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Fermion tunneling of charged particles from a non-static black hole in de Sitter space*

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Introducing a new coordinate system and choosing a set of appropriate matrices γ^{μ}, this paper attempts to investigate the fermion tunneling of charged particles across the event horizon from the Vaidya–Bonner de Sitter black hole. The result shows that the tunneling rate of the non-static black hole is related not only to the change of Bekenstein–Hawking entropy but also to the integral of the changing horizon, which violates unitary theory and is different from the stationary case.

Keywords: fermion, tunneling rate, energy conservation, charge conservation
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1. Introduction

Recently, a semi-classical tunneling method to describe Hawking radiation as a tunneling process, first put forward by Parikh and Wilczek,[1,2] has been developed.[3−17] Later, the Hamilton–Jacobi method[18,19] and the anomaly method[20,21] to derive the Hawking radiation have also already attracted many people's attention.[22−30] Although these methods are shown to be very robust to successfully derive tunneling radiation, most papers have only considered scalar particle radiation. In fact, a black hole can radiate any type of particle, including scalar particles and fermions with spin 1/2.

Very recently, Kerner and Mann have presented fermion radiation for Rindler space–time and for an uncharged spherically symmetric black hole.[31,32] Their recent research shows that its Hawking temperature and tunneling rate can be exactly obtained by the fermion tunneling method from its horizon. This method has been extended to the BTZ black hole,[33] Kerr black hole,[34] Kerr–Newman black hole,[35] de Sitter black hole,[36] and more complicated black rings.[37] In the current paper, we focus on fermion tunneling from the Vaidya–Bonner de Sitter black hole which is non-stationary.

2. Fermion tunneling of charged particles

The metric of the Vaidya–Bonner de Sitter black holein Eddington–Finkelstein coordinates is given by

$$\begin{aligned} \mathrm{d}s^2 = &-\left(1-\frac{2M(v)}{r}+\frac{Q^2(v)}{r^2}-\frac{1}{3}\lambda r^2\right)\mathrm{d}v^2 \\ &+2\mathrm{d}v\mathrm{d}r+r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2) \end{aligned} \tag{1}$$

with the electromagnetic potential

$$A_{\mu}=\left(\frac{Q(v)}{r},0,0,0\right), \tag{2}$$

where M and Q are the mass and charge of the black hole, respectively, λ is the positive cosmological constant. This line element is different from that of a stationary sphere-symmetrically charged black hole, and the mass M and charge Q of the black hole vary with time. The event horizon r_{H} is determined by the following equation

$$1-2\dot{r}_{\mathrm{H}}-\frac{2M(v)}{r_{\mathrm{H}}}+\frac{Q^2(v)}{r_{\mathrm{H}}^2}-\frac{1}{3}\lambda r_{\mathrm{H}}^2=0, \tag{3}$$

where $\dot{r}_{\mathrm{H}}=\dfrac{\mathrm{d}r_{\mathrm{H}}}{\mathrm{d}v}$. From the metric Eq.(1), we find that this coordinate system has many good properties, such as no singularity at the event horizon and complete convergence of the whole space–time manifold. However, in this coordinate system, the event horizon and the time-like limit surface are not located at the

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same place, which causes some difficulties in calculating the tunneling probability. In order to investigate the fermion tunneling through the event horizon, we introduce a new coordinate[38,39]

$$R = r - r_{\rm H}(v). \tag{4}$$

Then, we have $\mathrm{d}R = \mathrm{d}r - \dot{r}_{\rm H}\mathrm{d}v$. So the metric of Eq.(1) can be rewritten as

$$\begin{aligned}\mathrm{d}s^2 = &-\left(1-2\dot{r}_{\rm H}-\frac{2M(v)}{r}+\frac{Q^2(v)}{r^2}-\frac{1}{3}\lambda r^2\right)\mathrm{d}v^2\\ &+2\mathrm{d}v\mathrm{d}r+r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2).\end{aligned} \tag{5}$$

In the new coordinate system, the event horizon and the time-like limit surface locate at the same place, and the metric of Eq.(5) satisfies Landau's coordinate clock synchronization. These features of the new coordinate are of great importance for us to discuss the tunneling process.

