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Effects of periodic optical lattice potential on dark solitons in a Bose–Einstein condensate*

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This paper investigates the dynamics of dark solitons in a Bose–Einstein condensate with a magnetic trap and an optical lattice (OL) trap, and analyses the effects of the periodic OL potential on the dynamics by applying the variational approach based on the renormalized integrals of motion. The results show that the dark soliton becomes only a standing-wave and free propagation of the dark soliton is not possible when the periodic length of the OL potential is approximately equal to the effective width of the dark soliton. When the periodic length is very small or very large, the effects of the OL potential on the dark soliton will be sharply reduced. Finally, the numerical results confirm these theoretical findings.

Keywords: Bose–Einstein condensate, dark soliton, magnetic potential, optical lattice potential

PACC: 0530J, 0290, 1110L

1. Introduction

The current experimental and theoretical studies of Bose–Einstein condensates (BECs) have attracted a great deal of interest, and the behaviour of BECs crucially depends on the sign of the atomic interactions.[1] The mean-field description for the macroscopic BEC wave-function is constructed by using the Hartree–Fock approximation, and results in the Gross–Pitaevskii equation.[2] It is well known that the Gross–Pitaevskii equation supports solitonic solutions. Dark solitons (with positive scattering length) of Bose condensed atoms were experimentally observed few years ago, and bright solitons (with negative scattering length) have been detected by using an optical red-detuned laser beam along the axial direction of the sample to impose a transverse (radial) confinement, and studied in one-dimensional models.[3,4] A dark soliton in a BEC is a macroscopic excitation, and characterized by a local density minimum and a phase gradient of the wave function at the position of the minimum. The stability of dark solitons in a nonlinear system is a critical property, and the dynamics of dark solitons in the presence of a magnetic trap have been extensively studied.[5]

In recent experiments BECs have been successfully loaded in an optical lattice (OL)[6] and this field has attracted considerable attention. A periodic OL potential is created along the waveguide axis by interference patterns from multiple laser beams. The longitudinal envelope of the OL potential is determined by the profile of the laser beam waist, and the lattice wave-number can be tuned by the geometry of laser beams. The great flexibility of an OL potential arises from the fact that the experimental parameters can be tuned, providing precise control over the shape and time-variation of the external potential.[7,8]

Controlling the generation of dark soliton trains from periodic waves can be achieved by variation of the scattering length. Experiments have demonstrated that tuning of the s-wave scattering length can be achieved due to the Feshbach resonance.[9] This offers a possibility to vary the interaction strength in ultracold atomic gases simply by applying external fields. The strength of the interaction can be adjusted experimentally from large negative values to large positive values. As a result, experiments with trapped ultra cold atomic gases, where the s-wave scattering length fully determines the interaction effects, have an unprecedented high level of control over the interatomic interactions.[10] A relevant interesting issue is to learn how to control the motion of BEC dark soli-

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tons in the magnetic potential and the periodic OL potential, and the question of how one could affect or even guide their motion then arises. In this paper, the effects of the OL potential on the dynamics of BEC dark solitons in the two external potentials are investigated by means of the variational approach and numerical simulation, and some new results are obtained.

2. Analytical results

In this work, we consider a dark soliton in a BEC with repulsive interactions; the particle number of the BEC is not very large in the external trap potentials, and the describing equation is the following one-dimensional nonlinear Schrödinger equation:[10]

$$\mathrm{j}\frac{\partial u}{\partial t}+\frac{1}{2}\frac{\partial^2 u}{\partial z^2}-\alpha\left|u\right|^2 u-V(z)=0, \tag{1}$$

where u is the condensate wavefunction, and α is the effective nonlinear coefficient, $V(z)$ is the normalized potential of the external traps in the longitudinal direction (z direction). In order to study the dynamics of the BEC dark soliton in the magnetic potential and the periodic OL potential under the framework of Eq.(1), it is convenient to decompose the external potential $V(z)$ as follows:

$$V(z)=U_1(z)+U_2(z), \tag{2}$$

where $U_1(z)$ is the magnetic potential, $U_2(z)$ is the periodic OL potential, and their forms are

$$U_1(z)=\frac{1}{2}\Omega^2 z^2, \tag{3}$$

$$U_2(z)=V_0\cos^2\left(\frac{2\pi}{L_\mathrm{p}}z\right)=\frac{V_0}{2}\left[1+\cos\left(\frac{4\pi}{L_\mathrm{p}}z\right)\right], \tag{4}$$

where Ω is the harmonic frequency (trap coefficient) of the magnetic potential in the longitudinal direction (z direction). V_0 and L_p are the depth and the periodic spacing of the OL potential measured in units of the lattice recoil energy respectively,[8] and the periodic length of the OL potential is $L_\mathrm{p}/2$.

