Vol 18 No 11, November 2009
1674-1056/2009/18(11)/4738-04

Chinese Physics B

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The nonequilibrium phase transition and the symmetry revival induced by time delay in an asymmetric bistable system with correlated noises*

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(Received 2 March 2009; revised manuscript received 27 March 2009)

The nonequilibrium phase transition and the symmetry revival induced by time delay in a bistable system are investigated. The stationary probability distribution function (SPDF) of the bistable system with time delay and correlated noises are calculated by an analytical method and stochastic simulation respectively. The analytical and simulative results indicate that: (1) There is a certain value of λ(λ denotes the strength of correlations between the multiplicative and additive noises) to make the SPDF symmetric under some time delay; however, above or below the given value, the symmetry will be broken; (2) With the monotonic change of λ, the unimodal peak structure of SPDF becomes bimodal at the beginning, then it becomes unimodal again; this means that there is a reentrance phenomenon in the process; (3) There is a critical value of delay time, which makes the lower peak of SPDF equal to the higher one under the critical condition. This means that the symmetry revival phenomenon emerges.

Keywords: asymmetric bistable system, time delay, nonequilibrium phase transition, symmetry revival

PACC: 0540, 0250

1. Introduction

Time delay and noises are of great significance in investigating dynamics in bistable systems.[1−3] In particular, the presence of correlation between multiplicative and additive noise changes the dynamics of the system, including stationary properties,[4−6] transient properties,[7,8] stochastic resonance,[9−11] relaxation time[12] and correlation function.[13] There are many interesting results from previous works, such as the reentrance phenomenon[14,15] as well as resonant activation.[16]

Symmetry has had a major impact on the scientific community, even becoming the central concept of modern science such as crystallography, molecular science, chemistry, and physics. The symmetry breaking phenomenon induced by time delay in a stochastic bistable system has already been widely investigated,[3,17] but the symmetry revival induced by time delay has not been fully researched.

In this paper, we investigate a time-delayed bistable system driven by cross-correlated noises. Such an investigation is significant on the grounds that time delays are ubiquitous in nature and often change the dynamics of a system fundamentally. Time delays exist in many processes such as biophysiological controls[18] and signal transmissions in biological and artificial neuronal networks.[19] In recent years, several authors have made endeavors in the study of bistable systems with time-delayed feedbacks.[20−22] The symmetry revival phenomenon is also taken for some more investigation and understanding.

In Section 2, approximate analytical expressions of the stationary probability distribution function of the bistable system with time delay and correlated noises are derived. In Section 3, the effects of time delay and correlated noieses are presented and then a discussion of the results concludes the paper.

2. The stationary probability distribution

Consider a bistable system with time delay and correlated noises; a typical case is described by the following stochastic differential equation

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = r + x(t-\tau) - x^3(t) + x(t)\xi(t) + \eta(t), \quad (1)$$

where τ is the delay time, r is the asymmetric parameter, $\xi(t)$ and $\eta(t)$ are Gaussian white noises with zero

*Project supported by the National Natural Science Foundation of China (Grant No 10865006).

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mean and have the following statistical properties:

$$\langle \xi(t) \rangle = \langle \eta(t) \rangle = 0, \tag{2}$$

$$\langle \xi(t)\xi(t') \rangle = 2D\delta(t-t'), \tag{3}$$

$$\langle \eta(t)\eta(t') \rangle = 2\alpha\delta(t-t'), \tag{4}$$

$$\langle \xi(t)\eta(t') \rangle = \langle \eta(t)\xi(t') \rangle = 2\lambda\sqrt{D\alpha}\delta(t-t'). \tag{5}$$

Here D and α are the intensities of the multiplicative and additive noise, respectively, λ denotes the cross-correlated intensity between $\xi(t)$ and $\eta(t)$.