For a fermion particle, its equation of motion can be written as

$$\mathrm{i}\gamma^\mu\left(D_\mu+\frac{\mathrm{i}q}{\hbar}A_\mu\right)\psi+\frac{m}{\hbar}\psi=0, \tag{6}$$

where

$$\begin{aligned}&D_\mu=\partial_\mu+\frac{\mathrm{i}}{2}\Gamma_\mu^{\alpha\beta}\Sigma_{\alpha\beta},\\ &\Gamma_\mu^{\alpha\beta}=g^{\beta\gamma}\Gamma_{\mu\gamma}^{\alpha},\\ &\Sigma_{\alpha\beta}=\frac{\mathrm{i}}{4}\left[\gamma^\alpha,\gamma^\beta\right],\\ &\left\{\gamma^\alpha,\gamma^\beta\right\}=2g^{\alpha\beta}\times I.\end{aligned} \tag{7}$$

To deal with the Hawking radiation of fermions, we should first choose an appropriate γ^μ matrices. For simplicity, γ^μ matrices can be chosen as

$$\begin{aligned}\gamma^v&=\frac{1}{\sqrt{F(r,v)}}\begin{pmatrix}-\mathrm{i}&\sigma^3\\ \sigma^3&\mathrm{i}\end{pmatrix},\\ \gamma^R&=\sqrt{F(r,v)}\begin{pmatrix}0&\sigma^3\\ \sigma^3&0\end{pmatrix},\\ \gamma^\theta&=\frac{1}{r}\begin{pmatrix}0&\sigma^1\\ \sigma^1&0\end{pmatrix},\\ \gamma^\varphi&=\frac{1}{r\sin\theta}\begin{pmatrix}0&\sigma^2\\ \sigma^2&0\end{pmatrix},\end{aligned} \tag{8}$$

where

$$F(r,v)=1-2\dot{r}_{\rm H}-\frac{2M(v)}{r}+\frac{Q^2(v)}{r^2}-\frac{1}{3}\lambda r^2$$

and $\sigma^i(i=1,2,3)$ are Pauli matrices which are given by

$$\begin{aligned}&\sigma^1=\begin{pmatrix}0&1\\ 1&0\end{pmatrix},\quad \sigma^2=\begin{pmatrix}0&-\mathrm{i}\\ \mathrm{i}&0\end{pmatrix},\\ &\sigma^3=\begin{pmatrix}1&0\\ 0&-1\end{pmatrix},\end{aligned} \tag{9}$$

and we denote $\xi_{\uparrow/\downarrow}$ for the eigenvectors of σ^3. For 1/2 spin fermions, the wavefunction has two spin states. Now, we only refer to the spin field for the upper case $\xi_\uparrow$. (After a completely identical step for the spin down $\xi_\downarrow$ case, we can obtain the same result.) We employ the following ansatz as

$$\psi_\uparrow(v,R,\theta,\varphi)=\begin{pmatrix}C(v,R,\theta,\varphi)\\ 0\\ D(v,R,\theta,\varphi)\\ 0\end{pmatrix}\exp\left(\frac{\mathrm{i}}{\hbar}I_\uparrow(v,R,\theta,\varphi)\right). \tag{10}$$

Inserting the ansatz into the Dirac equation (6), dividing by the exponential term and multiplying by $\hbar$, we obtain the resulting equations to the leading order in $\hbar$.

$$\frac{1}{\sqrt{F(r,v)}}\begin{pmatrix}(\mathrm{i}C-D)\left(\partial_v I_\uparrow+\frac{Q(v)q}{r}\right)\\ 0\\ (-C-\mathrm{i}D)\left(\partial_v I_\uparrow+\frac{Q(v)q}{r}\right)\\ 0\end{pmatrix}+\sqrt{F(r,v)}\begin{pmatrix}-D\partial_R I_\uparrow\\ 0\\ -C\partial_R I_\uparrow\\ 0\end{pmatrix}$$

$$+\frac{1}{r\sin\theta}\begin{pmatrix}0\\-\mathrm{i}\,D\partial\varphi I_{\uparrow}\\0\\-\mathrm{i}\,C\partial\varphi I_{\uparrow}\end{pmatrix}+\frac{1}{r}\begin{pmatrix}0\\-D\partial_{\theta}I_{\uparrow}\\0\\-C\partial_{\theta}I_{\uparrow}\end{pmatrix}+m\begin{pmatrix}C\\0\\D\\0\end{pmatrix}=0. \tag{11}$$