A perturbation theory for dark solitons can be used on the renormalized integrals of motion.[11,12] The solution of Eq.(1) is given by

$$u\ (z,t)=\phi(z)\mathrm{e}^{\mathrm{j}\alpha t}\ \mathrm{v}(z,\ t), \tag{5}$$

where $\phi(z)$ is the background wave, and the solution to the dark soliton is

$$\mathrm{v}(z,\ t)=\ \mathrm{j}\cos\theta+\sin\theta\tanh[\sin\theta(z-\Delta)], \tag{6}$$

where $\Delta(t)$ is the central position of the dark soliton, $\theta(t)$ is the distributing angle, $\cos\theta$ is the soliton speed related to the speed of sound, and $\sin\theta$ is the soliton depth.

Substituting Eqs.(2) and (5) into Eq.(1), we get the following equation:

$$\begin{aligned}&\mathrm{j}\frac{\partial v}{\partial t}+\frac{1}{2}\frac{\partial^2 v}{\partial z^2}-\alpha\phi^2(|v|^2-1)v\\&+\frac{\partial\ln\phi}{\partial z}\frac{\partial v}{\partial z}-U_2(z)v=0.\end{aligned} \tag{7}$$

The background wave $\phi(z)$ can be approximated in the Thomas–Fermi (TF) framework as

$$\phi(z)=\sqrt{1-\frac{U_1(z)}{\alpha}}=\sqrt{1-\frac{1}{2\alpha}\Omega^2 z^2}, \tag{8}$$

in the region where $U_1(z)/\alpha<1$ and $\phi(z)=0$ outside. The atom density can be given under the TF framework as

$$n_\mathrm{d}(\alpha,\Omega)=\int_{-\infty}^{\infty}\left|\phi(z)\right|^2\mathrm{d}z. \tag{9}$$

The logarithmic derivative of the background wave $\phi(z)$ can be expressed as

$$\frac{\partial\ln\phi}{\partial z}=\frac{1}{\sqrt{1-U_1/\alpha}}\frac{\partial\sqrt{1-U_1/\alpha}}{\partial z}=-\frac{[1+(U_1/\alpha)/2+3(U_1/\alpha)^2/8+\cdots]}{2\alpha}\frac{\partial U_1}{\partial z}. \tag{10}$$

The logarithmic derivative is approximated as $\dfrac{\partial\ln\phi}{\partial z}\approx-\dfrac{1}{2\alpha}\dfrac{\partial U_1}{\partial z}$ due to $\Omega^2\ll 1$, and the renormalized equation can be expressed in the following form:

$$\mathrm{j}\frac{\partial v}{\partial t}+\frac{1}{2}\frac{\partial^2 v}{\partial z^2}-\alpha\phi^2(|v|^2-1)v-\frac{1}{2\alpha}\frac{\partial U_1}{\partial z}\frac{\partial v}{\partial z}-U_2(z)v=0. \tag{11}$$

The averaged Lagrangian of Eq.(11) can be obtained by applying the variational approach based on the

renormalized integrals of motion:

$$\begin{aligned}L(t)=&\int_{-\infty}^{\infty}\left[\frac{\mathrm{j}}{2}\left(v^*\frac{\partial v}{\partial t}-v\frac{\partial v^*}{\partial t}\right)\left(1-\frac{1}{|v|^2}\right)-\frac{1}{2}\left|\frac{\partial v}{\partial z}\right|^2-\frac{\alpha\phi^2}{2}(|v|^2-1)^2\right.\\&\left.-\frac{1}{2\alpha}\frac{\partial U_1}{\partial z}\left(v^*\frac{\partial v}{\partial z}+v\frac{\partial v^*}{\partial z}\right)-U_2(|v|^2-1)\right]\mathrm{d}z\\=&\frac{\mathrm{d}\Delta}{\mathrm{d}t}[\sin(2\theta)-2\theta]-\frac{2(1+\alpha)\sin^3\theta}{3}+\frac{A\Omega^2\sin\theta}{4}-\frac{\Omega^2\Delta^2\sin^3\theta}{2}\\&-\frac{B\Omega^2\sin\theta}{\alpha}+V_0\sin\theta+\frac{2\pi^2V_0}{L_\mathrm{p}}\operatorname{csch}\left(\frac{\pi^2}{L_\mathrm{p}\sin\theta}\right)\cos\left(\frac{4\pi\Delta}{L_\mathrm{p}}\right),\end{aligned}\tag{12}$$

where $A=\int_{-\infty}^{\infty}x^2\operatorname{sech}^4x\mathrm{d}x$ and $B=\int_{-\infty}^{\infty}x\tanh x\operatorname{sech}^2x\mathrm{d}x$.

