of m_c.

In the above discussion, the input voltage and ramp compensation slope of the current mode controlled buck-boost converter are assumed to vary continuously during operation, while the other parameters like inductance, output voltage, reference current and clock period are assumed constant. In this case, the buck-boost converter in DCM may undergo bifurcation routes. It is therefore necessary to study the bifurcation patterns over the parameter space of E and m_c.

Considering that the variation ranges of circuit parameters are $E = 4 - 10$ V and $m_c = 0 - 3000$, we can obtain a parameter space map, i.e., the two-parameter dynamic behaviour distribution map as shown in Fig.7(a), which depict their asymptotic behaviours with a colour over a grid of the parameter space. In Fig.7(a), the higher periodicities are depicted with darker grey levels. The white area means low period, and the darkest shade implies chaos. Figure 7(b) shows the corresponding regions of the orbit states of Fig.7(a). There

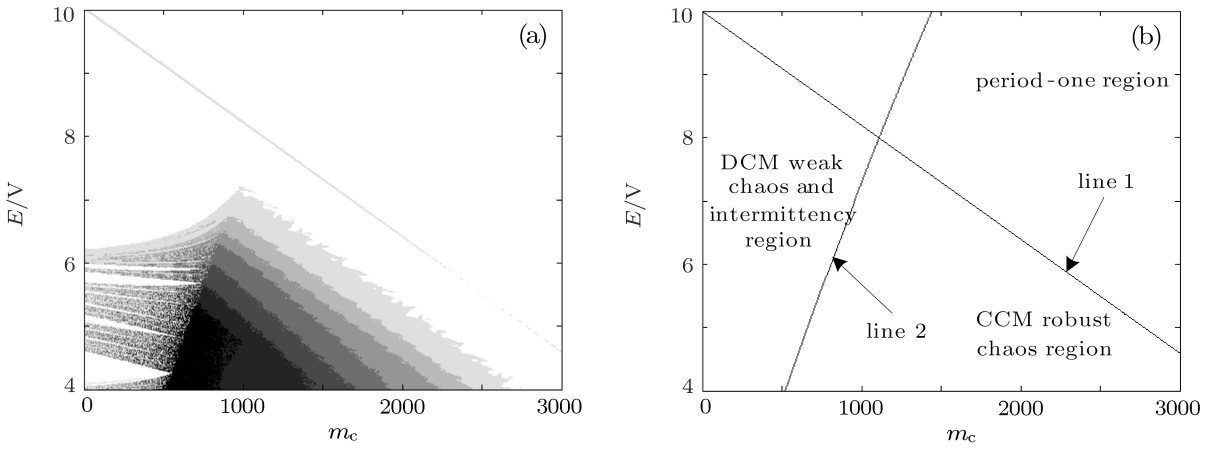


Fig.7. (a) Parameter space map showing regions of different periodicities; (b) its corresponding regions of the orbit states.

exist two borderlines: line 1: $E_c = 10 - 0.0018m_c$ and line 2: $E_{Mode} = 0.0081m_c(1 - 0.0001m_c)$, which are derived from Eqs.(13) and (16) respectively. Line 1 is the first period-doubling bifurcation borderline, the above

region is period-one, while the below region consists of unstable low period, chaos, intermittence and period window. Line 2 is the mode shifting borderline, the right region is CCM, while the left region is DCM. From the parameter space map, the stable operation range of the converter can clearly be demonstrated.

4. Conclusions

When the current mode controlled DC–DC converter shifts from CCM to DCM, the system trajectory is no longer strongly chaotic as in the case of CCM. If the current reaches zero value during at least one clock period within the period of the current waveform, the orbit becomes intermittently periodic even when the system is globally chaotic.

The introduction of a compensating ramp current can make the buck-boost converter shift from DCM to CCM, and can effectively control this system to operate at the stable period-one region. Under the variation of circuit parameters, the current mode controlled buck-boost converter in DCM can exhibit complex dynamical behaviours. Based on its discrete iterative map model, the performance of the switched buck-boost converter circuit utilizing a compensating ramp current has been investigated by using a bifurcation diagram, Lyapunov exponent spectrum, time-domain waveform and parameter space map.

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