It is difficult to directly get the expression of the action. Considering the properties of the black hole, we carry out the separation of variables as

$$I_{\uparrow}=-\omega v+W(R)+J(\theta,\varphi), \tag{12}$$

where ω is the energy of the emitted particle measured by an observer at infinity. Inserting Eq.(12) into Eq.(11), we have

$$\frac{-\mathrm{i}\,C}{\sqrt{F(r,v)}}\left(\omega-\frac{Q(v)q}{r}\right)+\frac{D}{\sqrt{F(r,v)}}\left(\omega-\frac{Q(v)q}{r}\right)-D\sqrt{F(r,v)}W'(R)+mC=0, \tag{13}$$

$$-\frac{D}{r}\left(J_{\theta}(\theta,\varphi)+\frac{\mathrm{i}}{\sin\theta}J_{\varphi}(\theta,\varphi)\right)=0, \tag{14}$$

$$\frac{C}{\sqrt{F(r,v)}}\left(\omega-\frac{Q(v)q}{r}\right)+\frac{\mathrm{i}\,D}{\sqrt{F(r,v)}}\left(\omega-\frac{Q(v)q}{r}\right)-C\sqrt{F(r,v)}W'(R)+mD=0, \tag{15}$$

$$-\frac{C}{r}\left(J_{\theta}(\theta,\varphi)+\frac{\mathrm{i}}{\sin\theta}J_{\varphi}(\theta,\varphi)\right)=0. \tag{16}$$

Equations (14) and (16) imply that $J(\theta,\varphi)$ is a complex function. The solution of $J(\theta,\varphi)$ is the same as the spin-down case, so its contribution to the rate emission is canceled out. Note that, whatever $m=0$ or $m\neq 0$, it does not affect the result. As regard the remaining equation, when $C=\mathrm{i}\,D$, we can obtain

$$W_{+}(R)=\int\frac{2\left(\omega-\dfrac{Qq}{r}\right)}{F(r,v)}\mathrm{d}R, \tag{17}$$

and when $C=-\mathrm{i}\,D$, then

$$W'_{-}(R)=0. \tag{18}$$

This corresponds to the incoming particle absorbed in the classical limit with probability $P(\text{absorption})=1$. From Eq.(17), we can obtain the imaginary part of the action

$$\begin{aligned}\mathrm{Im}\,I=\mathrm{Im}\,W_{+}(R)&=\mathrm{Im}\int\frac{2\left(\omega-\frac{Qq}{r}\right)}{F'(r,v)(r-r_{\mathrm{H}})}\mathrm{d}R\\&=\frac{\pi\left(\omega-\dfrac{Qq}{r_{\mathrm{H}}}\right)}{\dfrac{M}{r_{\mathrm{H}}^{2}}-\dfrac{Q^{2}}{r_{\mathrm{H}}^{3}}-\dfrac{1}{3}\lambda r_{\mathrm{H}}}.\end{aligned} \tag{19}$$

The above integral is divergent at $R=0$, which corresponds to the horizon $r=r_{\mathrm{H}}$. Making use of a prescription corresponding to the Feynman propagator, we can get the non-vanishing imaginary part of the action. When incorporating self-gravitational interaction into the charged black hole, not only should the energy conservation but also the charge conservation be taken into account. As Ren and Zhao mentioned,[38,39] since the fact that particle tunneling through a barrier is an instantaneous process, we should replace $r_{\mathrm{H}}(M,Q)$ with $r'_{\mathrm{H}}(M-\omega,Q-q)$, M with $M-\omega$, and Q with $Q-q$ when a particle with mass ω and charge q tunnels from the event horizon. Besides, as Medved and Vagenas mentioned,[40] because of the quantum uncertainty principle, it is unnatural to expect that the black hole mass and charge can jump, from M to $M-\omega$, Q to $Q-q$, in such a discontinuous manner. Furthermore, quantum blurring will require a 'gradual' transition (relating to whatever time scale is characteristic of the radiation process). So it is much more accurate to calculate with $M\to M-\omega$ and $Q\to Q-q$, and then suitably integrate over ω and q. So we rewrite the imaginary part of the action in a more concise form as follows:

$$\begin{aligned}\mathrm{Im}\, I' &= \mathrm{Im} \int_{(0,0)}^{(\omega,q)} I|_{\substack{M\to M-\omega \\ Q\to Q-q}} \left(\mathrm{d}\omega' - \frac{Q-q'}{r'_{\mathrm{H}}}\mathrm{d}q'\right) \\ &= -\mathrm{Im} \int_{(M,Q)}^{(M-\omega,Q-q)} \frac{\mathrm{i}\,\pi}{F'(r'_{\mathrm{H}},v)} \left(\mathrm{d}(M-\omega') - \frac{Q-q'}{r'_{\mathrm{H}}}\mathrm{d}(Q-q')\right). \end{aligned} \tag{20}$$