$$\int_{-\infty}^{\infty}\cos(ax)\operatorname{sech}^2[b(x-x_0)]\mathrm{d}x=\cos(2ax_0)\operatorname{csch}\left(\frac{a\pi}{b}\right)$$

is used in Eq.(12).[13,14]

The equations of motion for the distributing angle and the central position are obtained from the averaged Lagrangian using $\mathrm{d}L(t)/\mathrm{d}\sigma-\mathrm{d}[\mathrm{d}L(t)/\mathrm{d}(\mathrm{d}\sigma/\mathrm{d}t)]/\mathrm{d}t=0$ ($\sigma=\theta,\Delta$):

$$\begin{aligned}\frac{\mathrm{d}\theta}{\mathrm{d}t}=&\frac{\Delta\Omega^2}{4\sin^2\theta}+\frac{8\pi^3V_0}{L_\mathrm{p}^2\sin^2\theta}\operatorname{csch}\left(\frac{2\pi^2}{L_\mathrm{p}\sin\theta}\right)\sin\left(\frac{4\pi\Delta}{L_\mathrm{p}}\right),\\\frac{\mathrm{d}\Delta}{\mathrm{d}t}=&\frac{(1+\alpha)}{2}\cos\theta-\frac{A\Omega^2\cos\theta}{8\sin^2\theta}+\frac{3\Omega^2\Delta^2\cos\theta}{8}+\frac{B\Omega^2\cos\theta}{2\alpha\sin^2\theta}\\&-\frac{V_0\cos\theta}{4\sin^2\theta}-\frac{2\pi^4V_0\cos\theta}{L_\mathrm{p}^2\sin^2\theta}\operatorname{csch}\left(\frac{2\pi^2}{L_\mathrm{p}\sin\theta}\right)\coth\left(\frac{2\pi^2}{L_\mathrm{p}\sin\theta}\right)\cos\left(\frac{4\pi\Delta}{L_\mathrm{p}}\right).\end{aligned}\tag{13}$$

According to the equation $H=\partial\theta/\partial t\times\partial L/\partial(\partial\theta/\partial t)+\partial\Delta/\partial t\times\partial L/\partial(\partial\Delta/\partial t)-L$, the Hamiltonian of the BEC system is

$$\begin{aligned}H=&\frac{2(1+\alpha)\sin^3\theta}{3}-\frac{A\Omega^2\sin\theta}{4}+\frac{\Omega^2\Delta^2\sin^3\theta}{2}+\frac{B\Omega^2\sin\theta}{\alpha}\\&-V_0\sin\theta-\frac{2\pi^2V_0}{L_\mathrm{p}}\operatorname{csch}\left(\frac{\pi^2}{L_\mathrm{p}\sin\theta}\right)\cos\left(\frac{4\pi\Delta}{L_\mathrm{p}}\right).\end{aligned}\tag{14}$$

Equations (13) and (14) determine the dynamics of the dark soliton in a BEC with the magnetic potential and the OL potential.

From the equations of motion (13), we can see the equation of motion for the central position being $\mathrm{d}\Delta/\mathrm{d}t=\cos\theta$ in the absence of the magnetic potential and the periodic OL potential for $\alpha=1.0$. The special case coincides with the optical dark soliton solution.

The stationary states can be obtained by setting the time derivatives in Eq.(13) to zero, and the stationary states are

$$\theta_\mathrm{s}=0\ \text{ and }\ \Delta_\mathrm{s}\ \text{ of arbitrary value},\tag{15a}$$

$$\theta_\mathrm{s}=\pi/2\ \text{ and }\ \Delta_\mathrm{s}=0.\tag{15b}$$

The stationary state Eq.(15a) corresponds to $v(z,t)=1$, and this situation denotes the submergence of the dark soliton. The stationary state Eq.(15b) corresponds to $v(z,t)=\tanh(z)$, namely the dark soliton is the black soliton. The special kink case (both the speed and the central position of the dark soliton are zero) coincides with the optical vector dark soliton solution. The stationary state Eq.(15b) is stable by performing a standard linear stability analysis based on Eqs.(13) and (14).[15,16]