Substituting x_τ for $x(t-\tau)$, the delay Fokker–Planck equation corresponding to the Langevin equation can be written as[23]

$$\begin{aligned} &\frac{\partial P(x,t)}{\partial t} \\ &= -\frac{\partial}{\partial x}\int (x_\tau - x^3 + Dx + \lambda\sqrt{D\alpha}) \\ &\quad \times P(x,t;x_\tau,t-\tau)\,\mathrm{d}x_\tau \\ &\quad + \int \frac{\partial^2}{\partial x^2} g^2(x) P(x,t;x_\tau,t-\tau)\,\mathrm{d}x_\tau, \end{aligned} \tag{6}$$

where $P(x,t;x_\tau,t-\tau)$ is the joint probability density, and $g(x) = \sqrt{G(x)} = \sqrt{Dx^2 + 2\lambda\sqrt{\alpha D}x + \alpha}$.

The stationary probability distribution function (SPDF) of Eq.(1) is given by[24]

$$P_{\mathrm{st}}(x) = N g(x)^{\tau-1} \exp\left[-\frac{U(x)}{\alpha}\right], \tag{7}$$

where N is the normalization constant. Using the explicit forms of $f(x)$ and $G(x)$, the following generalized potential is obtained

$$\begin{aligned} U(x) &= -(1+\tau)\int^x \frac{r+y-y^3}{Ry^2 + 2\lambda\sqrt{R}y + 1}\,\mathrm{d}y \\ &= \frac{\beta_1 - r}{\sqrt{(1-\lambda^2)R}} \arctan\frac{\sqrt{R}x + \lambda}{\sqrt{1-\lambda^2}} \\ &\quad -\frac{2\lambda}{R\sqrt{R}}x + \frac{x^2}{2R} + \beta_2 \ln G(x), \end{aligned} \tag{8}$$

where

$$\begin{aligned} \beta_1 &= \frac{\lambda}{\sqrt{R}}\left(1 - \frac{4\lambda^2 - 3}{R}\right), \\ \beta_2 &= \frac{1}{2R}\left(\frac{4\lambda^2 - 1}{R} - 1\right). \end{aligned} \tag{9}$$

Here $R = D/\alpha$ is the ratio of noise intensities.

According to the expression for SPDF (Eq.(7)) of the system, the effects of both τ and λ on SPDF can be studied by numerical computation. The extrema of the SPDF (Eq.(7)) are determined by the following equation of third order[25,26]

$$x^3 - \frac{2\tau}{\tau+1}x + \frac{\lambda\sqrt{\alpha D} - \tau\lambda\sqrt{\alpha D} - \tau r - r}{\tau+1} = 0\ . \tag{10}$$

The critical curve separating the monostable and the bistable regions is determined by

$$\frac{1}{4}\left(\frac{\lambda\sqrt{\alpha D} - \tau\lambda\sqrt{\alpha D} - \tau r - r}{\tau+1}\right)^2 - \frac{1}{27}\left(\frac{2\tau}{\tau+1}\right)^3 = 0. \tag{11}$$

For the bistable region, when $-2/3\sqrt{3} < r < 2/3\sqrt{3}$, the SPDF has three extrema (two stable points and one unstable point).[27] It is interesting to give the phase diagrams among r, λ and τ. When any one of the three parameters is fixed, we can draw a phase diagram about the other two. Figures 1, 2 and 3 are phase diagrams in which the delay time τ, λ and r are fixed, respectively. The conclusions that can be drawn from these figures are as follows.

3. Discussion and conclusions

Figure 1 shows that the relation between r and λ is directly proportional. They play analogous roles in the extremum of SPDF. However, the line is close to a horizontal line when τ increases; this means that r affects the extremum of SPDF more strongly than λ; the change of SPDF for r becomes sensitive. In brief, r and λ affect the system in a similar way. Relatively speaking, in the case of a large time delay, r will prevail over λ.

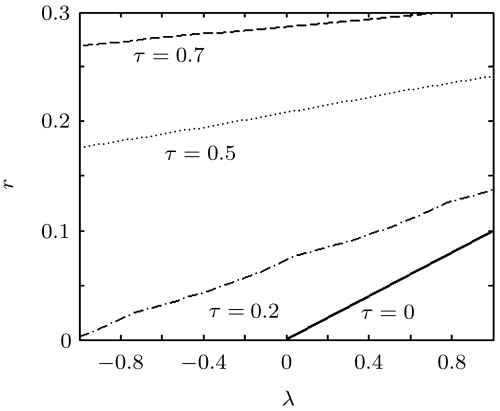


Fig.1. λ versus r calculated from Eq.(11) for $R = 1$, $\alpha = 0.1$, $D = 0.1$, and τ takes 0, 0.2, 0.5, 0.7.