The entropy of the black hole before emission is $S_{\mathrm{BH}}(M,Q) = \pi r_{\mathrm{H}}^2$, then the entropy after emission becomes $S_{\mathrm{BH}}(M-\omega', Q-q') = \pi r'^2_{\mathrm{H}}$, thus $\mathrm{d}S_{\mathrm{BH}}(M-\omega', Q-q') = 2\pi r'_{\mathrm{H}}\mathrm{d}r'_{\mathrm{H}}$. From the event horizon $F(r'_{\mathrm{H}}, v) = 0$, we can derive the following expression:

$$\mathrm{d}(M-\omega') - \frac{Q-q'}{r'_{\mathrm{H}}}\mathrm{d}(Q-q') = r'_{\mathrm{H}}\left(\frac{M-\omega'}{r'^2_{\mathrm{H}}} - \frac{1}{3}\lambda r'_{\mathrm{H}} - \frac{(Q-q')^2}{r'^3_{\mathrm{H}}}\right)\mathrm{d}r'_{\mathrm{H}} - r'_{\mathrm{H}}\mathrm{d}\dot{r}'_{\mathrm{H}}. \tag{21}$$

Thus, we can get

$$\begin{aligned}\mathrm{Im}\, I' &= -\frac{1}{2}\int_{(M,Q)}^{(M-\omega,Q-q)} 2\pi r'_{\mathrm{H}}\mathrm{d}r'_{\mathrm{H}} - \frac{2\pi r'_{\mathrm{H}}}{(M-\omega')/r'^2_{\mathrm{H}} - \lambda r'_{\mathrm{H}}/3 - (Q-q')^2/r'^3_{\mathrm{H}}}\mathrm{d}\dot{r}'_{\mathrm{H}} \\ &= -\frac{1}{2}\Delta S_{\mathrm{BH}} + \int_{(M,Q)}^{(M-\omega,Q-q)} \frac{S_{\mathrm{BH}}(M-\omega', Q-q')}{(M-\omega')/r'^2_{\mathrm{H}} - \lambda r'_{\mathrm{H}}/3 - (Q-q')^2/r'^3_{\mathrm{H}}}\mathrm{d}\dot{r}'_{\mathrm{H}}, \end{aligned} \tag{22}$$

where $\Delta S_{\mathrm{BH}} = S_{\mathrm{BH}}(M-\omega, Q-q) - S_{\mathrm{BH}}(M,Q)$ is the change of Bekenstein–Hawking entropy before and after the particle tunnels out of the event horizon. The tunneling rate of the black hole can then be expressed as

$$\begin{aligned}\Gamma &= \frac{P(\mathrm{emission})}{P(\mathrm{absorption})} = \exp(-2\mathrm{Im}\, I') \\ &= \exp\left(\Delta S_{\mathrm{BH}} - 2\int_{(M,Q)}^{(M-\omega,Q-q)} \frac{S_{\mathrm{BH}}(M-\omega', Q-q')}{(M-\omega')/r'^2_{\mathrm{H}} - \lambda r'_{\mathrm{H}}/3 - (Q-q')^2/r'^3_{\mathrm{H}}}\mathrm{d}\dot{r}'_{\mathrm{H}}\right). \end{aligned} \tag{23}$$

From Eq.(23) we can see that the tunneling probability of the fermion is not only related to the change of the Bekenstein–Hawking entropy, but also to the integral of the changing horizon, which is different from the case of stationary black holes. Because $\dot{r}'_{\mathrm{H}}$ is the function of v for the non-stationary black hole, the integral in Eq.(23) could not be worked out. However, we have presented the fermion tunneling of the non-static black hole in de Sitter space, and we could make the physical meaning clear by the tunneling method. The results reflect the dynamical change of the Vaidya–Bonner de Sitter black hole.

3. Conclusion

In this paper, we have successfully extended Kerner and Mann's fermion tunneling method to the emission of charged fermions from the Vaidya–Bonner de Sitter black hole. For the Vaidya–Bonner de Sitter black hole which is non-stationary, after performing coordinate transformation, we find that there are many good properties for us to discuss the tunneling process, such as the property that the event horizon and the time-like limit surface locate at the same place, and the property that the line element satisfies Landau's coordinate clock synchronization. This is convenient for us in choosing the appropriate matrices γ^μ and viewing the Hawking radiation of fermions. As a result, the true tunneling rate is well recovered by charged fermions tunneling from the black hole.

The result indicates that the tunneling rate of the non-static black hole is related to the integral of the changing horizon as well as the change of Bekenstein–Hawking entropy, which is different from the stationary

cases. It also essentially implies that unitary theory is violated. This is because the non-static black hole is non-stationary and cannot be treated as an isolated system.

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