Linearizing Eqs.(13) in θ and Δ around the stable stationary state of $\theta_\mathrm{s}=\pi/2$ and $\Delta_\mathrm{s}=0$, for the central position only we can approximately reduce the equation of motion to the very simple form:

$$\frac{\mathrm{d}^2\Delta}{\mathrm{d}t^2}+\left[\frac{(1+\alpha)}{2}-\frac{V_0}{4}-\frac{2\pi^4V_0}{L_\mathrm{p}^2}\,\mathrm{csch}\left(\frac{2\pi^2}{L_\mathrm{p}}\right)\coth\left(\frac{2\pi^2}{L_\mathrm{p}}\right)\right]\left[\frac{\Omega^2}{4}+\frac{8\pi^4V_0}{L_\mathrm{p}^3}\,\mathrm{csch}\left(\frac{2\pi^2}{L_\mathrm{p}}\right)\right]\Delta=\frac{\mathrm{d}^2\Delta}{\mathrm{d}t^2}+\omega^2\Delta=0, \tag{16}$$

where

$$\omega=\left\{\left[\frac{(1+\alpha)}{2}-\frac{V_0}{4}-\frac{2\pi^4V_0}{L_\mathrm{p}^2}\,\mathrm{csch}\left(\frac{2\pi^2}{L_\mathrm{p}}\right)\coth\left(\frac{2\pi^2}{L_\mathrm{p}}\right)\right]\left[\frac{\Omega^2}{4}+\frac{8\pi^4V_0}{L_\mathrm{p}^3}\,\mathrm{csch}\left(\frac{2\pi^2}{L_\mathrm{p}}\right)\right]\right\}^{1/2}$$

is the effective frequency of the reciprocal movement for the central position in the presence of the magnetic potential and the OL potential.

The soliton motion is the reciprocal movement around the stationary state of $\theta_\mathrm{s}=\pi/2$ and $\Delta_\mathrm{s}=0$, and the periodic time of the reciprocal movement is

$$T_\mathrm{p}=\frac{2\pi}{\omega}=\frac{2\pi}{\left\{\left[\frac{(1+\alpha)}{2}-\frac{V_0}{4}-\frac{2\pi^4V_0}{L_\mathrm{p}^2}\,\mathrm{csch}\left(\frac{2\pi^2}{L_\mathrm{p}}\right)\coth\left(\frac{2\pi^2}{L_\mathrm{p}}\right)\right]\left[\frac{\Omega^2}{4}+\frac{8\pi^4V_0}{L_\mathrm{p}^3}\,\mathrm{csch}\left(\frac{2\pi^2}{L_\mathrm{p}}\right)\right]\right\}^{1/2}}. \tag{17}$$

We can see that the periodic movement depends strictly on the effective nonlinear coefficient and the harmonic frequency, the depth and the periodic spacing (periodic length).

Specially, when the periodic spacing (periodic length) is very small (such as $L_\mathrm{p}\ll 2\pi^2$), or the periodic spacing (periodic length) is very large (such as $L_\mathrm{p}\gg 2\pi^2$), the periodic time Eq.(17) of the reciprocal movement is rewritten by

$$T_\mathrm{p}=\frac{2\pi}{\omega}=\frac{4\pi}{\left[\frac{(1+\alpha)}{2}-\frac{V_0}{4}\right]^{1/2}\Omega}. \tag{18}$$

So we can predict that the effects of the periodic OL potential on the dynamics of the dark soliton will be sharply reduced in the two special cases.

Furthermore, there is in-phase resonance[14] between the OL potential and the dark soliton when the periodic length ($L_\mathrm{p}/2$) of the OL potential is approximately equal to the effective width (1.76) of the black soliton. Namely, the periodic spacing satisfies $L_\mathrm{p}\approx 3.52$. In the case where only standing-wave solutions exist for the one-dimensional nonlinear Schrödinger equation (1), free propagation of the dark soliton is not possible. We can foresee that the OL potential plays serious roles in the evolution of the dark soliton.

3. Numerical results

We can perform a series of direct numerical simulations for the nonlinear Schrödinger equation (1) to discuss the dynamics of the dark soliton and the effects of the OL potential on the dynamics. The initially input condensate wave is $u(z,t=0)=\sqrt{1-\Omega^2z^2/2\alpha}\,\{\,\mathrm{j}\cos\theta+\sin\theta\tanh[\sin\theta(z-\Delta)]\,\}$ for an initial effective nonlinear coefficient of $\alpha=1.0$, an initial distributing angle of $\theta=\pi/2$ and an initial central position of $\Delta=2.0$ in simulations below.