The λ is fixed at –0.5, –0.2, 0, 0.2, 0.5 respectively in Fig.2. We can see that the degree of the correlation of the noises is an important factor for the SPDF. When the correlation between the noises is a negative association, τ and r play analogous roles in the extremum of SPDF. Under zero association, the variation of λ will not change the SPDF at first; however, when λ arrives at a certain value, τ and r play

the same role together. In the case of a positive association, they will play an opposing role first, and collaborate at last.

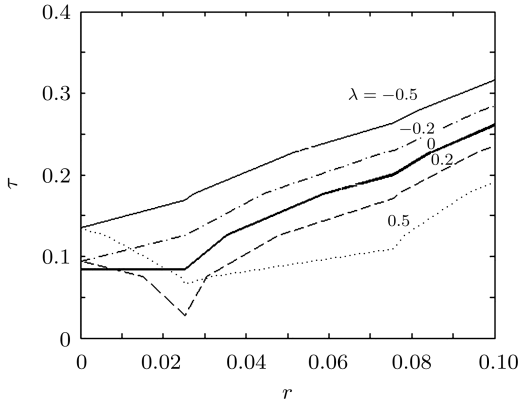


Fig.2. r versus τ calculated from Eq.(11) for $R = 1$, $\alpha = 0.1$, $D = 0.1$, and λ takes –0.5, –0.2, 0, 0.2, 0.5.

Obviously, Fig.3 certifies the rationality of Figs.1 and 2 again. If the asymmetric parameter is zero, that is, the system is symmetric, when the correlation between the noises is a negative association, τ and r play opposing roles. However, they play the same role under a negative association. The same regularity will be reflected in Figs.1 and 2 in the case of $r > 0$.

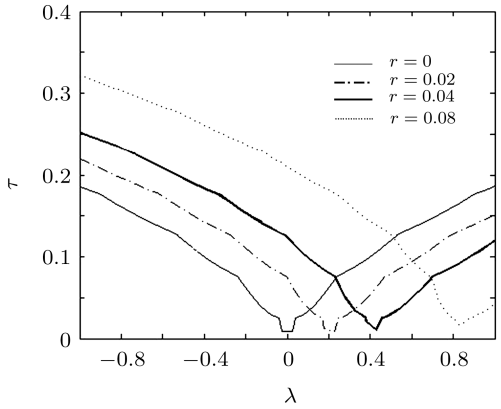


Fig.3. λ versus τ calculated from Eq.(11) for $R = 1$, $\alpha = 0.1$, $D = 0.1$, and r takes 0, 0.02, 0.04, 0.08.

The SPDF for different values of λ are plotted in Fig.4. Obviously, one can find that the strength of correlations affects the symmetric properties greatly according to Fig.4. The system is asymmetric at the beginning, then with decreasing λ, the left peak goes down and the right peak goes up gradually. At a certain time, the height of the left peak equals that of the right peak completely, and the structure of the peak becomes symmetric. This means that the previous unimodal structure becomes a bimodal one and the system becomes bistable. However, the bistable state is temporary: finally the descent of the left peak with the rise of the right peak results in a unimodal structure; in other words, the system becomes monostable again, which is the same as the initial state. Evidently, the change of the structure undergoes a repeated process; the state variable corresponding to it causes a nonequilibrium transition which is the so-called reentrance phenomenon.[14,15]

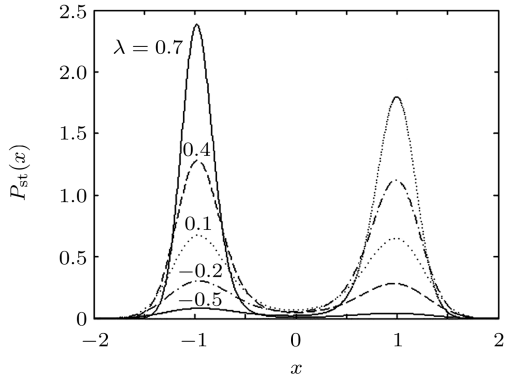


Fig.4. $P_{\rm st}(x)$ versus x calculated from the analytical expression (7) with Eq.(8) for $\tau = 0.3$, $R = 1$, $\alpha = 0.1$ and $r = 0.02$, and λ takes 0.7, 0.4, 0.1, –0.2, –0.5.