When the dark soliton is only in the magnetic potential, we show the intensity evolution ($|u(z,t)|^2$) of the condensate wave in the absence of the OL potential for the different harmonic frequencies $\Omega=0.17$, $\Omega=0.20$, or $\Omega=0.25$ in Fig.1. We can see that the dark soliton moves periodically around the centre ($z=0$) of the magnetic potential as time changes, and the evolution depends strictly on the magnetic potential. From the atom density Eq.(9), we can see that the density of atoms injected into the BEC is determined by the different harmonic frequencies, and the results mean that the dark soliton can be controlled by the number of atoms continuously injected into the BEC from the reservoir when we apply the magnetic field via Feshbach resonance in a BEC.[10] The dark soliton has good robust features, and may maintain a tanh-shape as the atoms are injected continuously into the BEC. The periodic time of the reciprocal movement depends strictly on the harmonic frequency. For example, the periodic time becomes shorter when the harmonic frequency becomes larger. Figure 2 is the periodic time of the reciprocal movement versus the harmonic frequency in the absence of the OL potential ($V_0=0$); we can see that results from the direct simulation by Eq.(1) and the calculation by Eqs.(13) and (17) are in good agreement for the large harmonic frequency.

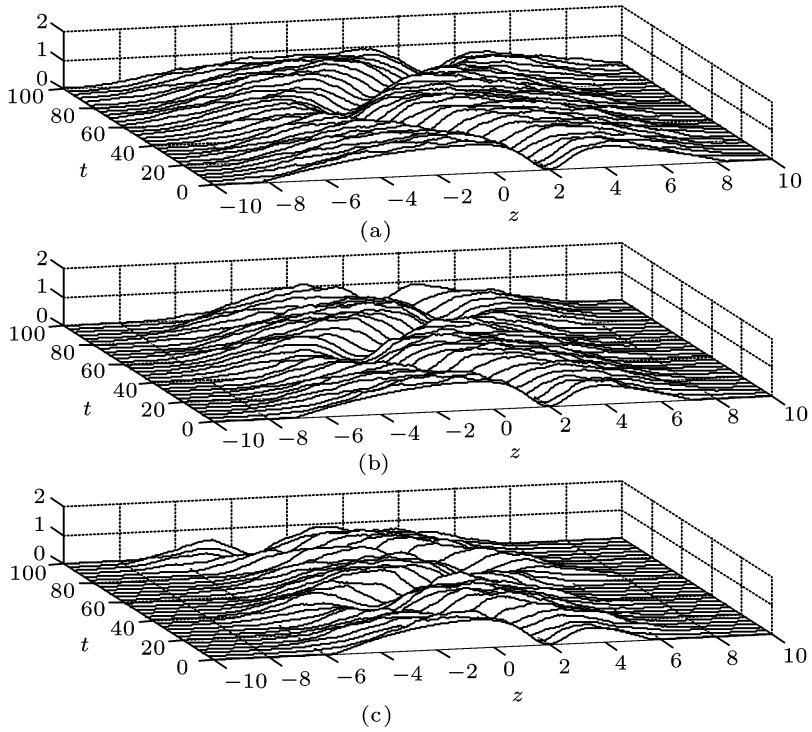


Fig.1. The intensity evolution ($|u(z,t)|^2$) of the condensate wave in the absence of the periodic OL potential for different harmonic frequencies: (a) $\Omega = 0.17$, (b) $\Omega = 0.20$ and (c) $\Omega = 0.25$.

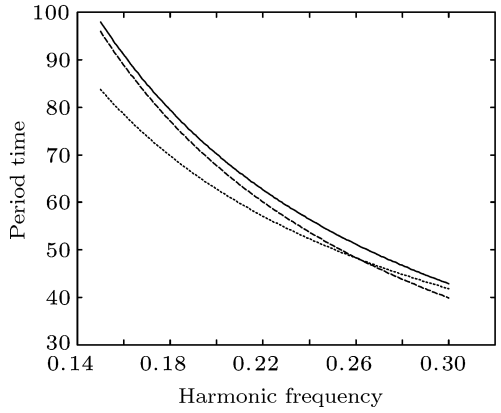


Fig.2. The period time of the reciprocal movement for the dark soliton versus the harmonic frequency in the absence of the OL potential ($V_0 = 0$). Solid line: direct simulation by Eq.(1). Dashed line: calculation by Eq.(13). Dotted line: calculation by Eq.(17).

To discuss the dynamics of the dark soliton in both the magnetic potential and the periodic OL potential, we show the intensity evolution ($|u(z,t)|^2$) of the condensate wave in the presence of the two external potentials for different harmonic frequencies $\Omega = 0.17$, $\Omega = 0.20$, and $\Omega = 0.25$ in Figs.3–5. The OL depth is $V_0 = 1.0$ and the periodic spacing is chosen as $L_\mathrm{p} = 0.5$ ($L_\mathrm{p} \ll 2\pi^2$), $L_\mathrm{p} = 20$ ($L_\mathrm{p} \approx 2\pi^2$) and $L_\mathrm{p} = 70$ ($L_\mathrm{p} \gg 2\pi^2$). When the period spacing is very small ($L_\mathrm{p} \ll 2\pi^2$) or very large ($L_\mathrm{p} \gg 2\pi^2$), the dark soliton moves periodically around the centre ($z = 0$) of the magnetic trap as the time changes, and the evolution becomes slightly dependent on the OL potential. There are obvious self-trapping effects on the dark soliton because of its good robust feature, so the depth of the dark soliton is approximately $\sin\theta \approx 1$ in the evolution. Figure 6 is the periodic time of the reciprocal movement for the dark soliton versus the harmonic frequency in the presence of the OL potential ($V_0 = 1.0$) for very small or very large periodic spacing (such as $L_\mathrm{p} = 0.5$ or $L_\mathrm{p} = 70$), and these results are from the direct simulation by Eq.(1) for $L_\mathrm{p} = 0.5$ and $L_\mathrm{p} = 70$, and the calculation by Eq.(17). The periodic time becomes longer in the presence of the periodic OL potential, the dark soliton maintains a tanh-shape in the evolution, and the numerical results confirm the theoretical analysis by Eq.(17) for a very small or very large periodic spacing. When the periodic spacing is $L_\mathrm{p} = 20$ ($L_\mathrm{p} \approx 2\pi^2$), we can see that the evolution depends on the OL potential, and the effects are obvious. For example, the dark soliton also moves periodically around the centre ($z = 0$) of the magnetic trap, but the dark soliton becomes distorted.

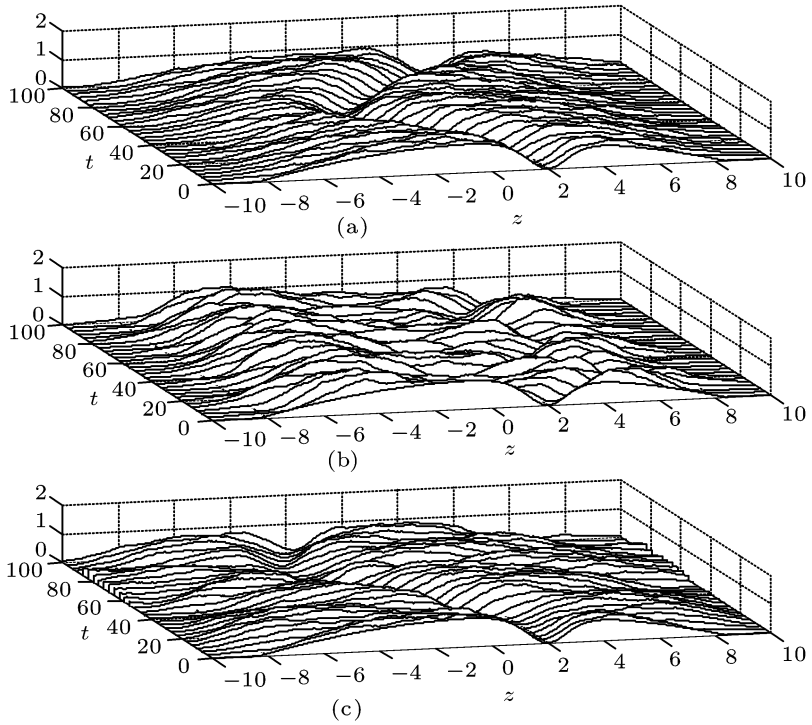


Fig.3. The intensity evolution ($|u(z,t)|^2$) of the condensate wave in the presence of the two external potentials for the harmonic frequency $\Omega = 0.17$: (a) $L_{\rm p} = 0.5$, (b) $L_{\rm p} = 20$ and (c) $L_{\rm p} = 70$.