Under conditions of small delay time and the first-order approximation, the SPDF with different values of τ according to Eq.(7) is plotted in Fig.5. Figure 5 is the analytical result of the SPDF; however, a stochastic simulation[28,29] is necessary in order to compare with the analytical result so as to preferably understand the symmetry revival phenomenon. In Fig.6, the SPDF can be obtained by means of the stochastic simulation of the Langevin equation (1).

One can see from Figs.5 and 6 that the time delay τ affects the symmetrical property of the two-peak structure. The height of the two peaks would fluctuate with the delay time τ.

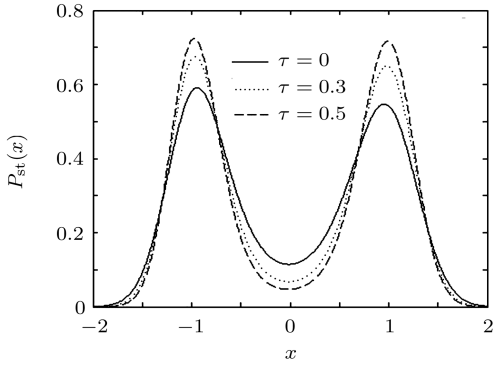


Fig.5. $P_{\rm st}(x)$ versus x calculated from the analytical expression (7) with Eq.(8) for $\lambda = 0.1$, $R = 1$, $\alpha = 0.1$ and $r = 0.02$, and τ takes 0, 0.3, 0.5.

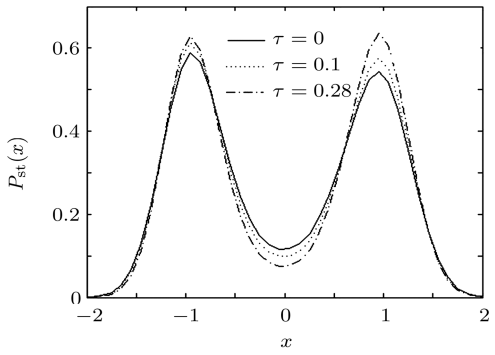


Fig.6. $P_{\rm st}(x)$ versus x by means of stochastic simulation from Eq.(1) for $\lambda = 0.1$, $R = 1$, $\alpha = 0.1$ and $r = 0.02$, and τ takes 0, 0.1, 0.28.

In Fig.5, for the case of $\tau = 0$, the height of the left peak is clearly higher than that of the right one; for the case of $\tau = 0.3$, the height of the left peak is still higher than the right one, but the height difference is not so remarkable. This means that the extent of the asymmetry is decreasing, in other words, the symmetry is in the process of revival. For the case of $\tau = 0.5$, the height of the left peak is subequal to the right one; this means that the symmetry revival phenomenon has emerged, and at the same time, the system becomes completely symmetric.

In Fig.6, for the case of $\tau = 0$, the height of the left peak is clearly higher than that of the right one; for the case of $\tau = 0.1$, the height of the left peak is still higher than the right one, but the height difference is not so remarkable; this means that the extent of the asymmetry is decreasing, in other words, the symmetry is in the process of revival. For the case of $\tau = 0.28$, the height of the left peak is subequal to the right one, indicating that the symmetry revival phenomenon has emerged, and at the same time, the system becomes completely symmetric.

Contracting the analytic result from the simulative result, the discrepancy that the symmetry will revive completely while $\tau = 0.5$ can easily be seen by means of the analytic approach; in comparison, the symmetry can revive completely just while $\tau = 0.28$. Such a discrepancy shows that the small delay time approximation approach in the analytic approach has some limitations. Though the analytic result cannot correspond perfectly to the numerical simulation, both the simulation result and the analytic result show that the SPDF of the bistable system could revive its symmetry completely.

To sum up, there is a critical phenomenon of the symmetric property in a time-delayed bistable system subjected to correlated noises. The SPDF of the bistable system becomes symmetric under the conditions of the delay time having a certain value, i.e., the symmetry revival phenomenon induced by time delay indeed exists.

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