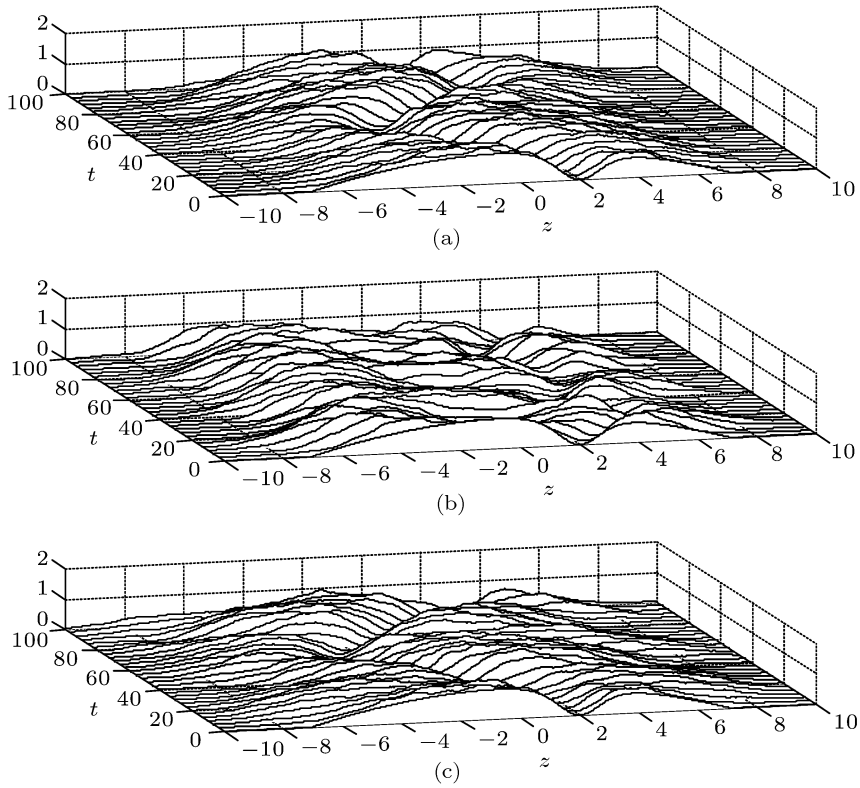


Fig.4. The intensity evolution ($|u(z,t)|^2$) of the condensate wave in the presence of the two external potentials for the harmonic frequency $\Omega = 0.20$: (a) $L_{\rm p} = 0.5$, (b) $L_{\rm p} = 20$ and (c) $L_{\rm p} = 70$.

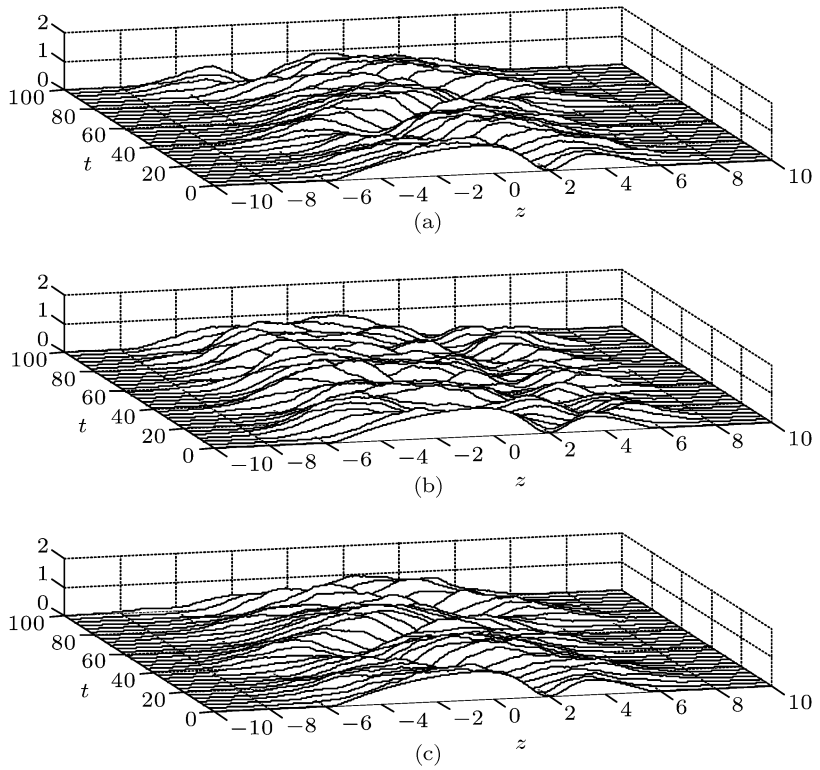


Fig.5. The intensity evolution ($|u(z,t)|^2$) of the condensate wave in the presence of the two external potentials for the harmonic frequency $\Omega = 0.25$, (a) $L_{\rm p} = 0.5$, (b) $L_{\rm p} = 20$ and (c) $L_{\rm p} = 70$.

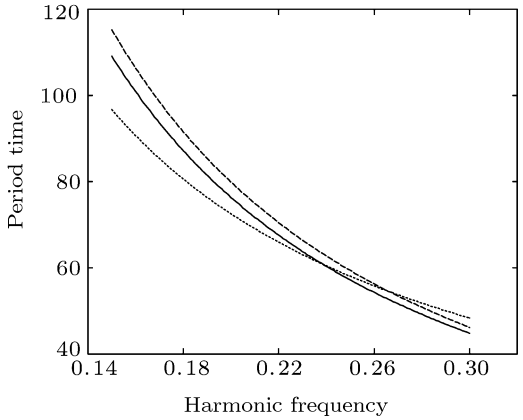


Fig.6. The period time of the reciprocal movement for the dark soliton versus the harmonic frequency in the presence of the OL potential ($V_0 = 1.0$) for a very small or very large period spacing (period length). Solid line: direct simulation by Eq.(1) for $L_{\rm p} = 0.5$. Dashed line: direct simulation by Eq.(1) for $L_{\rm p} = 70$. Dotted line: calculation by Eq.(18).

We maintain the same harmonic frequency of $\Omega = 0.20$ and the OL depth of $V_0 = 1.0$. The intensity evolution ($|u(z,t)|^2$) of the condensate wave is shown for the periodic spacing of $L_{\rm p} \approx 3.52$ in Fig.7. The effects of the OL potential on the reciprocal movement are very obvious, and the evolution depends strictly on the OL potential. The OL potential leads to the distortion and submergence of the dark soliton in the evolution, and the reciprocal movement of the dark soliton disappears around the centre ($z = 0$) of the magnetic trap when the OL depth is large enough (such as $V_0 = 1.0$). The results mean that the dark soliton does not maintain a tanh-shape when the periodic length of the OL potential is approximately equal to the effective width of the dark soliton. The dark soliton becomes only a standing-wave in the evolution, and free propagation of the dark soliton is not possible; the result is in agreement with that in Ref.[14]. The solution Eq.(6) of the dark soliton and the equations of motion (13) for the distributing angle and the central position are invalid in the special case. The reason is that there is in-phase resonance between the OL potential and the dark soliton, and the OL potential plays serious roles in the motion of the dark soliton.

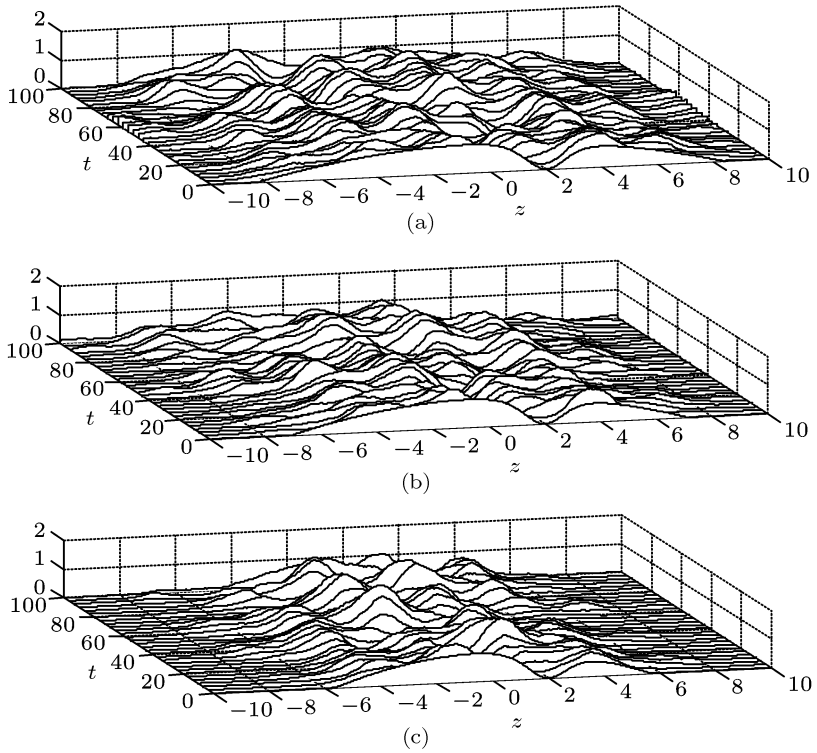


Fig.7. The intensity evolution ($|u(z,t)|^2$) of the condensate wave in the presence of the two external potentials for the period spacing of $L_{\rm p} = 3.52$: (a) $\Omega = 0.17$, (b) $\Omega = 0.20$ and (c) $\Omega = 0.25$.

4. Conclusion

The dynamics of dark solitons in a BEC with a magnetic potential and a periodic OL potential are investigated, and the equations of motion are obtained for the distributing angle and the central position by the variational approach based on the renormalized integrals of motion. The dynamics of the dark soliton are discussed by performing a linear analysis, and the effects of the OL potential on the dynamics are analysed. There are obvious self-trapping effects on the dark soliton around the stable stationary state, and the black soliton is stable when it has zero velocity and its centre position is in the centre of the magnetic trap. When the periodic spacing (periodic length) is very small or very large, the effects of the periodic OL potential on the dynamics of the dark soliton will be sharply reduced in the two special cases. There is in-phase resonance between the periodic OL potential and the dark soliton when the periodic length is approximately equal to the effective width of the dark soliton. The dark soliton becomes only a standing-wave and free propagation is not possible. We have found that our direct numerical simulations are in agreement with analytical findings.

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